

Thinking About the Long-Run Economic Costs of AIDS

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Chapter III of Markus Haacker (ed.), 2004, “The Macroeconomics of HIV/AIDS,”
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In his book *Plagues and Peoples*, McNeill (1976) views history as the interplay between an array of parasites and their human hosts—a struggle in which communicable diseases and human responses to them have profound social, economic, and cultural effects. Following the outbreak of the AIDS epidemic in the 1980s, humanity must now contend with a new great plague, the scale and character of which will surely put McNeill's thesis to the test. One vital lesson to be drawn from his account is that any attempt to understand the effects of the AIDS epidemic must take a long-term perspective. That is a salient feature of the approach we adopt here: we will argue that, from modest beginnings, the economic damage caused by AIDS can assume catastrophic proportions over the long run, and thereby threaten the social fabric itself.

The disease is selective, and its individual course is both lengthy and, until the end stages, largely free of symptoms. Yet when they do sicken, its victims are still overwhelmingly young adults or those in the early prime of life, the great majority of them with children to raise and care for. The nature of the long-term threat to social well-being now becomes clear. If parents sicken and die while their children are still young, then the means available with which to raise their children so that they can become productive and capable citizens will be greatly reduced. The affected families' lifetime income will shrink, and with it the means to finance their children's education, whether in the form of school fees or taxes. When a parent dies, moreover, the children lose the love, knowledge, and guidance that complement formal education. AIDS does much more,

therefore, than destroy the existing abilities and capacities—the human capital—embodied in its victims; it also weakens the mechanism through which human capital is formed in the next generation and beyond. These ramifications will take decades to make themselves fully felt: like the course of the disease in individuals, they are long drawn out and insidious. To this it should be added that the incessant reminders of the likelihood of an untimely death can seize both individuals and society with a pessimism that hinders provision for the future.

AIDS, like all causes of premature adult mortality, is also a potentially powerful generator of poverty and inequality. Some children will grow up enjoying the care and resources provided by two parents, others will have to make do with only one, and still others will suffer the loss of both. In societies based on nuclear families, special arrangements are required to support one-parent families, and full orphans must hope for adoption into a loving family as the alternative to an orphanage—or to life on the streets. Determined and compassionate policies can provide material security and education, but hardly love and affection. The ideal extended family, in which all children, natural and adopted alike, are valued and treated identically by the surviving adults, solves both problems of insurance. When premature adult mortality is rising, however, both systems come under increasing strain, as the burden on surviving adults grows. AIDS therefore poses a threat not only to the support of needy nuclear families, but also to the traditional extended family as an institution.

The above account assigns central roles, first, to the formation of human capital through a process in which childrearing and formal education combine to produce the wellspring of long-term economic growth, and, second, to premature adult mortality as the primary threat to that process. The setting is one in which the decision-making unit is the household, nuclear or extended, and premature adult mortality can result in inequality among households. These features are in sharp contrast to much of the existing literature on the long-run macroeconomic effects of AIDS, in which a “beneficial” lower rate of population growth vies in an aggregate world with the diversion of savings away from the formation of physical capital, with a heavy emphasis on steady-state analysis. The natural

choice of framework for our purposes is therefore the overlapping-generations (OLG) model, rather than the usual Solow (1956) model.

The plan of the chapter is as follows. We first motivate our approach by identifying the main economic consequences of AIDS and describing the sheer scale of the wave of premature adult mortality that now threatens to engulf many countries in eastern and southern Africa. A detailed treatment of the OLG model, which incorporates a wide array of these effects, is set out in the next section, followed by an analysis of the outbreak of the epidemic and its effects. We then examine the policy problem, namely, how to find the right combination of interventions to preserve economic growth in the face of the epidemic. In the penultimate section we discuss some of the economic effects of the epidemic that the earlier sections mostly neglected and show how they, too, can be incorporated into that framework. We draw together our conclusions in the final section.

[A]THE APPROACH AND ITS MOTIVATION

We begin by listing what are arguably the primary effects of morbidity and mortality in the age groups that AIDS typically strikes, namely, young and prime-aged adults. It is useful to order such effects, whether due to AIDS or to other causes, in the following way:

- (a) Morbidity reduces productivity on the job or results in outright absenteeism. If the worker dies, his or her skills and experience are destroyed.
- (b) Firms and the government lose trained workers on both counts and must replace them. In particular, many teachers die prematurely of AIDS.
- (c) Substantial expenditure, public and private alike, may be required to treat and care for those who become sick. (This is certainly so in the case of AIDS.)
- (d) Savings are also diverted out of net investment in physical and human capital into the treatment and replacement of workers who fall sick and die.
- (e) Lifetime family income is greatly reduced, and with it the family's means to invest.

- (f) Children lose the love, care, guidance, and knowledge of one or both parents, which plausibly weakens the transmission of knowledge and capacity from generation to generation.
- (g) The tax base shrinks.
- (h) Collateralization in credit markets becomes more difficult, and as a consequence credit markets function less well.
- (i) Social cohesion and social capital decline.

Most of the earlier work on the macroeconomic effects of AIDS focused on the first two effects, which are concerned with disruptions to the production process. These contributions have been based on variants of the Solow (1956) model, in which the level of productivity in the long run depends on thrift and the rate of population growth. In this framework a general increase in mortality, with unchanged fertility and thrift, will reduce the pressure of population on existing land and physical capital, and so increase productivity in both the short and the long run. When applied empirically to countries heavily afflicted by AIDS, the model yields predictable results, namely, that the epidemic tends to reduce the aggregate rate of growth—the estimates range from -0.3 to -1.5 percentage points a year—but to *increase* the rate of growth of GDP per capita.¹ The latter finding has driven some authors to tinker with other elements of the model, commonly in the form of diverting savings from the formation of physical capital into expenditure on health—effects (c) and (d) above—and of lowering the productivity of infected individuals in an attempt to overturn it; but these exertions often bring about only modest “corrections.”

This chapter will argue that a different sort of framework, with a wholly different emphasis, is needed. The center stage is given over to the formation of human capital as the main wellspring of economic growth, in which the transmission of capacities and knowledge across generations within nuclear or extended family structures plays a vital

¹ See for example, Cuddington (1993) and Over (1992). Multisector models of this genre are to be found in Kambou, Devarajan, and Over (1992) and Arndt and Lewis (2000).

role. Effects (e) through (g) weaken the mechanism through which human capital is accumulated, by depriving the victims' children of parental upbringing and, very likely, of as much education as they would otherwise have enjoyed. Expectations concerning the future level of premature adult mortality come into play here, because it affects the expected returns to investment in human capital. To the extent that the education both of children in general and of needy children in particular is supported by public expenditure, and that medical treatment and survivors' pensions are publicly provided, a reduction in tax revenue aggravates the problem. Completing the list, poorly functioning capital markets also hinder economic growth, as does a lack of social cohesion and social capital understood in the broad sense, for both form part of the larger structure within which transactions are made. Effects (h) and (i) therefore intensify those of (e) through (g).

The magnitudes of all the effects listed above clearly depend heavily on the levels of morbidity and premature adult mortality. The first step in any attempt to assess the economic effects of AIDS, therefore, is to establish the scale of such mortality before and after the outbreak of the epidemic. Mortality was already high among all age groups in sub-Saharan Africa in the 1950s and 1960s. It then began to fall, especially among infants and young children, so that by the middle of the 1980s great improvements in life expectancy at birth and substantial improvements at prime ages had been achieved. In most countries, however, premature adult mortality was still significant when the AIDS epidemic began to take its toll. Its impact on the profile of mortality is reduced by what some demographers call the substitution effect: some of those who contract AIDS would have died prematurely of other causes. The higher the level of preexisting mortality, the larger this effect will be. By studying countries such as South Africa and Zimbabwe, where premature adult mortality was comparatively low in the 1980s but is now very high, we should therefore be able to get a good idea of the disease's net effect on such mortality, and hence of the gravity of the threat it poses to economic progress.

It is common practice among demographers who deal with AIDS to define premature adult mortality as the probability of dying before the age of 60, conditional on surviving to the age of 15 (Feeney, 2001); this probability is denoted by ${}_{45}q_{15}$, that is, the probability of dying within 45 years, starting at age 15. This is evidently unsuitable for

purposes of studying the effects of mortality on childrearing; more natural choices are ${}_{20}q_{20}$ and ${}_{30}q_{20}$. The former better fits the cycle of childrearing, because new parents are typically between the ages of 20 and 40; the latter captures the substantial mortality due to AIDS among those, especially men, in their forties, when adults are still very much in their productive years. Yet whatever the measure adopted, premature adult mortality has risen dramatically in both South Africa and Zimbabwe following the full-scale outbreak of the epidemic (Table 1). The levels for 2010 forecast by Dorrington and others (2001) are grim; those of the U.S. Census Bureau, as reported by Dorrington and others, are grimmer still.

[Table 1 about here]

It should be emphasized that these measures refer to steady states, in the sense that the q for each year are calculated on the basis of the continuation of that mortality profile. Note, however, that the HIV/AIDS prevalence rate among adults in the age group 15-49 in South Africa rose from about 1 percent in 1990 to about 20 percent in 2000. In the latter year the rate in Zimbabwe had reached 25 percent. According to Dorrington and others (2001), the rate in South Africa will peak in about 2006. The above estimates of q for 2010 can therefore be thought of as the values that will prevail if the disease establishes itself in the population at that level for good. The observed values for 1990 (1986 for Zimbabwe) correspond to the counterfactual case in which there is no epidemic at all.

That this developing wave of morbidity and mortality will considerably slow or even reverse the growth in the numbers of those of working age in the decades to come is clear. Yet this effect on the labor force fails by a wide margin to convey the force of the prospective effects on families with children, and hence on the formation of human capital through the mechanism discussed above. It is one thing when 11 percent of 20-year-old males, and 4 percent of 20-year-old females, will fail to see their fortieth birthdays (the case in South Africa in 1990); it is a problem of an altogether different kind when those proportions are 36 and 54 percent, yet this is the appalling prospect confronting South Africa in the coming two decades. Seen in this light, the loss of

workers in their most productive years signals only the beginning of the damage the epidemic will eventually wreak upon the economy and society.

[A]AN OVERLAPPING-GENERATIONS FRAMEWORK

The structure that follows draws upon and in some ways extends that in Bell, Devarajan, and Gersbach (2003). For simplicity we confine the exposition to the case of two overlapping generations, each going through two periods of life, which we call childhood and adulthood.² Rather than provide an exhaustive technical description of the model, here we seek to provide an accessible discussion of the factors listed in the previous section and how they relate to one another. Even so, a precise account demands a certain degree of formality. Since some readers may find the flavor of the main exposition unduly formal, we begin with a narrative sketch of the main idea.

The basic economic unit is the family, nuclear or extended, in which the adult members decide how current resources available to the family are to be allocated between consumption and the children's education. The level of those resources is heavily determined by the parents' human capital and their survival rate through their offspring's early childhood and school years, but the children themselves can also contribute labor instead of attending school. How much of childhood, if any, is spent at school depends not only on the family's available resources, but also on three other factors. The first is the strength of the parents' altruism, which expresses itself in their willingness to forgo some current consumption in favor of investment in their children's schooling, and hence in favor of the children's human capital when they attain adulthood in the next period. Second is the efficiency with which schooling is transformed into human capital, and this arguably depends on the quality both of the school system and of childrearing within the family; the latter is expected to improve with the parents' own human capital (if indeed they survive this phase of life). Third, the returns to the investment in any child will be

² An extensive literature analyzes the main differences and equivalences between models populated by immortal representative agents and those by mortal overlapping generations in terms of determinacy, Pareto optimality, and existence and dynamics in general, which is relevant for the type of model in this paper; see Aiyagari (1987, 1992), Huo (1987), Woodford (1986), and Lovo (2000).

effectively destroyed if that child dies prematurely in adulthood. This implies that the *expected* returns to education depend on parents' (subjective) assessment of the probability that their children will meet an untimely death. At the end of each generation, or "period" in the model, the surviving adults die in old age, and the story continues as the children become adults in their turn.

Dynamic elements are at work in this story through several channels. The levels of human capital and premature adult mortality in the present generation play a key role in determining the level of human capital attained by the next generation, as do present expectations concerning premature adult mortality in the future. Dynamic systems of this kind can exhibit multiple equilibria and regimes of behavior. If current levels of human capital are low, families may be so poor that parents are not prepared to make the investments in schooling that would raise their children out of poverty upon reaching adulthood. In this case poverty can perpetuate itself. At much higher levels of human capital, in contrast, affluence will yield a fine upbringing and education, and so beget still more affluence. High levels of premature adult mortality can give rise to a poverty trap, with an increasing share of the population mired in poverty and facing little or no prospect of upward mobility, and may dampen the accumulation of human capital even under conditions of affluence. The outbreak of an epidemic may even pitch what was a growing system into the widening jaws of a poverty trap.

[b]The Basic Model

The place to start is with the formation of families and of human capital. For the present, let the family structure be nuclear. At the beginning of each period (generation) t , every young adult chooses a partner with the same level of human capital—that is to say, there is assortative mating. Each cohort begins with equal numbers of young men and women, and all find a partner. All couples have their children soon afterward. With AIDS very much in mind, premature adult mortality is assumed to occur about a decade into full adulthood, this being the median time from infection to death in the absence of treatment with antiretroviral drugs. Thus, when the children have just started school, some parents

sicken and then die, leaving their children as half or full orphans. At this stage, therefore, the family will find itself in one of the following four states:

- both parents survive into old age ($s_t = 1$),
- the father has died ($s_t = 2$),
- the mother has died ($s_t = 3$),
- both parents have died ($s_t = 4$).

Let $\Lambda_t(s_t)$ denote the surviving adults' total human capital when the family is in state s_t , so that

$$\Lambda_t(1) = 2\lambda_t, \Lambda_t(2) = \Lambda_t(3) = \lambda_t, \Lambda_t(4) = 0,$$

where λ_t denotes the level of human capital possessed by each parent. Let the probability that a family formed at the beginning of period t winds up in state s_t be denoted by $\pi(s_t)$. Given the assumption that each cohort begins adulthood with equal numbers of males and females, the proportion of adults surviving into old age is given by

$$\kappa_t = [2\pi(1) + \pi(2) + \pi(3)]/2 = [1 + \pi(1) - \pi(4)]/2. \quad (1)$$

The relationship among between $\pi(s_t)$, κ_t , and the mortality statistic q is deferred to a later section.

Apart from innate ability, the two main factors that influence the level of human capital a young adult attains are the quality of childrearing and formal education. The former involves not only care and a loving upbringing, but also the transfer of knowledge. As a rule, it is surely both increasing in the parents' human capital and complementary with formal education. Let the latter be represented by the fraction of childhood, $e_t \in [0, 1]$, spent in school, where this phase of childhood may be thought of as spanning the period from 6 to 18 years of age. Then the process whereby these factors yield human capital in adulthood in period $t + 1$ for a child born in period t can be represented by

$$\lambda_{t+1} = z(s_t)g(e_t)\Lambda_t(s_t) + 1, \text{ with } s_t = 1, 2, 3, 4. \quad (2)$$

The term $z(s_t)$ may be thought of as a transmission factor, in the sense that its magnitude expresses the strength with which the parents' human capital creates in their children a potential capacity to acquire human capital themselves. If, as is plausible, fathers and mothers are not perfect substitutes for one another, then having both parents will be better than having only one: formally, $z(1) > \max[z(2)/2, z(3)/2]$. It is plausible, too, that z depends on the number of children within the family, but we defer discussion of this point for the moment. The function $g(e_t)$ represents the educational technology, where g is increasing in e_t . That $g(e_t)$ and $\Lambda_t(s_t)$ enter into equation (2) multiplicatively expresses the complementarity between the quality of childrearing and formal education: the stronger the transmission factor and the greater the parents' human capital, the more productive of human capital is any given level of the child's schooling. Now suppose further that some formal education is needed if a child is to realize at least some of the potential $z(s_t) \cdot \Lambda_t(s_t)$ created by childrearing: formally, $g(0) = 0$. It then follows that any child deprived of all formal schooling will attain $\lambda = 1$ as an adult, whatever the parents' level of human capital, whereby the value $\lambda = 1$ is simply a convenient normalization.

As it stands, the difference equation (2) is a purely "technical" relationship, in the sense that it yields the resulting formation of human capital for any given level of education but says nothing about how that level is chosen. This difference equation is also a stochastic one, in that the child of a union formed in period t with human capital $2\lambda_t$ can attain any of four arguably different levels of human capital as an adult one generation later, depending on the incidence of premature mortality among the child's parents in period t . Given the state s_t , and hence $\Lambda_t(s_t)$, the choice of e_t determines the outcome for the child, in the form of the level of λ_{t+1} .

How, then, is e_t chosen? One possibility is that school attendance is rigorously enforced by the authorities, and that full orphans are taken into first-rate care. Remedial measures might also be needed to offset the disadvantages suffered by half orphans. Such a policy would ensure the continued formation of human capital in society at large, while holding inequality within tolerable bounds. Yet the chances of actually implementing it in most poor countries are remote, to say nothing of the financial demands on the national treasury. In view of these difficulties, it seems much more compelling to treat e_t as the

parents' decision, which they make in light of the resources available to them and the expected returns to education.

We start with output and income. As in the Solow model, there is an aggregate consumption good, which is taken to be the numéraire, and there are constant returns to scale in its production, but the only input is labor, which is measured in efficiency units. In this setting it is natural to define an adult's endowment of labor so measured as λ_t , which he or she supplies completely inelastically. Children will be less productive workers than their parents, and, given the reasoning underlying equation (2), it seems plausible to assume that a child could supply at most $\gamma (< 1)$ efficiency units of labor to production. A family with N_t children then has the following level of full income, measured in units of the aggregate consumption good, in state s_t :

$$\Omega_t(\Lambda_t, N_t, s_t) \equiv \alpha[\Lambda_t(s_t) + N_t\gamma], \quad \text{with } s_t = 1, 2, 3, 4, \quad (3)$$

where the positive scalar α denotes the productivity of human capital, measured in units of the numéraire, and the expression in brackets is the total labor, measured in efficiency units, that the household can supply. It is seen at once that we are employing what is known in the literature on economic growth as an AK model. The sole means of production is human capital, which is itself produced through a process involving childrearing and formal education, and each unit of input of human capital in the production process yields α units of output.

The allocation of full income among competing uses lies in the parents' hands, so long as at least one of them survives into old age. We rule out bequests at death, so that full income is spent on the consumption good and the children's education. For simplicity, let the adults behave as equal partners, and let each child receive the fraction $\beta \in (0, 1)$ of a surviving adult's consumption. Since children of school age can also work, let this be the alternative to attending school, and let all siblings be treated in the same way. For the present, let the only costs of schooling be the opportunity cost of the children's time. Then, in the absence of taxes or subsidies, the household's budget line in the "goods" $c_t(s_t)$ and $e_t(s_t)$ may be written as

$$[(3 - s_t) + \beta N_t]c_t(s_t) + \alpha \gamma N_t e_t(s_t) = \Omega_t(s_t), \quad \text{with } s_t = 1, 2 \quad (4)$$

where $c_t(s_t)$ denotes the level of each adult's consumption. The assumption of assortative mating implies that states 2 and 3 are identical in this regard, so that equation (4) implicitly covers state 3. Observe that, given λ_t and N_t , single-parent families have less full income and face a higher relative price of education than do two-parent families. Full orphans ($s_t = 4$) are left to fend for themselves: they do not attend school and they consume whatever income they earn as child laborers.

To complete this account of the family's decision problem, we must specify its preferences. Let mothers and fathers have identical preferences over their consumption of the aggregate good and their children's welfare as adults, the level of which they can influence by choosing the level of schooling $e_t(s_t)$.³ It is clear from equation (4) that they will maximize their own consumption by using the children as full-time workers, so that one can say that their altruism toward their children is operative only when they choose $e_t(s_t) > 0$. When both parents survive, let there be no "joint" aspect of the bundle $(c_t(1), e_t(1))$: each adult enjoys $c_t(1)$ as a private good, whereas the children's resulting level of human capital as adults, λ_{t+1} , as given by equation (2), is a public good within the marriage. Since all their children will attain that value of λ_{t+1} , the only uncertainty that arises concerns the number of children who will die prematurely as adults in period $t + 1$, each such death being regarded as a "wasted" investment. To be exact, we assume that parents in period t form expectations about the premature mortality that will afflict their children as adults in period $t + 1$ and that they take the average number of survivors in weighting the payoff to schooling in the form of λ_{t+1} .

The appendix sets out a formal statement of the household's decision problem. Let $((c_t^0(s_t), e_t^0(s_t)))$ denote the household's optimum bundle of current consumption and

³ This implicitly assumes that adults are concerned directly only with their own consumption and the size of their children's budget sets, whereby the grandchildren's opportunities appear in the latter, and so on. Thus all generations are effectively connected. For further discussion of this formulation in the context of this OLG structure, see Bell and Gersbach (2003).

schooling. It can be shown that, under weak assumptions, $e_t^0(s_t)$ is increasing in λ_t whenever $0 < e_t^0(s_t) < 1$. For any given value of λ_t , (i) children in two-parent families receive at least as much schooling as those in single-parent families, and strictly more if the latter choose some, but not full, schooling; (ii) children in two-parent families attain, as adults, at least as much human capital as those in one-parent families, and strictly more if fathers and mothers are not perfect substitutes in childrearing; and (iii) an increase in expected premature mortality in period $t + 1$ will reduce schooling in period t if $0 < e_t^0(s_t) < 1$, and may do so if $e_t^0(s_t) = 1$ ($s_t = 1, 2, 3$). All these results accord with elementary intuition. To complete matters, we introduce a further, plausible assumption, namely, that uneducated couples ($\Lambda_t = 2$) are so poor that, in the absence of compulsory education, they choose not to educate their children; that is, $e_t^0(s_t) = 0$, even if neither dies prematurely.

[b]Dynamics

The next step is to investigate the system's dynamic behavior. By replacing the nonspecific e_t in equation (2) with $e_t^0(s_t)$, we obtain the system's equation of motion, as governed by the rational, forward-looking behavior of individual households in the technical and mortality environment in which they find themselves:

$$\lambda_{t+1} = z(s_t)g\left(e_t^0(\Lambda_t(s_t), s_t, \kappa_{t+1})\right)\Lambda_t(s_t) + 1, \quad \text{with } s_t = 1, 2, 3, 4, \quad (5)$$

where κ_{t+1} is a measure of the children's survival chances after they have reached adulthood in period $t + 1$, as assessed by their parents in period t . Like equation (2), equation (5) is a stochastic difference equation, a full treatment of which would go well beyond the scope of this chapter. Note that there are 16 possible cases: a child in any of the four family states in period t may wind up, as an adult, in any of the same four states in period $t + 1$. What follows, therefore, is an intuitive sketch of the main idea.

Full orphans ($s_t = 4$) can be dealt with at once. In the absence of support, they do not attend school, and each will marry another uneducated individual. In the absence of support or compulsion, the offspring of these unions will also go uneducated, and so on. Observe that any premature adult mortality will produce a new crop of orphaned children

in each period and that these lineages will fall into poverty and illiteracy, even if they were not in that condition before. Hence, as time progresses, a steadily increasing proportion of the whole population finds itself in poverty. Caring for orphans is not, of course, a new problem for humankind, and societies have devised various ways of dealing with it. Whether these arrangements can withstand the burden of an epidemic like AIDS, however, remains to be seen. We return to this question below.

At the other extreme, consider children who have the good fortune not only to see both parents survive into old age, but also to experience the same outcome themselves in adulthood (this is the case where $s_t = s_{t+1} = 1$). If premature adult mortality is not too high, this is the typical case, the essentials of which are captured in the phase diagram in Figure 1. Let $\Lambda^d(1) (> 2)$ be the parents' endowment of human capital such that, for all larger values, their children will receive some schooling, but otherwise none. Observe that $\Lambda^d(1)$ is determined by the parents' altruism, family income, and the expected level of premature adult mortality in the next generation. Similarly, let $\Lambda^a(1) (= 2\lambda^a)$ be the smallest value of the parents' human capital such that their children will receive complete schooling. As the value of $\Lambda_t(1)$ rises from 2 to $\Lambda^d(1)$, the children will remain wholly uneducated. As it rises further, from $\Lambda^d(1)$ to $\Lambda^a(1)$, increasing affluence will cause $e_t^0(1)$ to rise from zero to unity, so that, from equation (5), λ_{t+1} will increase from unity to $[z(1)g(1)\Lambda^a(1) + 1]$. Suppose further that $[z(1)g(1)\Lambda^a(1) + 1] > \Lambda^a(1)/2$, that is, that every child of couples with $\Lambda^a(1)$ attains a level higher than $\Lambda^a(1)/2$, the human capital of each such parent. Now consider the graph of $\Lambda_{t+1}(1) (= 2\lambda_{t+1})$ and $\Lambda_t(1)$. For all $\Lambda_t(1) \in [2, \Lambda^d(1)]$, the children will not attend school, with the outcome $\Lambda_{t+1}(1) = 2$. For all $\Lambda_t(1) \in [\Lambda^d(1), \Lambda^a(1)]$, $\Lambda_{t+1}(1)$ is increasing in $\Lambda_t(1)$, and its graph cuts the 45-degree line through the origin at least once, by virtue of the assumption that $[z(1)g(1)\Lambda^a(1) + 1] > \Lambda^a(1)/2$. Suppose it does so just once, at $\Lambda_t(1) = \Lambda^*(1)$.⁴ Then the system possesses two stationary equilibria: $\Lambda_t(1) = 2$ and $\Lambda_t(1) = \Lambda^*(1)$. It is clear from Figure 1 that the former is stable, that the latter is unstable, and that the system exhibits a poverty trap. We show in the appendix that when the transmission factor and educational technology

combine to ensure that the condition $2z(1)g(1) > 1$ holds, long-term growth is, in principle at least, possible. In what follows we assume that this condition indeed holds, and Figure 1 is drawn accordingly.

[Figure 1 about here]

Children in single-parent families ($s_t = 2, 3$) face a less favorable situation. All of $\Lambda^d(s_t)$, $\Lambda^a(s_t)$, and $\Lambda^*(s_t)$ will be larger than their counterparts $\Lambda^d(1)$, $\Lambda^a(1)$ and $\Lambda^*(1)$, respectively; in other words, the size of the trapdoor into poverty will be correspondingly larger. The long-term rate of growth will also be lower if fathers and mothers are not perfect substitutes in childrearing, for if they are not, then $2z(1) > \max[z(2), z(3)]$. Indeed, unbounded growth may not be possible at all for one or both of these subgroups, even when it is so for two-parent groups. Summing up, premature adult mortality in a nuclear family setting is a powerful force for inequality in the future as well as in the present.

[b]The Social Response to Premature Adult Mortality

The death of parents while their children are still young poses such grave problems in strictly nuclear families that societies have been driven to find solutions to them. In Africa the widespread practice of fostering and adoption within the circle of kinship is such a response. In effect, this is a collective, or pooling, arrangement to deal with the individual risks of premature adult mortality as they affect the rearing of children and hence the future well-being of the group or society. Since any such pooling arrangement also needs rules, one can suppose that one such rule is that all children must be treated alike. We now analyze such an arrangement in a starkly simplified form.

Let there be complete pooling, in the sense that all the surviving adults take on responsibility for all children. This family structure, or “state,” will be denoted by $s_t = 0$. The rule that all children be treated the same then ensures that there will be no inequality within each generation. The proportion of all adults surviving into old age in period t is denoted by κ_t . For simplicity, let premature mortality afflict men and women equally, so that each surviving “couple” will raise, not N_t children, but

⁴ A sufficient condition for this to hold is that the graph be convex over the specified interval. For a full discussion see

(continued...)

$$N_t(0) = N_t/\kappa_t. \quad (6)$$

The couple's budget constraint is

$$[2 + \beta N_t / \kappa_t]c_t(0) + \alpha\gamma(N_t / \kappa_t)e_t(0) = \alpha[2\lambda_t + (N_t / \kappa_t)\gamma] \equiv \Omega_t(0). \quad (7)$$

A comparison of equation (7) with equation (4) reveals that, relative to an otherwise identical two-parent nuclear family, the burden of pooling implies, first, a lower relative price of current consumption and, second, a lower level of full income, measured in units of an adult's consumption, provided $\beta > \gamma$. Both effects work in the direction of reducing schooling relative to what would occur in the two-parent nuclear family.

Turning to preferences, let the couple go beyond the requirements of the social rule of equal treatment and regard all the children in their care, natural and adopted alike, with the same degree of altruism. Altruism in this degree therefore furthers investment in education by increasing the weight attached to the payoff in the form λ_{t+1} , since $N_t(0) > N_t$. Whether altruism or even equal treatment prevails in practice will be taken up in later in the chapter.

Given the burden of rearing $N_t(0)$ as opposed to N_t children, it is natural to ask whether the transmission factor will not be correspondingly weakened. To allow for this possibility, we write the latter as $z(0, \kappa_t)$, where it is plausible that $z(0, \kappa_t)$ is increasing in κ_t . Note also that, in the absence of premature adult mortality, pooling will be superfluous and all children will be raised by their natural parents, so that $z(0, 1) = z(1)$. The fundamental difference equation now takes the form

$$\lambda_{t+1}(0) = z(0, \kappa_t)g\left(e_t^0(\Lambda_t(0), 0, \kappa_t, \kappa_{t+1})\right)\Lambda_t(0) + 1, \quad (8)$$

where it should be noted that the current level of premature adult mortality influences both investment in schooling and the transmission factor.

The dynamics are comparatively simple, in the sense that there is only one family state in all periods—so long as the institution of pooling can bear the weight of mortality among young adults. For any given κ_t , there is effectively a two-parent family with $N_t(0)$ children and transmission factor $z(0, \kappa_t)$, and Figure 1 may be used once more. Whether the trapdoor into poverty is larger than its counterpart in the corresponding nuclear family setting depends on whether the effects of pooling on the family's budget line and the transmission factor outweigh those of altruism. In any event the long-term rate of growth will be lower, since $z(0, \kappa_t) < z(1)$ whenever $\kappa_t < 1$. When all the single-parent households and full orphans are brought into the reckoning, however, the average growth rate of a society of nuclear families may well be smaller than the growth rate under pooling, namely, $[2z(0, \kappa_t)g(1) - 1]$.⁵

[b]Calibration

In order to apply the model, certain qualitative choices must be made and a whole variety of parameters estimated. How all this is to be done depends, naturally enough, on the data and resources available. One thing, however, is immediately clear: the system is almost certainly heavily underidentified, so that a resort to calibration is unavoidable. That is, the functional forms are chosen exogenously, and the values of the parameters are then derived by solving the system in an exactly identified form so that it reproduces some set of past configurations. There is no cookbook recipe for this procedure. What follows is a brief description of how we accomplished this task for South Africa, with the aim of conveying the nature of the difficulties we encountered and how we attempted to overcome them.⁶

The phenomena of interest are inherently of a time-series nature, the available data for which are overwhelmingly of the aggregate kind: GDP, labor force, average

⁵ Note that, with a representative family unit of this kind, the OLG model is equivalent to a Solow model in which output is proportional to inputs of efficiency units of labor. The difference is that whereas the OLG model provides the structure that yields the evolution of human capital, the latter must be supplied exogenously to the Solow equivalent, which is then, in effect, the fifth wheel of the coach.

⁶ Readers interested in the detailed derivations are directed to Bell, Devarajan, and Gersbach (2003).

years of schooling completed, total fertility rate, and so forth. This simple fact makes an approach based on a representative unit virtually unavoidable, so that the choice of institutional arrangement falls on what we have called pooling, with all the advantages of simplification it brings. It must be conceded that, especially in light of South Africa's history of *apartheid*, the assumption of pooling is also nothing short of heroic. Indeed, the calibration is based on the period 1960-90, at whose close *apartheid* was about to collapse and the epidemic was in its very early stages.

With this much decided, we can work with various aggregates. At any point in time, total output (GDP) is defined as the product of the unknown parameter α and the total input of labor, measured in efficiency units. If we ignore open unemployment and child labor, the total input of efficiency units of labor is the product of the labor force, which we can observe, and the average value of λ_t , which we need to estimate. Since a (quinquennial) series for average years of schooling completed among the adult population is available for 1960-90 (Barro and Lee, 1996), the next step is to employ the difference equation (2) with $s_t = 0$ as a purely technical relationship. To save on degrees of freedom, two further assumptions are now needed: first, that the function $z(0, \kappa_t)$ is a constant throughout this period, and second, that the educational technology, as defined by the function $g(e_t)$, takes the very simple form $g(e_t) = e_t$. Repeated application of equation (2), with appropriate rescaling to allow for the fact that, when but two overlap, a single generation spans 30 years and that some of the data are annual and others quinquennial, permits us, at length, to coax out the following estimates: $\alpha = 3,419$, $z = 0.818$, and $\lambda_{1960} = 2.62$.

These estimates yield the following implications. First, a wholly uneducated adult, who by definition possesses 1 efficiency unit of labor ($\lambda = 1$), will produce \$3,419 a year (in 1995 dollars). Second, the starting value of human capital (λ_{1960}) is not only much larger than this reference value, but also placed South Africa well out of reach of the poverty trap that existed at the levels of premature adult mortality prevailing at that time. Third, the value of z is such that, when all children receive a full education, output per capita will eventually grow at the rate of $(2 \times 0.818) - 1 = 0.636$ per generation, or about

1.64 percent annually. Although this rate is hardly in the East Asian league, it does promise considerable prosperity within two to three generations.

The remaining steps involve treating the Barro and Lee series for average years of schooling as the outcome of rational decisions made by an extended family, as set out above and in the appendix. That is to say, a representative couple in 1960 chose e_{60} such that individuals born between 1935 and 1965 attained, on average, the Barro-Lee estimate of five years of schooling for adults aged 25 years and older in 1990. The couple's preferences over current consumption and the level of human capital attained by each of their children in adulthood are assumed to be additively separable in logarithms—a formulation in keeping with common practice throughout the macroeconomics literature. The parameters β and γ are set exogenously, the former at around one-half, the latter at one-fifth of the value of α . The total fertility rate N_t is taken from World Development Indicators (World Bank, 2002), and values for κ_t are derived from a procedure that we describe in the next section in connection with the outbreak of the epidemic. With the “taste” and mortality parameters thus determined, the family's choice $e_t^0(\Lambda_t(0), 0, \kappa_t, \kappa_{t+1})$ can be derived in every given situation.

[A] THE OUTBREAK OF AN EPIDEMIC

What happens when the outbreak of a hitherto unknown disease brings about a dramatic rise in premature adult mortality, albeit with a lag of a decade or so? At first, very little, because those infected show no symptoms. As time wears on, however, they begin to sicken and die, and the survivors begin to revise their assessments of the chances that their children will die prematurely on reaching adulthood. The first wave of deaths leaves behind orphans on a scale not seen in earlier generations.

A careful distinction between nuclear and collective family structures is needed. Given the nature of AIDS, infection of one partner in a marriage is likely to be followed by infection of the other, so that the proportion of full orphans in the child population will rise dramatically as well. Yet the effects of the adverse shift in the proportions of nuclear families falling into the four states will make themselves fully felt only in the next

generation and beyond, in the form of lower levels of human capital averaged over the population as a whole. Under pooling, the fall in κ_t has both an immediate, adverse effect on the common budget set, by increasing $N_t(0)$, and a damaging long-term effect on the accumulation of human capital.

If the disease persists—or, rather, if the adults expect it to do so—there will be a further effect in the present, certainly adverse and perhaps devastating. If families are nuclear, a fall in the expected level of κ_{t+1} will cause $\Lambda^d(s_t)$, $\Lambda^a(s_t)$, and $\Lambda^*(s_t)$ to increase for two-parent and single-parent families alike, for it will reduce the expected returns to education. In other words, it will make the trapdoor larger, so that groups that were enjoying sustained growth before the outbreak could slide into poverty. Again, this effect will make itself felt only with a long lag, but if the (expected) increase in mortality is large enough, the whole system could switch regimes from one generation to the next. Under pooling, the sharing of resources and responsibilities will stave off such a collapse if neither κ_t nor the expected level of κ_{t+1} falls too far, even though a permanent fall in κ will tend to reduce the long-run rate of growth by weakening the transmission factor. Otherwise the entire group will slide into poverty together, a disaster that may well undermine the institution's rules and even the institution itself.

We now use the model, as calibrated to South Africa, to investigate the epidemic's effects on human capital formation and economic growth. Although a mere glance at Table 1 reveals that the situation is already grave and threatens to become worse, what we actually need in order to apply the OLG model are the probabilities of each of the four family states, that is, the $\pi(s_t)$, which, through equation (1), also yield the survival statistic κ needed in the pooling case. In order to derive the $\pi(s_t)$ from the mortality measures q , an assumption has to be made about the occurrence of mortality within marital unions. In the absence of AIDS, one could perhaps make a case for treating the premature deaths of spouses as independent events, at least as a working approximation for the population as a whole. Given the nature of the disease, however, it is tempting to assume that the infection of one partner outside the relationship will soon be followed by the infection of the other within it. Viewed in a time frame of 20 or 30 years, single-parent households would become rather rare.

In fact, the probability of transmission within a union appears to be of the order of 10 percent a year under the conditions now prevailing in East Africa (Marseille, Hoffman, and Kahn, 2002). Cumulated over the median course of the disease from infection to death, namely, about a decade, this implies that the probability that both partners will become infected, conditional on one of them getting infected outside the relationship, is about 0.65; this is high, but still far removed from infection being perfectly correlated within a union. Since it would take anywhere from one to two decades for both to die, the chances that all their children will lose both parents before reaching the end of adolescence are correspondingly reduced. This has a strong bearing on deriving the state probabilities using ${}_{20}q_{20}$, which is the natural measure in connection with analyzing the effects of premature adult mortality on the distribution of family types. To err on the side of caution where the numbers of full orphans are concerned, let us therefore assume that infection is indeed an independent event within a union. This yields the state probabilities corresponding to any choice of q as follows:

$$\pi(1) = (1 - q(M)) \cdot (1 - q(F))$$

$$\pi(2) = q(M) \cdot (1 - q(F))$$

$$\pi(3) = (1 - q(M)) \cdot q(F)$$

$$\pi(4) = q(M) \cdot q(F).$$

Various constellations, together with the corresponding values of κ_i , are set out in Table 2.

[Table 2 about here.]

The effects of the AIDS epidemic on families are appalling to contemplate. In its absence, about 86 percent of South African children in nuclear families would have grown up in the happy circumstances of having both parents to care for them, and fewer than 1 percent would have been completely orphaned. In the mature phase of the epidemic, as described by the steady state corresponding to 2010, these proportions will

lie close together, at 29 and 19 percent, respectively, and just over half of all children will be raised by a single parent. The fraction of adults surviving beyond the age of 40 would have been 93 percent; instead, only about 55 percent will do so, which speaks eloquently of the burden already beginning to fall on survivors under pooling arrangements. In Zimbabwe, where the epidemic broke out earlier, the observed shift over the period 1986-97 is scarcely less dramatic, and the epidemic still had not peaked at the end of that period. One should not, of course, make too much of a few percentage points here or there, but the broad qualitative nature of the results is surely robust to any reasonable amendments to the underlying mortality profiles. Viewed in the light of how human capital is formed, these drastic shifts in family state probabilities and adult survival rates contain the very real threat of an economic collapse if the epidemic continues unabated on its present course.

To pursue this possibility, the model is run from 1960 onward in three variants. The first is the counterfactual, in which the AIDS epidemic never occurs and the mortality profile in 1990 continues on into the indefinite future. In this happy event, the South African economy would have enjoyed modest economic growth, with almost universal and complete primary education (of 10 years) attained in one generation, that is, by 2020. Income per family, $y(0)$, which includes the contribution of child labor, if any, would have quadrupled over its 1990 level in three generations (Table 3).

With the outbreak of the AIDS epidemic, and with no interventions to stem it, this salutary path will be interrupted. In the second variant not only do the surviving parents have to cope with a very heavy burden of child dependency from 1990 onward, but they also immediately revise their expectations about future mortality: they correctly forecast current mortality in their generation, as described by the steady-state value of κ in 2010 and denoted formally by κ_{90} , and they expect that level to rule in the next generation, too: formally, $E_{90}(\kappa_{120}) = \kappa_{90}$. With current resources much reduced and the outlook grim, primary education levels begin to decline, and if this scale of mortality continues, the society will be full of uneducated adults in two generations. Economic performance declines accordingly, and income per family, instead of quadrupling by 2080, declines to about half its 1990 value.

It might be argued that the second variant constitutes too harsh an estimate of the conditions prevailing in 1990-2020, and that, in particular, the revision of expectations will take time. Suppose, therefore, that this revision does not occur until the very start of the next generation, when the childhood experience of death among their parents will be vivid in the minds of the next cohort of young adults: their firm expectations are $E_{120}(\kappa_{150}) = \kappa_{90}$. Suppose, further, that these expectations are realized and that this scale of mortality persists into the future. The happy, but false, expectations about future mortality in 1990, coupled with what is assumed to be the rich altruism of full pooling, induce adults to invest heavily in the children's education, despite the heavy reductions in resources caused by the outbreak of the epidemic. Yet although the generation of adults starting out in 2020 are every bit as well endowed with human capital as they would have been in the absence of the epidemic, their expectations concerning their children's future are so bleak as to induce them to roll back investment in schooling to levels not seen since the middle of the 20th century, and the result is to send the entire system into a progressive decline. Income per capita peaks in the period starting in 2020, and two generations later the fresh cohort of adults will be scarcely more productive than their forebears in 1960. Only a revival of optimism about the future—and the resumption of low levels of premature adult mortality to confirm it—will stave off a complete collapse.⁷

[Table 3 about here]

[A]POLICY

The real possibility that the economy can have two strikingly different equilibria—a poverty trap and steady growth—means that there is a job for the government to do, namely, to bring about the latter state by ensuring the reproduction of human capital and its accumulation. Yet although the need for intervention to secure such an outcome is clear enough, the rationale for it must be sharpened if the government is to intervene in the right way. The first reason is paternalistic, for the failure to attain, or maintain, growth may stem from parents' weak altruism. In the formulation of private preferences

⁷ The fact that false expectations can be helpful in overcoming shocks raises some delicate questions concerning the

(continued...)

chosen by Bell, Devarajan, and Gersbach (2003), weak altruism expresses itself in the form of parents assigning a low value to their children's level of human capital in adulthood, which effectively competes with their own current consumption. A closely related consideration is that although parents are not necessarily myopic, inasmuch as they recognize that their children will, in turn, care about their own children, and so on, they may not *directly* value any generation's welfare beyond their children's. All else being equal, the fewer the number of future generations whose well-being today's parents directly value, the weaker are the incentives to invest in education today. Although it is hard to distinguish between these aspects of altruism in a practical calibration, the distinction is still important.

The second reason for intervention is that AIDS, as a communicable disease, involves an externality whose full ramifications can be enormously damaging. The third reason has to do with information: the course of the disease is long and insidious, and in many communities knowledge of how it is transmitted and how to prevent it is often sketchy and sometimes woefully wanting and distorted. A fourth reason is that there is—or should be—a social aversion to inequality, a condition that premature adult mortality does much to promote.

In formulating policies it will be useful to begin by drawing a distinction between preventive and remedial measures, with a firm emphasis on their economic and social consequences. A second, more conventional distinction is among the socioeconomic sectors in which intervention is undertaken: education, health, and support of the needy. We take them up in that order. Once formulated, the spending programs must be financed, and raising the additional revenue will itself have an effect on the accumulation of human capital through the channels identified above. The framework laid out above enables us to treat interventions in a way that reveals their full ramifications in a long-run setting.

Premature adult mortality in the present and expectations concerning its level in the future emerged above as key factors influencing long-term economic performance. The outbreak of a disease like AIDS calls for containment on both scores. The first *economic* preventive measure is, almost tautologically, the vigorous pursuit of public health measures to stem the spread of the epidemic, ideally in the very early stages. (AIDS is, after all, very much a preventable disease.) The second such measure, however, involves medical treatment of the infected, to maintain their productivity and prolong their lives, and so enable them to provide more of the vital things their children need to become productive members of society. Although there is yet no cure for AIDS, treating opportunistic infections and providing antiretroviral therapies still enter into the reckoning, at least potentially. The second class of measures is unquestionably much more expensive than the first, but both directly further the formation of human capital and equality in the next generation.

Premature adult mortality cannot be eliminated, however, and therefore remedial action in the form of supporting the survivors, adults and children alike, is also essential. In a nuclear family setting this will involve making payments to single-parent and foster families, as well as establishing orphanages for those children who would otherwise lack a home. In extended family settings the aim must be to support the institution itself, while seeing to it that all children enjoy equal treatment. Note that all these measures are also preventive, to the extent that they further investment in the children's education and so help them to become productive adults.

It is here that social and educational policy overlap. The case for the government to intervene rests heavily on market failures that result in socially suboptimal levels of schooling, especially when the family structure is nuclear. Received theory tells us that distortions of this kind should be attacked as close to their source as possible, which suggests that the right form of intervention is a subsidy payable to the family contingent on each child's attendance at school. In poor societies this intervention is generally desirable even in the absence of premature adult mortality (Bell and Gersbach, 2003; Siemers, 2002); in the face of the AIDS epidemic, such a sharp instrument is surely of great importance. It is quite possible, of course, that the government is unable to

implement such a policy, in which case it might have to fall back on unconditional transfers to all needy households. But this is an administratively troublesome policy, too.

What, then, is the right balance between preventive and remedial measures? This is a complicated question to which there is no ready, general answer. The need to finance any bundle of policies selected from the above menu also enters into the reckoning, and, as in all questions involving public economics, the means available will exert an influence on the right choice. Foreign aid, in the form of outright grants, will relieve the burden, but no society can expect help on a scale that will make increases in domestic taxation unnecessary. For clarity of exposition, therefore, we go to the other extreme of self-reliance. We treat pooling first, since this is relatively straightforward, and then outline the additional considerations that arise when families are nuclear. The framework presented earlier will enable us to formulate the problem in such a way as to gain qualitative insights. We then follow this discussion with a sketch of how to estimate the relationship between public spending and premature adult mortality for purposes of incorporating it into the framework, and then we offer a brief account of the quantitative results obtained for South Africa.

[b]Optimal Policy Under Pooling

There is neither scope nor need for redistribution among families under pooling, and therefore the government can impose poll taxes to finance its expenditure. Let each “couple” pay the amount τ_t in period t . For the moment let us rule out school attendance subsidies, so that τ_t will be used solely for “economic” preventive measures, that is, effectively to increase the survival statistic κ_t . To emphasize the connections, we write $\kappa_t = \kappa_t(\tau_t)$, and hence $z(0) = z(0, \kappa_t(\tau_t))$, both of which are increasing in τ_t . These are two of the factors yielding benefits, whereby diminishing returns will set in at some point. The drawback is the reduction in full income in the amount τ_t . The budget constraint in equation (7) becomes

$$\left[2 + \beta \frac{N_t}{\kappa_t(\tau_t)}\right] c_t(0) + \frac{\alpha \gamma N_t}{\kappa_t(\tau_t)} e_t(0) = \alpha \left[2\lambda_t + \frac{\gamma N_t}{\kappa_t(\tau_t)}\right] - \tau_t \equiv \Omega_t(0, \tau_t). \quad (9)$$

Thus parents must form expectations not only about the general mortality environment in the next period, but also about the level of the government's expenditure (equals tax revenue) on mitigating it. Hopes of a less dangerous future will induce more schooling; pessimism will reduce it. A failure to act swiftly and publicly can therefore do enormous damage through this channel alone.

Armed with this pair of instruments, the government's task is to choose a sequence of taxes, or "plan," over some time horizon starting in period 0, $\{\tau_t\}_{t=0}^T$, with the aim of maximizing social welfare. This is a difficult problem, for not only are all the periods connected through the formation of parents' (rational) expectations about the course of future policy, but in addition there may be problems of credibility if the government is unable to commit itself to a certain future course of action. This is not the place to go into the details. A sketchy account is given in the appendix, and the interested reader is referred to Bell, Devarajan, and Gersbach (2003).

To complete the formulation of the government's decision problem, we must define social welfare. Since there is no inequality within each generation, but the threat of a collapse is ever present, it is intuitively appealing to put a heavy emphasis on accumulating human capital rapidly. Bell, Devarajan, and Gersbach (2003) argue that the following claim is valid when each generation covers a sufficiently long period:

The pooling case: If, in each and every period, the government chooses the level of the tax in that period so as to maximize the level of human capital attained by a child on reaching adulthood in the next, the resulting plan will be "good," in the sense of not departing very far from the optimum.

That is to say, it is enough for the government to look just one generation ahead in order to attain a "good" result, given the expenditure instruments at its disposal. The government's problem is set out formally in the appendix.

We now introduce school attendance subsidies, which, if available, will yield a further improvement. Let the amount paid per child in period t be σ_t for each unit of time the child spends in school. The fact that the government must allocate its revenue

between combating mortality and subsidizing schooling directly calls for a little additional notation. Denote total spending on the former by η_t , where this expenditure should be thought of as producing a public good within the family, so that $\kappa_t(\eta_t)$ replaces $\kappa_t(\tau_t)$. The family's budget constraint then reads

$$\left[2 + \beta \frac{N_t}{\kappa_t(\eta_t)}\right]c_t(0) + (\alpha\gamma - \sigma_t) \frac{N_t}{\kappa_t(\eta_t)} e_t(0) = \alpha \left[2\lambda_t + \frac{N_t}{\kappa_t(\eta_t)} \gamma\right] - \tau_t \equiv \Omega_t(0, \eta_t, \tau_t), \quad (10)$$

a comparison of which with equation (9) reveals that the subsidy works by reducing the opportunity cost of the child's time, and so encourages investment in schooling directly. The government's budget constraint will always bind at the optimum, so that the ministries of health and education will be competing for funds.

The trade-off is subtle and complex. For any level of taxation, additional spending on reducing mortality will increase not only current full income, but also positive expectations about future mortality. Both work to increase schooling and so offset the reduction in schooling brought about by the correspondingly smaller subsidy on school attendance. Finding the right balance between them involves solving problem (A9) in the appendix; but it is intuitively clear that, when a society is threatened by very high mortality, the best policy will always involve a fairly substantial effort to combat it if the available measures are even moderately effective.

[b]Optimal Policies with Nuclear Families

In societies with strictly nuclear family structures, premature adult mortality brings about inequality within each generation unless there is countervailing action by communities or government. In this respect, therefore, the government's task is more complicated than under pooling. Indeed, avoiding such inequality may not be possible, depending on the instruments available and the pressures exerted by the need to ensure long-term growth. Yet most of the elements that make up the policy program are clear. Suppose, for simplicity, that the authorities are able to observe each family's status. The tax base is normally provided by two-parent households, for only under conditions of some affluence will single-parent households have any taxable capacity when their children

attend school full-time. The society's needy, therefore, are the members of single-parent households and full orphans, who must be cared for in special institutions. The budget constraints for households with adults can be expressed as before, taking into account whatever taxes and subsidies are payable. Suitable standards must be drawn up for orphanages, whose staffing must suffice to provide decent care for their charges, and whose staff must be paid their opportunity cost in the production of the aggregate consumption good. At the very least, the children should receive the package of consumption and education enjoyed by their counterparts in single-parent households. The government's budget constraint is written out accordingly.

The absence of pooling as a form of insurance and as an instrument to ensure equality within each generation requires a reformulation of the policy program. Bell, Devarajan, and Gersbach (2003) argue that one way of arriving at a "good" program is as follows:

The nuclear family case: In each and every period, choose a tax and expenditure plan so as to maximize the society's expected taxable capacity in the next period.

Observe the switch from the future attainment of the representative child under pooling to future *aggregate* taxable capacity. The intuition here is that, given the need sooner or later to undertake redistribution within a generation, it is this capacity that ultimately determines whether the whole society can eventually escape from want and illiteracy. In dire circumstances, however, it may happen in some periods that it is not optimal to grant support to all those in need. The full problem is written out formally in Bell, Devarajan, and Gersbach (2003) and will not be repeated here. Suffice it to say that the tension between providing direct support to families (whether conditional on school attendance or otherwise) and combating mortality necessarily arises once more, with the further twist that avoiding premature mortality in period t has an immediate effect both on the tax base and on the numbers and types of the needy, through its influence on the distribution of family types in period t .

[b]Estimating the Costs and Effects of Policies

We introduced above public spending on combating premature adult mortality from all sources as an instrument to improve economic performance and well-being. What especially concerns us in this chapter, of course, is spending on combating AIDS, and here we know much less than we would like about the effects of the disease on the level of mortality. This obstacle notwithstanding, we need to estimate the function $q(\eta)$ if we are to reach defensible conclusions about the right policies to be pursued. There follow a discussion of the elements of the approach and a short summary of the results it yields when applied to South Africa.

We begin with two useful reference cases. The first is the counterfactual, in which the human AIDS virus never came into existence; we denote this disease environment by $D = 0$. This, as we argued above, can be taken as corresponding to the mortality profile that prevails in the very early stages of the epidemic, as exemplified by Zimbabwe in 1986 and South Africa in 1990. The second step involves the second reference case, which arises when the epidemic simply runs its course, unhindered by public action of any kind. Specifying this alternative poses various problems, for even here the course of the epidemic depends on individual behavior, which may, in its own turn, respond to the experience of the epidemic, and on whether the virus adapts to its human hosts by becoming less virulent over the longer run. There is, unfortunately, not a single historical example of this particular epidemic simply running its full course in some society to offer any guidance on this matter. In neither South Africa nor Zimbabwe had the epidemic reached maturity by the end of the 1990s, grim though the situation had already become in both countries. Describing this second case is therefore a task more for epidemiologists and virologists than for social scientists. For South Africa the forecast by Dorrington and others (2001) for 2010 reflects such considerations, and it seems to be a good working approximation of what is needed, with the reservation that palliative care of the sick and the treatment of opportunistic infections are already absorbing resources on a substantial scale.

The next step is to connect these two reference cases through the plausible assertion that very heavy spending on combating the disease would restore the status quo ante profile of mortality. If this much is granted, then the probability of premature mortality, viewed as a function of spending on combating the disease, namely, $q(\eta; D = 1)$, will be anchored at both ends of the spending range. To supply the shape of the curve in between, it is natural, for economists at least, to appeal to diminishing returns; thus $q(\eta; D = 1)$ would be downward-sloping and convex, steep when η is small and flat when it is large. It is also tempting to associate these endpoints of the spending range with preventive measures and antiretroviral treatments, respectively.

One intervention that commends itself in connection with the control of all sexually transmitted diseases is to target prostitutes and their clients; in this context the use of condoms is also strongly promoted. Marseille, Hofmann, and Kahn (2002) give the corresponding cost of averting a single case of AIDS in Kenya, for example, as \$8 to \$12. This is cheap indeed, but given the nature of the disease and of people, it must be inferred that this is an expenditure that will recur annually. They also present evidence that other preventive measures, such as ensuring a safe blood supply and treating mothers at birth with nevirapine, are less cost-effective by a factor of 10 or more. Choosing a bundle of diverse preventive measures, they estimate the resulting cost per disability-adjusted life-year (DALY) so saved at \$12.50.

At the other end of the range, the overwhelming bulk of expenditure is devoted to treating those with the disease. Such treatment covers not only opportunistic infections, especially in the later stages of the disease, but also antiretroviral therapies. These measures keep infected individuals healthier and can extend their lives for a few years, thereby raising lifetime family income and improving parental care. It seems perfectly defensible, therefore, to interpret these gains as equivalent to a reduction in q within the OLG framework. Marseille, Hofmann, and Kahn (2002) put the cost of saving one DALY by these means at \$395, on the very conservative assumption that the drugs take the form of low-cost generics and that the costs of the technical and human infrastructure needed to support an effective HAART (highly active antiretroviral therapy) regimen of this kind can be wholly neglected.

At this point the reader may be forgiven for wondering how much a comprehensive HAART program might cost. The elements of an estimate for Burkina Faso, a poor West African country in which the AIDS prevalence rate is about 8 percent, are set out in World Bank (2003). If generic antiretroviral drugs can be purchased from Indian firms, the annual cost of treating each individual would be about \$810; under the next-best, negotiated alternative, they would more than double, to \$1,730. At the prevailing prevalence rate, the lower of the two estimates translates into an aggregate outlay roughly equivalent to 80 percent of the health ministry's current budget, or about 1.8 percent of GDP. In Kenya, where the prevalence rate is about 15 percent and GDP per capita is similar, the aggregate outlay would be roughly twice as large. These are sobering numbers, but broadly in line with the values of $\{\eta_t\}$ that emerge from the "good" programs derived in Bell, Devarajan, and Gersbach (2003) for South Africa, which we now summarize.

In the pooling case without school attendance subsidies, the sequential procedure described earlier yields a "good" plan $\{\eta_t\}$ in which total expenditure per family or subfamily is \$960 in 1990 and \$1,030 in 2020 and thereafter, which correspond to about 4.5 and 3.6 percent of GDP, respectively. This represents a very considerable fiscal effort, over and above that already implicitly contained in the calibration for the period 1960-90, which must also continue into the future to maintain the validity of the parameter values so derived. If this effort is politically possible, not only would it stave off a collapse and virtually restore the status quo ante with respect to premature adult mortality, but it would also leave λ in 2080 only about 12 percent lower than that in the counterfactual case without AIDS.

Such satisfactory results where long-run growth is concerned are not attainable under a nuclear family system, if inequality in individual levels of human capital is to be avoided in each and every generation. The two-parent families that form the tax base must finance spending programs not only to combat the epidemic, but also to support one-parent families and full orphans. Recall that, since there is no extended family to ensure equality within each generation, formulating a "good" plan under these circumstances involves maximizing a different objective function, namely, aggregate

taxable capacity in the next period. The corresponding sequential procedure described there yields expenditure on combating the epidemic that is about 15 percent higher than under pooling; in addition, each one-parent family receives a (lump-sum) subsidy that is about twice as large as η_t . To finance all this, each two-parent family pays about twice the amount of the subsidy as a lump-sum tax. The higher levels of spending to combat the disease in each period are warranted because they lead to fewer needy families and orphans in that period, and hence to less inequality among adults in the next generation. Put slightly differently, these results confirm the importance of keeping premature mortality among adults low, so that parents can provide and care for their children. This clutch of ambitious interventions does stave off collapse and the emergence of inequality, but at the cost of much slower growth. Full education for all is reached only in 2080, and λ in 2080 is less than half its level in the counterfactual case without AIDS. These findings point to the macroeconomic significance of informal social insurance through extended family structures in the face of increasing adult mortality.

[A] OTHER ECONOMIC LOSSES

Certain of the primary effects of morbidity and mortality among young adults listed at the outset have received rather short shrift thus far. Here, therefore, we show how they can be given a home in the framework developed above.

An adult who dies prematurely will, all else being equal, produce less over the life cycle than one who does not. This loss of output and all its consequences are fully taken into account in the OLG framework, and there is no real need to estimate it independently in its own right. Much is sometimes made of the loss of trained workers in particular. Here, too, to the extent that productivity in adulthood depends only on the quality of childrearing and formal education, this loss of human capital is captured in full in the above framework. To the extent that workers acquire specific skills through training on the job, however, premature mortality among them will indeed entail losses for which that framework does not allow, and so the results derived from it will be on the optimistic side. Assuming that firms are rational in their investment in workers through such training, the costs of training replacements places a lower bound on such losses.

Some idea of the order of magnitude of these losses in the case of financial services companies in South Africa can be gained from estimates by Schneider and Kelly (2003). Their analysis covers the following items: main risks costs (for example, the costs of death-related and disability benefits), defined pension benefits, replacement and retraining, sick leave, economic costs of absences, maternity benefits, and ancillary insured benefits. In the absence of AIDS the combined costs of these items to a hypothetical company in the financial services sector are estimated to account for 24.8 percent of the firm's outlay on basic payroll. (The contribution of replacement and retraining is a very modest 1.1 percent.) In the current phase of the epidemic, this combined total is estimated to have risen by 2.8, 2.9, and 2.3 percent of basic payroll in Gauteng, KwaZulu-Natal, and Western Cape provinces, respectively. Of these increases, only 0.1, 0.2, and 0.1 percentage point, respectively, arise from additional replacement and retraining costs (Schneider and Kelly, 2003, pp. 9-10). For a hypothetical manufacturing company in Gauteng province, the contribution of replacement and retraining costs is not only larger in the absence of AIDS (1.4 percent of basic payroll) but also more sensitive to the epidemic, which induces an increase of 0.9 percentage point (Schneider and Kelly, 2003, p. 75). None of these estimates is strikingly large, but they are costs all the same.

What we have called the transmission factor $z(s_t)$, $s_t = 0, 1, 2, 3, 4$, and the educational technology $g(\cdot)$ play a vital role in determining the dynamic behavior of the system, especially where its long-run rate of growth, $2zg(1) - 1$, is concerned. Our direct empirical knowledge of these elements, so formulated, is limited, but something can be said about the direction of the epidemic's effects. For strictly nuclear families, the value of $z(s_t)$ is given, and changes in mortality work their effects by changing the state probabilities $\pi_t(s_t)$. If, however, surviving couples take in orphans—pooling arrangements provide an extreme “ideal”—then the sheer burden of caring for more children will at some point surely reduce the quality of childrearing they can provide, although the magnitude of the effect remains a matter for speculation. Given the related financial stress, it is only to be expected that these parents might favor their natural children over their adopted or foster ones, not only in matters of nutrition, education, and health but

also in that vital intangible, loving care and attention. Some recent empirical work supports the view that these adverse effects are indeed at work. A study of Indonesian children, for example, yields the finding that orphans are less healthy, less likely to go to school, and overall less prepared for life (Gertler, Levine, and Martinez, 2003). Case, Paxson, and Ableidinger (2002) find, in a group of African countries, that the schooling of orphans depends heavily on how closely related they are to the head of the adopting household. In another recent study of 28 countries, 22 of them African, Ainsworth and Filmer (2002) arrive at a more cautious conclusion.

Although enrollment rates in the majority of the countries studied are lower among orphans than among children with two living parents, the differences are frequently modest in comparison with those between children from rich and poor households, so that targeting on the basis of orphan status is not always obviously the right option. As Ainsworth and Filmer emphasize, moreover, the ultimate aim is not enrollment in itself, important though that is, but rather learning; yet we know little about how orphans perform compared with children with two living parents. As the numbers of orphans swell with the wave of adult mortality now beginning to sweep through sub-Saharan Africa, gaining such knowledge has become a pressing need.

Turning to the educational technology or, more broadly, the supply side of education, much has been made of the very high mortality among teachers, of its grave consequences if replacements are not trained or found, and of the costs of those replacements. None of these considerations appears in this chapter's analysis thus far, but they are readily introduced into the OLG framework. The essentials are fully captured by looking at pooling arrangements, which have evident expositional advantages. Since a teacher's time in the classroom is spread over the children present, it bears a relation to the average value of e_t in generation t . For simplicity, let it be a fixed fraction r thereof for each child. Teachers in period t , like all other adults, are endowed with human capital λ_t and are correspondingly paid $\alpha\lambda_t$. Equation (7), the extended family's budget constraint when normalized to a single "couple," becomes

$$\left[2 + \beta \frac{N_t}{\kappa_t(\tau_t)}\right]c_t(0) + \alpha(\gamma + r\lambda_t) \frac{N_t}{\kappa_t(\tau_t)} e_t(0) = \alpha\left[2\lambda_t + \frac{N_t}{\kappa_t(\tau_t)}\gamma\right] - \tau_t \equiv \Omega_t(0, \tau_t), \quad (11)$$

from which it is seen that the need for teachers to bring about learning expresses itself as the component $r\alpha\lambda_t(N_t/\kappa_t(\tau_t))$ of the total “price” of education. This component is increasing in the current levels of both productivity and premature mortality among adults. Thus, even if mortality among teachers is no different from that in the rest of the population, it still discourages investment in education. If, further, teachers require special training, then the costs discussed above compound the problem by imposing an additional burden on the national treasury, and hence on families through taxation.

It was argued above that parents effectively choose the level of schooling, so that e_t is influenced by the whole range of factors discussed above, including premature adult mortality. Disentangling them empirically is a very tall order indeed, but one can still attempt to establish whether there is an association between such mortality and schooling, and thereby provide indirect support for the approach chosen here. This has been undertaken by Hamoudi and Birdsall (2002), using data from Demographic and Health Surveys conducted in 23 sub-Saharan African countries. Employing two specifications, they settle on the estimate that a reduction in life expectancy at birth of 10 years is associated with a fall of 0.6 year in the average schooling attained by that cohort (Hamoudi and Birdsall, 2002, p. 23). Given that life expectancy at birth in most countries in southern and East Africa fell by 10 years or more from 1985 to 2000 (Dorrington and Schneider, 2001), and that average schooling among the population aged 25-49 was in the modest range of three to six years, this is a disturbing finding. In the light of the OLG framework, it would be useful to know whether the general magnitude of this effect also holds good for the more pertinent indicators ${}_{20}q_{20}$ and ${}_{30}q_{20}$, but no such results appear to be available.

[A] CONCLUSION

Like the Black Death of the 1300s (Cohn, 2002), AIDS has the potential to transform the societies in which its victims live. But unlike that great plague, AIDS can have this effect largely by undermining the transfer of human capital from one generation to the next—

arguably the core mechanism by which societies grow and flourish. The reason is that, in contrast with other epidemics, AIDS is overwhelmingly a fatal disease of young adults. Not only does AIDS cause unspeakable human suffering, but it also makes it difficult for these young men and women to provide for the education of their children, not to mention offer the love and care they need to complement their formal schooling. The result is possibly a whole generation of undereducated and hence underproductive youth, who in adulthood will find it difficult to provide for *their* children's education, and so on. In this way an otherwise growing economy could, when hit with an enduring and sufficiently severe AIDS epidemic, spiral downward into a low-level subsistence economy in three or four generations.

This threat of a progressive collapse of the economy is particularly insidious because the effects will not be felt immediately. Thus estimates of the economic impact of AIDS that look only at the short- to medium-term effects of reductions in labor supply are dangerously misleading. They risk lulling policymakers, especially those concerned with short-term economic fluctuations, into a sense of complacency. As this chapter has shown, it is possible to avert the downward spiral, but only with an aggressive set of policies aimed at shoring up the faltering mechanisms of human capital transmission between generations—policies that prevent AIDS, prolong the lives of its victims, and support the education of their children. These policies are expensive, but, when viewed against the specter of a collapse of the economy and possibly of society itself, they seem like a bargain.

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[A]APPENDIX: SOME TECHNICAL NOTES

[b]The Household's Decision Problem

Let preferences be separable in c_t and λ_{t+1} , with representation in terms of the (expected) utility function

$$EU_t(s_t) = (3 - s_t)[u(c_t(s_t)) + E_t A_{t+1} v(\lambda_{t+1})], \quad \text{with } s_t = 1, 2, \quad (\text{A1})$$

where the random variable $A_{t+1} \in \{0, 1, \dots, N_t\}$ is the number of the N_t children born in period t who survive into old age in period $t + 1$, and E_t is the expectations operator. To put it somewhat differently, parents in period t form expectations about the premature mortality that will afflict their children as adults in period $t + 1$, and they consider the average number of survivors in weighting the payoff $v(\lambda_{t+1})$.

We are now in a position to write out the household's problem formally. As a preliminary, we substitute for λ_{t+1} in equation (A1) using (2), which yields

$$EU_t(s_t) = (3 - s_t)[u(c_t(s_t)) + E_t A_{t+1} v(z(s_t)g(e_t)\lambda_t + 1)], \quad \text{with } s_t = 1, 2. \quad (\text{A2})$$

A family in state $s_t (=1, 2, 3)$ chooses the bundle $(c_t(s_t), e_t(s_t))$ so as to

$$\text{maximize } EU_t, \text{ subject to equation (4), } c_t(s_t) \geq 0, \text{ and } e_t(s_t) \in [0, 1]. \quad (\text{A3})$$

A full analysis of problem (A3) is provided in Bell, Devarajan, and Gersbach (2003).

[b]Dynamics

To establish whether unbounded growth is possible, consider the interval $[\Lambda^a(1), \infty)$, in which $e_t^0(1) = 1$. The growth rate of human capital in period t in those families such that $s_t = s_{t+1} = 1$ is

$$\frac{\Lambda_{t+1}(1)}{\Lambda_t(1)} - 1 = [2z(1)g(1) - 1] + \frac{2}{\Lambda_t(1)}.$$

Hence, for all families in the group such that $s_t = s_{t+1} = 1$ for all t and $\Lambda_t(1) > \Lambda^*(1)$ at some point in time, human capital per capita will indeed grow without bound if and only if $2z(1)g(1) \geq 1$, the asymptotic growth rate being $2z(1)g(1) - 1$.

[b] Preferences Under Pooling

Given the assumptions about altruism in the text, the “couple’s” preferences are represented as

$$EU_t(0) = 2[u(c_t(0)) + E_t A_{t+1}(0)v(\lambda_{t+1})], \quad (\text{A4})$$

where the random variable $A_{t+1}(0) \in \{0, 1, \dots, N_t(0)\}$ is the number of the $N_t(0)$ children born in period t who survive into old age in period $t + 1$. The term $E_t A_{t+1}$ depends on the parents’ expectations in period t about premature adult mortality among their children in period $t + 1$. In the presence of the poll tax τ_t , the variable $N_t(0)$ must be rewritten accordingly: the random variable $A_{t+1}(0) \in \{0, 1, \dots, N_t(0, \kappa_t(\tau_t))\}$ and

$$E_t A_{t+1} = [E_t \kappa_{t+1}(\tau_{t+1})] \cdot N_t / \kappa_t(\tau_t). \quad (\text{A5})$$

Thus the parents must form expectations about not only the mortality environment in the next period, but also the level of the government’s expenditure (equals taxation) on mitigating it. If these expectations are stationary, that is, that the future will be like the present, then equation (A5) will specialize to the simple form

$$E_t A_{t+1} = N_t. \quad (\text{A6})$$

[b] Optimum Policy

Where expectations are concerned, Bell, Devarajan, and Gersbach (2003) argue that a good approximation to a full optimum can be achieved as follows. Suppose there are stationary expectations, so that equation (A6) holds and the forward connection among periods is cut. Then, starting from period t , the sequence $\{\tau_t\}_{t=0}^T$ can be constructed as a series of taxes, each element of which is derived independently of all future values as the optimum for the period in question. There is, however, an important connection with the

immediately preceding period: the condition $\tau_{t-1} \leq \tau_t$ must not be violated, for otherwise the extended family will have chosen the level of education in period $t - 1$ on the basis of falsely optimistic expectations about mortality in period 1. Observe that if the sequence settles down into a stationary one, it will also involve a rational expectations equilibrium.

Formally stated, the government's problem is as follows. Starting in period 0,

$$\begin{aligned} \max_{\tau_t} [2z(0, \kappa(\tau_t))g(e_t^0(\Omega_t(0, \tau_t), 0, \kappa(\tau_t)))\lambda_t + 1], \\ \text{subject to } \tau_t \geq 0 \text{ and } \tau_t \geq \tau_{t-1} \quad \forall t \end{aligned} \quad (\text{A7})$$

Observe that the potential problem of credibility is implicitly assumed away: when the family forms its (stationary) expectations in period t , the government has found some way to commit itself to $\tau_{t+1} = \tau_t$.

Following the introduction of school attendance subsidies as an additional instrument, the government's own budget constraint reads

$$\eta_t + \sigma_t(N_t / \kappa_t(\eta_t))e_t^0(\Omega_t(0, \eta_t, \tau_t), 0, \sigma_t, \kappa_t(\eta_t)) \leq \tau_t. \quad (\text{A8})$$

Problem (A7) becomes

$$\begin{aligned} \max_{\eta_t, \sigma_t, \tau_t} [2z(0, \kappa(\eta_t))g(e_t^0(\Omega_t(0, \eta_t, \tau_t), 0, \sigma_t, \kappa(\eta_t)))\lambda_t + 1], \\ \text{subject to } \tau_t \geq 0, \tau_t \geq \tau_{t-1}, \text{ and (A8)} \quad \forall t. \end{aligned} \quad (\text{A9})$$

Table 1. Adult mortality by sex in South Africa and Zimbabwe

| Country | Year | ${}_{45}q_{15}$ | ${}_{45}q_{15}$ | ${}_{20}q_{20}$ | ${}_{20}q_{20}$ | ${}_{30}q_{20}$ | ${}_{30}q_{20}$ |
|--------------|------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| | | (Male) | (Female) | (Male) | (Female) | (Male) | (Female) |
| South Africa | 1990 | 0.265 | 0.265 | 0.106 | 0.040 | 0.182 | 0.098 |
| | 2000 | 0.419 | 0.419 | ... | ... | ... | ... |
| | 2010 | 0.790 | 0.790 | 0.359 | 0.541 | 0.616 | 0.707 |
| Zimbabwe | 1986 | 0.310 | 0.195 | ... | ... | 0.169 | 0.107 |
| | 1997 | 0.553 | 0.417 | ... | ... | 0.414 | 0.331 |

Sources : For South Africa, ${}_{45}q_{15}$ is from Dorrington et al. (2001), who report only the average over both sexes ; ${}_{20}q_{20}$ and ${}_{30}q_{20}$ are based on author's calculations, using data from Dorrington et al. (2001). For Zimbabwe, Feeney (2001, Table 1), who reports ${}_{35}q_{15}$ instead of ${}_{30}q_{20}$.

Table 2. Family state probabilities and survival rates

| Country | Year | $\pi(1)$ | $\pi(2)$ | $\pi(3)$ | $\pi(4)$ | κ |
|-----------|------|----------|----------|----------|----------|----------|
| S. Africa | 1990 | 0.855 | 0.101 | 0.039 | 0.005 | 0.925 |
| | 2010 | 0.294 | 0.168 | 0.347 | 0.194 | 0.550 |
| Zimbabwe | 1986 | 0.742 | 0.151 | 0.089 | 0.018 | 0.862 |
| | 1997 | 0.392 | 0.277 | 0.194 | 0.137 | 0.628 |

Source: Authors' calculations, based on, ${}_{20}q_{20}$ for South Africa, and ${}_{35}q_{15}$ for Zimbabwe (see Table 1).

Table 3. Three growth paths for the South African Economy

| Year | No AIDS | | | AIDS | | | | | |
|------|-----------|------|-----------|--------------------------------------|------|-----------|---------------------------------------|------|-----------|
| | λ | e | $y(0)$ 1/ | $E_{90}(\kappa_{120}) = \kappa_{90}$ | | | $E_{120}(\kappa_{150}) = \kappa_{90}$ | | |
| | | | | λ | e | $y(0)$ 1/ | λ | e | $y(0)$ 1/ |
| 1960 | 2.62 | 0.50 | 19,500 | 2.62 | 0.50 | 19,500 | 2.62 | 0.50 | 19,500 |
| 1990 | 3.14 | 0.64 | 22,340 | 3.14 | 0.20 | 26,370 | 3.14 | 0.68 | 23,400 |
| 2020 | 4.32 | 0.97 | 29,590 | 2.01 | 0.0 | 17,770 | 4.52 | 0.39 | 34,690 |
| 2050 | 7.86 | 1.0 | 53,720 | 1.0 | 0.0 | 12,900 | 3.80 | 0.29 | 30,280 |
| 2080 | 13.85 | 1.0 | 94,720 | 1.0 | 0.0 | 12,900 | 2.78 | 0.14 | 24,250 |

Source: Bell, Devarajan and Gersbach (2003). All results are based on 30q20. 1/ From 1990 onwards, a representative 'family' comprises two surviving adults and 3.49 children in the absence of AIDS, and two surviving adults and 8.87 children in its presence.

Figure 1. The phase diagram in the absence of premature adult mortality

