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A Framework for the Analysis of Financial Reforms and the Cost of Official Safety Nets

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Abstract

This paper builds a multiperiod, general equilibrium framework for analyzing the macroeconomic effects of financial reforms in developing countries and the costs of maintaining official safety nets under the financial system during such reforms.

While a financial liberalization yields efficiency gains, adverse macroeconomic effects can arise if the creditworthiness of the nonfinancial sector is weak. In this situation, financial liberalization may also increase the authorities' expected deposit insurance funding obligations even with strong prudential supervision. Moreover, given the distortions in a repressed financial system, an increase in the required bank capital-asset ratio may increase the funding obligations associated with deposit insurance, particularly when the debt-servicing capacity of nonfinancial firms is low.

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Summary

This paper builds an analytic framework for analyzing the effects of financial reforms in developing countries and the costs of maintaining official safety nets under the financial system during such reforms. A multiperiod general equilibrium framework is used to explore the interactions between three types of economic agents--firms, which borrow to finance production; households, which provide labor and hold deposits; and banks, which accept deposits and make loans--in the presence of production uncertainty, financial repression, and an official safety net that encompasses deposit insurance provided (explicitly or implicitly) by the authorities. Both the credit risks incurred by banks and the expected deposit insurance obligations of the official sector that arise during a financial reform are linked explicitly to the degree of production uncertainty, the financial positions of nonfinancial firms, and the nature of prudential supervision.

The analysis suggests a number of policy conclusions. First, the macroeconomic effects of a financial liberalization will depend critically on the perceived creditworthiness of the nonfinancial sector. If banks regard lending to firms as highly risky, for example, then increasing or removing ceilings on loan interest rates may reduce both the scale of financial intermediation and economic activity. Second, even with strong prudential supervision, financial liberalization may increase the authorities' expected deposit insurance funding obligations, especially when the creditworthiness of the nonfinancial sector is low. Third, given the distortions that are likely to exist in a repressed financial system, an increase in the required capital asset ratio may have the perverse effect of increasing the expected funding obligations associated with deposit insurance, particularly when the debt-servicing capacity of nonfinancial firms is low. The paper finds that, even with good prudential supervision and enhanced capital adequacy requirements, countries undertaking financial reforms may confront a trade-off between financial efficiency and the risk of larger safety net funding obligations.

I. Introduction

Many Fund-supported adjustment programs for developing countries have included structural reforms aimed at strengthening the financial system. Countries often start such reforms with extensive financial restrictions, including interest rate ceilings for both deposits and loans, limitations on competition and entry into the financial sector, high required reserve ratios, and various credit allocation rules. Usually, these countries also have weak systems for prudential supervision of the financial sector. As a result, the financial reforms have often initially focused on removing or raising ceiling interest rates, reducing required reserve ratios, permitting freer entry into the financial system, and strengthening prudential supervision.

As noted by Díaz-Alejandro (1985), however, a number of reform efforts have ended in periods of financial instability that required extensive restructuring of both the corporate and financial sectors and created large public sector funding obligations as the authorities provided emergency lending to enterprises and financial institutions. These experiences have raised the issues of what factors contribute to financial instability 1/ and whether a financial reform is likely to expose the authorities to new credit risks through the operation of any official safety net underpinning the domestic financial system.

In general, these official safety nets have been designed to prevent financial disturbances from creating disruptions in either the payments system or the intermediation of credit that would have large spillover effects on the real economy. The provision of short-term emergency liquidity assistance by the central bank, some form of private or official deposit insurance, and direct short- or medium-term emergency assistance for large troubled financial institutions have often been key elements in such safety nets. While such provisions have helped contain the effects of financial crises, they have also exposed the authorities to credit risks through lending to troubled financial institutions, either directly or through the central bank's discount window, and through the fulfillment of insurance obligations to depositors.

1/ The contributing factors suggested by analyses of these episodes of financial distress (e.g., see Villanueva and Mirakhor (1990)) include: (1) inconsistencies between the financial reforms and the accompanying macroeconomic stabilization programs (especially when a lack of fiscal control led to rapid inflation); (2) the emergence of destabilizing capital inflows which contributed to an appreciation of the real exchange rate; (3) inadequate prudential supervision which allowed some financial institutions to acquire undiversified and risky loan portfolios; and (4) inappropriate pricing of the (explicit or implicit) deposit insurance guarantees offered by the authorities.

In developing a framework for analyzing the effects of financial reforms on the cost of official safety nets, it is important to recognize that the desire to maintain a stable financial system and to limit the authorities' exposure to the credit risks associated with official safety nets has been a primary motivation for policies specifying minimum capital adequacy standards, systems of prudential supervision, limits to risk-taking by institutions and individuals, restrictions on competition and activities and, in some countries, ceilings on interest rates. 1/ Such limitations on activities, portfolio choices, and interest rates have been perceived as a means of promoting stability by creating financial institutions with strong market and capital positions, limiting speculative behavior, and restricting competition. These considerations suggest that a useful direction for developing a better understanding of why financial reforms sometimes lead to instability and impose large costs on official safety nets is to construct an analytic framework with microeconomic foundations that focus explicitly on how depositors, borrowers, and financial intermediaries are likely to respond to the relaxation or elimination of various types of financial restrictions. 2/

This paper presents an optimizing model that focuses explicitly on the conditions under which firms and financial intermediaries become bankrupt, on how the incidence of failure is affected by various financial reforms, and on the size of the funding obligations the government incurs in a system with an official safety net containing explicit or implicit deposit insurance. A multiperiod general equilibrium framework is developed that includes three types of optimizing agents: households, firms, and banks. The linkages between financial reforms and the authorities' funding obligations reflect the fact that, since firms face uncertain production shocks, there is the possibility that they will default on their debt-service obligations to the banks. To reduce the likelihood of defaults, banks will spend resources to monitor the firms' investment plans and production outcomes. Under some contingencies, however, the number of firms that default may leave a bank unable to service its deposit payment obligations, in which case the authorities will incur funding obligations through the deposit insurance system.

1/ As noted by McKinnon (1973), Shaw (1973), and Fry (1988), other motivations for such restrictions have been to direct credit toward preferred customers and activities and to provide a basis for taxing (either explicitly or implicitly) the financial system. For example, high required reserve ratios can increase the flow of seignorage resources to the public sector by creating a large demand for base money.

2/ The transmission mechanism for monetary policy in developing countries, and the analysis of why financial reforms fail, have received considerable attention in recent years; see, for example, Díaz-Alejandro (1985), McKinnon (1988), Cho and Khatkhate (1989), Villanueva and Mirakhor (1990), and Montiel (1991). This paper attempts to break new ground by building an analytic framework with more explicit microeconomic foundations.

One key determinant of the economy's response to a financial liberalization is the extent to which banks ration credit to firms. The amount of credit that banks make available to firms may be rationed in our model for two reasons. In the first place, the presence of official constraints on financial activities may leave banks unable to attract sufficient deposits to satisfy fully the prevailing demands for credit. This type of situation is often referred to as "disequilibrium credit rationing;" both suppliers and rationed demanders (of deposits and/or credit) have incentives to bid up interest rates, but official restrictions prevent interest rates from rising to market-clearing levels. In addition, the amount of credit that banks find it optimal to extend to firms may be a backward bending function of the loan interest rate, since an increase in the loan rate may increase the probability that the borrower will default and, beyond some point, may reduce the banks' expected rate of return. This represents a form of "equilibrium credit rationing." 1/

A large part of the paper is devoted to developing the analytic framework we require (Section II) and to describing its long-run properties through comparative "steady-state" analysis (Section III). We also devote specific attention to the size of the funding obligations the government can expect to incur when it provides deposit insurance (Section IV). Further analysis of short-run dynamics, perhaps including simulation experiments, has been left for subsequent papers.

Although the analysis undertaken in this paper does not focus on the short-run consequences of financial reforms, the comparative static responses of macroeconomic variables suggest a number of important policy conclusions. Whatever the system of prudential supervision, it is clear that the responses of macroeconomic variables to financial liberalization measures depend critically on the perceived creditworthiness of the nonfinancial sector. If banks regard firms as highly risky, the elevation or removal of ceilings on loan interest rates, for example, may actually reduce the scale of financial intermediation and the level of economic activity. Accordingly, one important policy implication is that any financial difficulties or creditworthiness problems of the nonfinancial

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1/ Situations of "equilibrium credit rationing" are said to exist when rational and unconstrained intermediaries maintain loan rates below the market-clearing level on a continuing basis. Following Stiglitz and Weiss (1981), most of the literature on this subject has focused on phenomena that lead intermediaries to restrict the number of their loans. This subset of the literature has identified moral hazard and adverse selection as phenomena that can give rise to equilibrium credit rationing when borrowers and lenders have asymmetrical information; see Jaffee (1987). In the present paper, intermediaries are able to obtain full information on firms through monitoring, moral hazard problems are limited, and equilibrium credit rationing takes the form of reducing the size of loans as the loan interest rate increases; see Wu and Gray (1991) for a more streamlined model that also investigates this form of equilibrium credit rationing.

sector should be addressed at an early stage: otherwise, financial reforms may well be contractionary.

A second conclusion pertains to systems in which the authorities provide implicit or explicit deposit insurance. In such systems, financial liberalization may increase the expected funding obligations of the government, especially when the creditworthiness of the nonfinancial sector is low. This is likely to be true even if the authorities put in place a strong system of prudential supervision.

A third conclusion is that, given the distortions that are likely to exist in a repressed financial system, an increase in the required capital-asset ratios of banks may have the perverse effect of increasing the expected funding obligations associated with deposit insurance, especially when the nonfinancial firms have weak financial positions. This effect reflects the fact that the presence of deposit insurance may encourage banks to lend fully against whatever capital they have in place.

In general, our analysis indicates that, even with good prudential supervision and enhanced capital adequacy requirements, countries undertaking or contemplating financial reform confront a tradeoff between financial efficiency and the risk of larger safety net funding obligations. While a financial reform can increase efficiency, it may also burden the authorities with greater risks via the official safety net. The authorities can improve on this tradeoff, however, by acting at an early stage in the reform process to strengthen the financial positions of nonfinancial firms and the prudential supervision system.

II. A Model of Financial Repression

In this section we characterize the optimizing behavior of each of the three types of private economic agents: firms, households, and banks. The authorities influence behavior through the restrictions they impose on the domestic financial system and through their control over the rate of expansion of high powered money, which banks must hold to fulfill their reserve requirements. The restrictions imposed on the financial system in our model are those that are most typically encountered in financially repressed systems in developing countries: namely, ceilings on loan and deposit interest rates, a required reserve ratio, and a minimum capital-asset ratio.

1. Firms

The production sector of the economy is composed of competitive firms, each of which is operated by an entrepreneur who maximizes the expected discounted utility of his planned consumption over time. For simplicity, assume that each entrepreneur (denoted by j) is risk neutral, and that his utility during period t ($U_{j,t}$) is linearly related to the level of his consumption ($C_{j,t}$). Thus:

$$(1) \quad U_{j,t} = \gamma_j^F C_{j,t}$$

with γ_j^F representing the marginal utility of consumption.

Each entrepreneur owns a firm and produces output (Y_j) of a single homogenous good, using capital (K_j) and labor (L_j). A production lag implies that output available for sale in period t is produced using capital and labor employed in the previous period. Output produced by each firm is subject to a random shock or productivity factor (λ_j). While all entrepreneurs are assumed to know the distribution of shocks, they do not know the actual value of the shocks that will impinge upon their output during the current and future periods. Specifically,

$$(2) \quad Y_{j,t} = \lambda_{j,t} f_{j,t} (K_{j,t-1}, L_{j,t-1})$$

with $f_{j,t}$ exhibiting decreasing returns to scale 1/ and $\lambda_{j,t}$ being uniformly distributed between 0 and 1 for all firms. 2/

The rationale for borrowing in our model is to finance production, and given our traditional neoclassical production function with no material inputs, it is convenient to assume that labor must be paid at the beginning of each production period. 3/ The funds that entrepreneurs borrow from banks to finance their wage bills during period t ($B_{j,t}$) are obtained at the ceiling interest rate $R_{j,t}$ and are repaid at the beginning of the next period when the firm sells its output. 4/ The entrepreneur has the constraint that:

1/ As will be clarified below, if $f_{j,t}$ was linear homogeneous, an optimizing credit-rationed firm would choose to expand its capital and labor equi-proportionately with any expansion in credit. This would make the bank's expected return on credit independent of the volume of lending and, hence, for risk neutral banks, the optimal scale of lending would be indeterminate.

2/ Firms thus have identical density functions describing their production uncertainty ex ante but will generally differ in terms of the shocks that are realized. Firms' owners also have different consumption preferences, which may lead them to choose different amounts of capital and labor inputs.

3/ The model could be modified to interpret L as capital rented from households at the rental rate W, and K as capital owned by firms. This would require a respecification of the production function.

4/ Under this paradigm, but without loss of generality, all transactions during any period are viewed to take place at the beginning of the period, when all markets are open simultaneously.

(3) $B_{j,t} = W_t \, L_{j,t}$

where W_t is the wage rate in period t. In addition, the entrepreneur knows that, when interest rate ceilings are binding, the amount of credit that will be made available to him will be rationed by any bank he approaches. Thus:

(4) $B_{j,t} \leq \bar{B}_{j,t}$

where $\bar{B}_{j,t}$ is the maximum amount of credit that banks will supply to firm j. 1/ As will be shown, $\bar{B}_{j,t}$ is determined by the bank's credit rationing decision, which is influenced by interest rate ceilings and other prevailing financial policies.

In addition to placing demands for labor and credit, entrepreneurs formulate plans for consumption ($C_{j,t}$) and investment ($K_{j,t} - K_{j,t-1}$). For simplicity, it is assumed that capital does not depreciate over time, but that once purchased it cannot be resold, either because it is bolted into place or because it is otherwise transformed into plant and equipment that can only be used productively by the specific firm to which it belongs. The firm's budget constraint can be written as

(5) $C_{j,t} + K_{j,t} - K_{j,t-1} = Y_{j,t} - (1+R_{t-1}) \frac{W_{t-1}}{P_t} L_{j,t-1}$

where P_t is the price of goods in period t and it is understood that $C_{j,t}$ and $K_{j,t}$-$K_{j,t-1}$ must each be non-negative. 2/

In formulating his plans, the entrepreneur recognizes that some production shocks will leave the firm unable to service its debt obligations out of the proceeds from the sale of its output. Let $\lambda^*_{j,t+1}$ denote the

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1/ It is assumed that all banks would treat the firm exactly the same in terms of the total amount of credit that would be made available to the firm. It is also assumed that banks have complete information about the loans extended by other banks, and that, even if the firm borrowed from several banks, the total credit available would remain fixed, since no individual bank's loan would be regarded as "senior" to any other bank's loan.

2/ We do not, however, rule out reductions in K when we analyze in Section III the implications of policy changes on the steady state level of the capital stock. This implicitly assumes that entrepreneurs can abandon capital that is no longer economically viable, but cannot sell capital to increase their consumption.

scale of the shock or productivity factor for which the market value of the firm's entire output is just sufficient to meet its debt obligations. Thus:

$$(6)\quad \lambda^{*}_{j,t+1} = \frac{(1+R_t)(B_{j,t}/P_{t+1})}{f_{j,t+1}}$$

When the firm experiences a productivity factor less than $\lambda^{*}_{j,t+1}$, the firm is considered to be in "default" and the entrepreneur "dies"--that is, his current and future consumption levels are zero. 1/

The entrepreneur's optimal choice of consumption, capital, and labor in this uncertain environment corresponds to the plan that maximizes the value function (indirect utility function) defined by:

$$(7)\quad V_j(\cdot_t) = \max E_t[U_{j,t} + \beta^F_j V_j(\cdot_{t+1})] + \phi_j \left[\frac{\bar{B}_{j,t}}{P_{t+1}} - \frac{W_t L_{j,t}}{P_{t+1}} \right]$$

subject to the budget constraint given in (5), the production function given in (2), and the utility function described in (1), where $V_j(0) = 0$ and the argument of the value function is given by:

$$(8)\quad V_j(\cdot_t) = V_j\ (C_{j,t} + K_{j,t})$$

In equation (7), β^F_j is the firm's discount factor ($0 < \beta^F_j < 1$), E is the expectations operator, ϕ_j reflects the shadow price of relaxing the credit rationing constraint given in (4), and the working capital requirement described by (3) has been used to substitute for $B_{j,t}$.

As shown in Appendix II, the first order conditions for a maximum imply that the expected marginal product of capital, conditional on the firm's survival, must equal the reciprocal of the entrepreneur's marginal rate of time preference:

$$(9)\quad \frac{1}{\beta^F_j} = \int_{\lambda^{*}_{j,t+1}}^{1} \left[1 + \lambda_{j,t+1} \frac{\partial f_{j,t+1}}{\partial K_{j,t}}\right] d\lambda_{j,t+1}$$

1/ If the firm could sell its capital for a fraction ξ of the price of output, the critical value of the productivity factor would be $\lambda^{*}_{j,t+1} = [(1+R_t)(B_{j,t}/P_{t+1}) - \xi K_{j,t}]/f_{j,t+1}$.

When the credit rationing constraint is binding, the entrepreneur's constrained demand for labor is simply $L_{j,t} = \bar{B}_{j,t}/W_t$. Equation (9) then implies that the entrepreneur's demand for capital can be written as a negative function of both the expected real wage (W_t/P_{t+1}) and the loan rate $(1+R_t)$ and, under normal conditions, a positive function of the real stock of credit made available to the firm. Thus:

$$(10)\quad K_{j,t} = K_{j,t}\left(\overset{(-)}{\frac{W_t}{P_{t+1}}},\ \overset{(-)}{1+R_t},\ \overset{(+)}{\frac{\bar{B}_{j,t}}{P_{t+1}}}\right) \quad \text{for } \bar{B}_{j,t} \text{ binding}$$

where (+) or (-) above a variable indicates the sign of the partial derivative of $K_{j,t}$ with respect to that variable. If the credit rationing constraint was not binding, the entrepreneur's behavior would be described by his notional demands for both capital and labor, which normally are negatively related to both the expected real wage and the loan rate.

$$(11)\quad K_{j,t} = K_{j,t}\left[\overset{(-)}{\frac{W_t}{P_{t+1}}},\ \overset{(-)}{1+R_t}\right] \quad \text{for } \bar{B}_{j,t} \text{ not binding}$$

$$(12)\quad L_{j,t} = L_{j,t}\left[\overset{(-)}{\frac{W_t}{P_{t+1}}},\ \overset{(-)}{1+R_t}\right] \quad \text{for } \bar{B}_{j,t} \text{ not binding}$$

2. Banks

Banks facilitate the savings and investment process by providing entrepreneurs with working capital and by providing the savings instrument (deposits) that allows households to transfer consumption over time. Loans and deposits constitute the only financial instruments in our economy; workers do not acquire equity claims on the firms' capital and firms do not issue debt securities. These simplifying assumptions seem in line with the observed limited development of markets for equities and securities in most financially-repressed developing countries.

The owners of banks must make decisions regarding the optimal scales of their intermediation activities during the current period as well as intertemporal consumption decisions. 1/ At the beginning of period t, the owner of bank i inherits an equity position, $S_{i,t-1}$, a stock of loans to firms made at the beginning of the previous period, $\sum_{j=1}^{n} B_{i,j,t-1}$, and a stock of deposits accepted at the beginning of the previous period, $D_{i,t-1}$. It is assumed that the authorities require that a fraction k of all deposits must be held in the form of noninterest-bearing reserves at the central bank. Since the owners of the banks are assumed to be risk neutral, they will not hold any excess reserves (see Appendix III). The balance sheet constraint for period t-1 is thus

$$(13) \quad \sum_{j=1}^{n} B_{i,j,t-1} = (1-k)\ D_{i,t-1} + S_{i,t-1}.$$

At the beginning of period t, firms will use their proceeds from selling output to service the debts incurred during period t-1 (as well as to fund their consumption and investment purchases). 2/ From those firms that have sufficient output to meet their full debt payments and thus avoid default, the bank will receive interest income and loan repayments equal (in real terms) to $\sum_{j\epsilon N_1} (1+R_{t-1})(B_{i,j,t-1}/P_t)$, where N_1 is the set of firms that do not default. For those firms that default (i.e., for which $\lambda_{j,t} < \lambda^*_{j,t}$), the bank receives the output of the firms $\sum_{j\epsilon N_2} \left[\lambda_{j,t}\ f_{j,t}\right]$, where N_2 is the set of firms that default.

In making loans to entrepreneurs, the banks engage in both ex ante evaluation and ex post monitoring of their borrowers. When a bank receives a loan application from an entrepreneur, it evaluates the firm's production and investment plans so as to determine its vulnerability to potential production shocks. It is assumed that this requires an expenditure of resources, through which ex ante evaluation the bank essentially acquires

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1/ We simplify by assuming that each bank has a single owner and each owner owns only one bank.

2/ The analysis is simplified by assuming that all debts mature in one period.

full information about the firm. The bank is thus able to use equation (6) to calculate the range of shocks that would lead the entrepreneur to default on his debt-servicing obligations, but it does not know ex ante the actual shock that will occur for the firm during the period. In addition, when an entrepreneur reports at the end of the period that he cannot meet his debt-service obligations, the bank engages in ex post monitoring to ensure that it is being provided with accurate information. Such ex post monitoring, however, is unnecessary for those firms that meet their entire debt-servicing obligations. 1/ The sum of the evaluation and monitoring costs that bank i incurs can be represented, in real terms, as:

$$(14) \quad M_i = m_0 + m_1 \sum_{j=1}^{n} \frac{B_{i,j,t-1}}{P_t} + m_2 n_2$$

where n is the total number of loans and n_2 is the number of firms that default (i.e., the number of firms in the set N_2). The first two terms on the right side of (14) represent the fixed and proportionate costs of ex ante evaluation. The third term represents the cost associated with ex post monitoring. 2/

Banks must fully service their deposit obligations in each period. During period t, this involves payment of interest and repayment of principal totalling $(1+r_{t-1})(D_{i,t-1}/P_t)$, where r_{t-1} is the interest rate on the deposits. 3/ To meet part of these payments, bank i can make use of its reserve holdings $(kD_{i,t-1}/P_t)$. The remainder must come from loan payments received from firms. Any profits from the bank's operations are used by the owner to purchase consumption goods $(C_{i,t})$ and to add to his real equity $(S_{i,t}/P_t)$ in the bank. Equity funds can be lent out to firms, which provides the bank's owner with an incentive to accumulate such funds, especially if his ability to attract deposits is limited by a ceiling on deposit interest rates.

The net profit or loss position that bank i experiences at the beginning of period t $(\Pi_{i,t})$ reflects both the financial decisions taken

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1/ Ex post monitoring, with penalties for firms caught cheating, solves a moral hazard problem since it gives entrepreneurs the incentive to reveal their true outcomes.

2/ Economies of scale in monitoring, as reflected in the fixed cost term m_0, provide a rationale for why many economic agents (including those without sufficient savings to meet minimum equity requirements) do not engage in intermediation activities.

3/ The assumption that banks are obligated to repay deposits fully at the beginning of each period is symmetric with the assumption that all loans mature in one period.

during period t-1 and the values of production shocks realized in period t. This net position can be written as:

$$(15) \quad \Pi_{i,t} = \sum_{j \epsilon N_1} (1+R_{t-1}) \left(\frac{B_{i,j,t-1}}{P_t}\right) + \sum_{j \epsilon N_2} \left[\lambda_{j,t} f_{j,t} \right]$$

$$- m_0 - m_1 \sum_{j=1}^{n} \frac{B_{i,j,t-1}}{P_t} - m_2 n_2 - (1+r_{t-1}-k)(D_{i,t-1}/P_t)$$

For simplicity, it is assumed that the owner's utility during period t is linearly related to the level of his consumption:

$$(16) \quad U_{i,t} = \gamma_i^I C_{i,t}$$

The choices that the owner must make in period t, subject to various constraints, are his current level of consumption ($C_{i,t}$), his equity in the bank ($S_{i,t}$), the amount of deposits to raise ($D_{i,t}$), and the amount of lending that he will make to each of the firms ($B_{i,j,t}$). The owner's consumption level and equity holdings must be non-negative and are thus jointly constrained by: 1/

$$(17) \quad C_{i,t} + \frac{S_{i,t}}{P_t} = \max \left[0, \Pi_{i,t}\right]$$

The financial variables he must choose are subject to his balance sheet constraint (condition 13) and a regulatory requirement that his equity exceed some minimum proportion s of his loans:

$$(18) \quad S_{i,t} \geq s \sum_{j=1}^{n} B_{i,j,t}$$

1/ Note from (15) and (13) that the bank's net profit position $\Pi_{i,t}$ reflects its equity holdings in the previous period, $S_{i,t-1}$. The bank's owner must thus allocate profits between consumption flows and the accumulation of equity stocks over time, just as the firm's entrepreneur allocates profits between consumption and the accumulation of physical capital stocks.

Moreover, in the presence of financial market restrictions, the bank knows that the amount of deposits it can raise will sometimes be limited to a ceiling level $\bar{D}_{i,t}$, such that

$$(19) \quad D_{i,t} \le \bar{D}_{i,t}$$

In general, the ceiling level of deposits that the bank can raise from households will depend on the levels of the parameters that the government controls, including in particular the ceiling interest rate on deposits. 1/

Ex ante, the profit or loss that bank i expects in period t+1 ($E_t\Pi_{i,t+1}$) can be described as:

$$(20) \quad E_t\Pi_{i,t+1} = \sum_{j=1}^{n} (1+R_t)\int_{\lambda^*_{j,t+1}}^{1} \frac{B_{i,j,t}}{P_{t+1}} \, d\lambda_{j,t+1}$$

$$+ \sum_{j=1}^{n} \int_{0}^{\lambda^*_{j,t+1}} [\lambda_{j,t+1} \, f_{j,t+1}] \, d\lambda_{j,t+1}$$

$$- (1+r_t-k) \frac{D_{i,t}}{P_{t+1}} - m_o - m_1 \sum_{j=1}^{n} \frac{B_{i,j,t}}{P_{t+1}} - m_2 \sum_{j=1}^{n} \lambda^*_{j,t+1}$$

where $\sum_{j=1}^{n} \lambda^*_{j,t+1}$ represents the expected number of defaults. This expected profit or loss position reflects both the choices that the bank makes in period t and the probability distributions of the production shocks that will be experienced by the firms to which the bank lends. The first two terms on the right hand side of equation (20) represent the expected revenues from lending. These terms reflect full repayment from any firm j that experiences a shock in the range $\lambda^*_{j,t+1}$ to 1 and partial repayment

1/ It is implicitly assumed that banks raise deposits from fixed and mutually exclusive sets of households, and thus do not effectively compete with each other for deposits. This assumption might be rationalized as an equilibrium outcome in the presence of transactions costs. It may also be noted that firms have no incentives to hold deposits when R > r.

(equal to $\lambda_{j,t+1} f_{j,t+1}$) where shocks are in the range 0 to $\lambda^*_{j,t+1}$. The third term represents principal and interest payments that must be made to depositors, minus the bank's required reserve holdings (which are available to repay deposits). The final three terms are the expected costs of evaluation and monitoring.

For purposes of this paper we restrict attention to the case in which households' deposits with banks are insured by the government, but in a manner that limits any moral hazard problem for banks. Accordingly, it is assumed that the authorities supervise each bank and impose a "prudent man rule" which effectively forces the bank's owner to maximize the present discounted value of a utility function that gives equal weight to his private consumption and any losses that his activities might force the government to absorb through the deposit insurance system. More precisely, the "prudent man rule" completely internalizes the negative externalities by equating the marginal disutility of government losses to the marginal utility of the bank owner's private gains: 1/

$$(21) \quad \tilde{U}_{i,t} = \begin{cases} \gamma_i^I C_{i,t} & \text{when } \Pi_{i,t} > 0 \\ \gamma_i^I \Pi_{i,t} & \text{when } \Pi_{i,t} \leq 0 \end{cases}$$

This rule is therefore equivalent to establishing an appropriate risk-based pricing scheme for deposit insurance.

The owner's optimal consumption, equity, and financial intermediation decisions under the "prudent-man" supervisory system are those that maximize the value function (indirect utility function) defined by:

1/ This prudent man rule effectively requires that, when the bank's owner is calculating the expected return from lending to a given firm, he must take into account the full range of losses that could occur, even if some of these losses were to be such that the authorities would have to step in and rescue the bank or protect the depositors.

We plan to consider two alternative supervisory standards and the appropriate pricing of deposit insurance in future work. One standard is the case in which the government insures deposits without imposing a prudent man rule, and in which the bank faces no penalty for incurring a loss in period t+1 (other than realizing $C_{i,t+1} = S_{i,t+1} = 0$); in this case, expected-utility maximization will typically lead the bank to lend more than in the case with no moral hazard problem. The second standard is the case in which the government insures deposits without imposing a prudent man rule, but the bank faces the "death penalty" (i.e., $C_{i,\tau} = S_{i,\tau} = 0$ for all $\tau \geq t+1$) for incurring a loss in period t+1.

$$(22) \quad V_i(\cdot_t) = \max E_t(\tilde{U}_{i,t} + \beta_i^I V_i(\cdot_{t+1})) - \phi_i^S \left(s \sum_{j=1}^{n} \frac{B_{i,j,t}}{P_{t+1}} - \frac{S_{i,t}}{P_{t+1}} \right)$$

$$+ \phi_i^D \left(\frac{\bar{D}_{i,t}}{P_{t+1}} - \sum_{j=1}^{n} \frac{B_{i,j,t}}{(1-k)P_{t+1}} + \frac{S_{i,t}}{(1-k)P_{t+1}} \right)$$

where the argument of the value function is given by:

$$(23) \quad V_i(\cdot_t) = V_i(\Pi_{i,t})$$

and $V_i(0)=0$. The terms ϕ_i^S and ϕ_i^D are Lagrangian multipliers associated with the minimum equity requirement (18) and the upper bound on deposits (19). 1/

The bank monitors each firm's investment and output plans when a loan application is received, and it is assumed that the bank's owner maximizes his value function with full information about the ex ante decisions that firms will make under different conditions. We restrict attention in this paper to the analysis of financially repressed economies in which the existence of interest rate ceilings and other constraints leads to disequilibrium credit rationing; this is the case in which constraint (4) is binding and the behavior of firm j is characterized by (3), (4), and (10). The level of credit that firm j receives in period t is in this case one of the choice variables of bank i ($\bar{B}_{j,t} = B_{i,j,t}$). 2/ The bank, in maximizing its value function, essentially uses the information summarized by (3) and (10) to evaluate how the choice of its decision variables will influence its expected profits (as described by equation (20)). For the case of disequilibrium credit rationing, it is convenient to characterize the bank's behavior in terms of its choices for the $B_{i,j,t}/P_{t+1}$ and $S_{i,t}/P_{t+1}$. The implied level of deposits that the bank must raise is then

1/ We have substituted for $D_{i,t}$ using (13). Note also that when $\Pi_{i,t} \geq 0$, equation (17) implies $(\cdot_t) = \Pi_{i,t} = C_{i,t} + S_{i,t}/P_t$.

2/ Recall that, for simplification, we assume that each firm deals with only one particular bank. Since banks have full information and are essentially identical, the matching between firms and banks is arbitrary, and information sharing by banks would prevent any firm from borrowing more from two separate banks than it could borrow from either individually.

described by (13), 1/ and the implied level of the owner's consumption is described by (17).

It should be emphasized that, even though each firm deals with only one bank, and even though the bank through its ex ante evaluation activities obtains full information about the ex ante behavior of firms, the financial market environment reflects competitive conditions. 2/ Banks offer the standard type of loan contracts found in competitive markets, rather than seeking to extract all the profits that borrowers can earn. Indeed, in choosing the levels of its loans and deposits, the individual bank takes as given the ceiling interest rates on loans and deposits. 3/

The first-order conditions for bank's optimal level of real lending to each firm j ($B_{i,j,t}/P_{t+1}$) and real equity position $\left(\frac{S_{i,t}}{P_{t+1}}\right)$ can be written as

$$(24)\quad 0 = \frac{\partial V_i(\cdot_t)}{\partial\left(\frac{B_{i,j,t}}{P_{t+1}}\right)} = \beta_i^I E_t \left\{\frac{\partial V_i(\cdot_{t+1})}{\partial\left(\frac{B_{i,j,t}}{P_{t+1}}\right)}\right\} - \frac{\phi_i^D}{1-k} - s\,\phi_i^S$$

$$(25)\quad 0 = \frac{\partial V_i(\cdot_t)}{\partial\left(\frac{S_{i,t}}{P_{t+1}}\right)} = -\gamma_i^I(1+\rho_{t+1}) + \beta_i^I E_t \left\{\frac{\partial V_i(\cdot_{t+1})}{\partial\left(\frac{S_{i,t}}{P_{t+1}}\right)}\right\} + \frac{\phi_i^D}{(1-k)} + \phi_i^S$$

where $1+\rho_{t+1} = P_{t+1}/P_t$ denotes the expected rate of inflation and letting z denote either choice variable: 4/ 5/

1/ Note, however, that in the presence of a binding ceiling on the deposit interest rate, the bank's choices will be constrained by an upper bound on the quantity of deposits it can raise, as expressed in (19).

2/ There are many banks and, implicitly, if one bank attempted to exercise market power, its borrowers could apply for loans from other banks.

3/ We would be inclined to define the bank's set of choice variables differently for the case in which the financial system is not repressed, since in the absence of disequilibrium credit rationing the firm's behavior would be characterized by (3), (11), and (12) rather than (3), (10) and the equality in (4).

4/ This uses the Benveniste-Scheinkman condition, as discussed by Sargent (1987), pp. 21-22.

$$(26) \quad E_t \left\{ \frac{\partial V_i(\cdot_{t+1})}{\partial Z} \right\} = E_t \left\{ \frac{\partial V_i(\cdot_{t+1})}{\partial (\cdot_{t+1})} \frac{\partial (\cdot_{t+1})}{\partial Z} \right\} = \gamma_i^I E_t \left\{ \frac{\partial \Pi_{i,t+1}}{\partial Z} \right\}$$

As shown in Appendix III, these conditions imply: 1/

$$(27) \quad 0 = \gamma_i^I \beta_i^I \Bigg\{ (1-\lambda^*_{j,t+1})(1+R_t) + \frac{\lambda^{*2}_{j,t+1}}{2} \frac{\partial f_{j,t+1}}{\partial L_{j,t}} \frac{P_{t+1}}{W_t}$$

$$+ \frac{\lambda^{*2}_{j,t+1}}{2} \frac{\partial f_{j,t+1}}{\partial K_{j,t}} \frac{\partial K_{j,t}}{\partial (B_{j,t}/P_{t+1})}$$

$$- \frac{1+r_t-k}{1-k} - m_1 - m_2 \frac{\partial \lambda^*_{j,t+1}}{\partial (B_{j,t}/P_{t+1})} \Bigg\} - s\,\phi_i^S - \frac{\phi_i^D}{1-k}$$

and

$$(28) \quad 0 = \gamma_i^I \beta_i^I \frac{(1+r_t-k)}{(1-k)} + \phi_i^S + \frac{\phi_i^D}{1-k} - \gamma_i^I (1+\rho_{t+1}).$$

Equations (27) and (28) can be explored under different combinations of the constraints that may be binding on the bank. When the structure of official constraints leads to both a binding ceiling on deposit availability ($\phi_i^D > 0$) and creates an incentive to hold only the minimum level of equity

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5/ (...continued)

5/ Recalling the footnote accompanying equation (21), note that, in the absence of a prudent man rule and the presence of a death penalty for incurring a loss, the utility function relevant to the optimization problem is (16) rather than (21), but the relevant argument in $V_i(\cdot_t)$ is $(\cdot_t) = C_{i,t} + (S_{i,t}/P_t) - \max[0, \Pi_{i,t}]$ rather than $\Pi_{i,t}$. Thus, the first-order conditions depend on $E\left\{\frac{\partial \Pi_{i,t+1}}{\partial Z} \,|\Pi_{i,t+1} \geq 0\right\}$, and the calculus of characterizing the bank's optimal behavior becomes much more complex. In Section IV we make strong simplifying assumptions to avoid this difficulty. In general, however, a bank operating in such an environment would probably have to base its lending on reasonable ad hoc rules rather than a complete optimization calculation.

1/ The derivations use (6). In addition, since firm j only receives loans from bank i, $B_{i,j,t} = B_{j,t}$.

($\phi_i^S > 0$), 1/ then the total amount of lending by the bank will be constrained as:

$$(29) \quad \sum_j \frac{B_{i,j,t}}{P_{t+1}} = \frac{(1-k)}{(1-s)} \frac{\bar{D}_{i,t}}{P_{t+1}} \qquad \text{for } \phi_i^D > 0,\ \phi_i^S > 0.$$

In this situation, the amount of lending to each individual firm will differ from the notional amount that the bank would lend (at the prevailing ceiling loan interest rate) if it could obtain all the deposits it wished at the prevailing deposit interest rate. The actual lending to each firm will be such that the shadow price of an additional dollar of lending to any firm will be equalized across the portfolio.

Disequilibrium credit rationing can also emerge when the bank faces a binding ceiling on deposit availability ($\phi_i^D > 0$) but nevertheless has an incentive to expand its equity position beyond the required minimum level ($\phi_i^S=0$). Equation (27) indicates that the bank will increase its lending to firm j until the discounted marginal revenue from lending an additional dollar just matches the marginal cost (in terms of the consumption foregone to raise an additional dollar of equity). An extra dollar of lending increases the expected revenue of the bank since it allows the credit constrained firm to expand its output by employing more labor. Moreover, as shown in the previous section, a larger amount of credit would also induce the owners of firms to increase the firm's capital stock and thereby its output. Since the production shock for firm j is uniformly distributed between 0 and 1, $1-\lambda^*_{j,t+1}$ represents the ex ante probability that the bank will be fully repaid on its loan; the associated expected marginal revenue would be $(1-\lambda^*_{j,t+1})(1+R_t)$. If the firm defaults, however, the bank would obtain whatever output is produced. The second and third terms inside the brackets in equation (27) represent the effect of additional lending on the expected value of the output that would be available if the firm defaults. 2/ An additional real dollar of lending also affects expected monitoring costs both immediately (the m_1 term), and by altering the probability of default by $\partial\lambda^*_{j,t+1}/\partial(B_{j,t}/P_{t+1})$. It is easily shown by differentiating (6) that $\partial\lambda^*_{j,t+1}/\partial(B_{j,t}/P_{t+1}) > 0$ for the "normal case" in

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1/ This reflects a situation where the expected profit that can be made from creating an extra dollar of equity (which is the rate at which consumption can be transferred from t to t+1) is less than the owner's internal rate of discount.

2/ The second and third terms equal

$$\int_0^{\lambda^*_{j,t+1}} \lambda_{j,t+1} \frac{\partial f_{j,t+1}}{\partial(B_{j,t}/P_{t+1})} d\lambda_{j,t+1} \text{ where } B_{j,t}/P_{t+1} = (\frac{W_t}{P_{t+1}}) L_{j,t}.$$

which the total elasticity of output with respect to real credit availability is less than one. 1/

Equation (27) implies that, when banks hold more equity than the required minimum level, the constrained supply of loans to each firm is a function of the loan interest rate, the real wage, the monitoring cost parameters, and the expected rate of inflation: 2/

$$(30)\quad \frac{B_{j,t}}{P_{t+1}} = \frac{B_{j,t}}{P_{t+1}}\left[\overset{(?)}{1+R_t},\ \overset{(-)}{\frac{W_t}{P_{t+1}}},\ \overset{(-)}{m_1},\ \overset{(-)}{m_2},\ \overset{(-)}{\rho_{t+1}}\right] \text{ for } \phi_i^D > 0,\ \phi_i^S = 0.$$

The signs of the partial derivatives are derived in Appendix III.

In this situation, default risk implies that the bank's supply of credit to each particular firm is likely to be a backward bending function of the loan rate. At any given loan rate, the slope depends on whether the revenue associated with a larger loan (at a higher interest rate) would be offset (in an expected value sense) by a higher probability that the entrepreneur would default (which would imply a loss of revenue and higher ex post monitoring costs). At a sufficiently high loan rate, the bank would not be willing to lend additional funds to an entrepreneur, and might even reduce the amount of lending relative to the desired level of constrained lending at a lower interest rate--i.e., the constrained supply of loans becomes backward bending. 3/ Notice that, since entrepreneurs may differ in the scales of their investment and production plans (for example, due to

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1/ This will be the case whenever the partial elasticities of output with respect to both labor and capital, along with the elasticity of the firm's demand for capital with respect to real credit availability, are all less than one. Most models of the demand for bank loans in developing countries assume that the demand for working capital is inelastic in the short run with regard to all of its arguments.

2/ Just as the firm's constrained demands for capital and labor differ from its notional demands (compare, e.g., (10) and (11), the bank's constrained supply of loans differs from the notional supply it would make available if it could raise an unlimited amount of deposits under prevailing market conditions.

3/ The backward bending relationship between $B_{j,t}/P_{t+1}$ and $1+R_t$ plays an important role in our analysis, and in the analysis of credit rationing behavior in general. Although the derivation is lengthy (see Appendix III), some intuition can be obtained by focusing on $(1-\lambda^*_{j,t+1})(1+R_t)$, the contractual payment due to the bank multiplied by the probability of non-default. It is easy to show that this component of the bank's expected revenue is backward bending: as R_t rises, the probability of default also rises, and beyond some point $(1-\lambda^*_{j,t+1})(1+R_t)$ begins to decline.

different marginal utilities of consumption), the amounts that a bank is willing to lend to different entrepreneurs at a given interest rate also will differ.

Although the partial derivative of $B_{j,t}/P_{t+1}$ with respect to W_t/P_{t+1} is ambiguous, under reasonable assumptions, particularly when the probability of default ($\lambda^*_{j,t+1}$) is relatively small, a higher real wage will reduce the bank's desired amount of lending. Higher monitoring costs will also reduce the bank's desired amount of lending. A higher real wage effectively increases the probability that the firm will default on its debt-service obligations, whereas higher monitoring costs imply lower net returns from lending.

A higher expected rate of inflation will also reduce the attractiveness of additional lending to any firm, essentially by increasing the amount of nominal equity that will be needed to fund a loan. Since the bank focuses on providing a loan whose real value ($B_{i,j,t}/P_{t+1}$) is measured in terms of the price level in period t+1, a higher expected rate of inflation means that the bank will have to accumulate a greater stock of equity at time t. Since this implies a lower level of consumption in period t, there is an incentive to reduce real lending as expected inflation rises.

3. Households

Households supply labor services to firms, hold deposits with banks, and attempt to maximize the expected discounted value of the utility of their consumption over time. The representative household's utility during period t ($U_{h,t}$) is taken as a positive function of the level of its consumption ($C_{h,t}$) a negative function of the labor services ($L_{h,t}$) it supplies, and a positive function of the real value of its deposit holdings. 1/ Thus:

$$(31) \quad U_{h,t} = U_{h,t}(C_{h,t},\ L_{h,t},\ D_{h,t}/P_{t+1})$$

with $\dfrac{\partial U_{h,t}}{\partial C_{h,t}} > 0,\ \dfrac{\partial U_{h,t}}{\partial L_{h,t}} < 0,\ \dfrac{\partial U_{h,t}}{\partial (D_{h,t}/P_{t+1})} > 0$

1/ The assumption that utility is generated by real money holdings is sometimes justified by emphasizing that the use of money reduces transactions costs relative to a situation of barter. An alternative justification is based on the view that the household's utility in period t depends not only on its present consumption, but also on the degree of security it has about its future consumption possibilities; in this context, savings set aside in the form of deposits provides current utility (or a sense of well being), even though the deposit will not be spent on consumption goods until the future.

$$\frac{\partial^2 U_{h,t}}{\partial C^2_{h,t}} < 0, \; \frac{\partial^2 U_{h,t}}{\partial L^2_{h,t}} < 0, \; \frac{\partial^2 U_{h,t}}{\partial \left(\frac{D_{h,t}}{P_{t+1}}\right)^2} < 0, \; \frac{\partial^2 U_{h,t}}{\partial C_{h,t} \partial L_{h,t}} = 0$$

$$\frac{\partial^2 U_{h,t}}{\partial C_{h,t} \partial (D_{h,t}/P_{t+1})} = 0, \; \frac{\partial^2 U_{h,t}}{\partial L_{h,t} \partial (D_{h,t}/P_{t+1})} = 0$$

The household's budget constraint implies that its consumption plus whatever additions it makes to its deposit holdings during period t must equal the sum of the interest it earned on its deposits during period t-1 and its wage income.

$$(32) \quad C_{h,t} + \frac{D_{h,t}}{P_t} - \frac{D_{h,t-1}}{P_t} = r_{t-1} \frac{D_{h,t-1}}{P_t} + \frac{W_t}{P_t} L_t$$

The household's optimal consumption, labor supply, and saving decisions are those that maximize the value function

$$(33) \quad V_h(\cdot_t) = \max E_t \left\{ U_{h,t}(C_{h,t}, L_{h,t}, \frac{D_{h,t}}{P_{t+1}}) + \beta^H_h V_h(\cdot_{t+1}) \right\}$$

where:

$$(34) \quad V_h(\cdot_t) = V_h \left(C_{h,t} + \frac{D_{h,t}}{P_t} - \frac{W_t L_{h,t}}{P_t}\right) = V_h \left((1+r_{t-1}) \frac{D_{h,t-1}}{P_t}\right)$$

The first order conditions for a maximum are:

$$(35) \quad \frac{\partial V_h(\cdot_t)}{\partial L_{h,t}} = 0 = \frac{\partial U_{h,t}}{\partial C_{h,t}} \frac{W_t}{P_t} + \frac{\partial U_{h,t}}{\partial L_{h,t}}$$

$$(36) \quad \frac{\partial V_h(\cdot_t)}{\partial \left(\frac{D_{h,t}}{P_{t+1}}\right)} = 0 = -(1+\rho_{t+1}) \frac{\partial U_{h,t}}{\partial C_{h,t}} + \beta^H_h (1+r_t) \frac{\partial U_{h,t+1}}{\partial C_{h,t+1}} + \frac{\partial U_{h,t}}{\partial (D_{h,t}/P_{t+1})}$$

As shown in Appendix IV, these conditions imply that the household's steady state real deposit holdings will be a positive function of both the deposit interest rate and the real wage, and a negative function of the expected rate of inflation. Moreover, its steady state demand for consumption goods will be a positive function of the real wage. The effects of changes in the deposit interest rate and the expected rate of inflation on steady state consumption are ambiguous, however, when there is a binding ceiling on the deposit interest rate. Similarly, the desired supply of labor services will normally be a positive function of the real wage but, under a binding deposit rate ceiling, could be either a positive or negative function of the deposit interest rate and the expected rate of inflation. Thus,

$$(37)\quad \frac{D_{h,t}}{P_{t+1}} = \frac{D_{h,t}}{P_{t+1}}\left(\overset{(+)}{r_t}, \overset{(+)}{\frac{W_t}{P_t}}, \overset{(-)}{\rho_{t+1}}\right)$$

$$(38)\quad C_{h,t} = C_{h,t}\left(\overset{(?)}{r_t}, \overset{(+)}{\frac{W_t}{P_t}}, \overset{(?)}{\rho_{t+1}}\right)$$

$$(39)\quad L_{h,t} = L_{h,t}\left(\overset{(?)}{r_t}, \overset{(+)}{\frac{W_t}{P_t}}, \overset{(?)}{\rho_{t+1}}\right)$$

If the deposit rate exceeds the expected rate of inflation, $C_{h,t}$ will depend positively on r_t and negatively on ρ_{t+1}, as the income effect will outweigh the substitution effect, while $L_{h,t}$ will depend negatively on r_t and positively on ρ_{t+1}.

III. Steady State Solutions

Financial regulations and creditworthiness considerations will be key determinants of the long-run behavior of a financially repressed economy. Since our analysis focuses on an economy where the authorities establish ceiling loan and deposit interest rates that are below market clearing levels, entrepreneurs, bank owners, and households will not all simultaneously achieve their desired spending and portfolio plans. In particular, firms that are credit rationed will be unable to employ the level of labor that they would find profitable to use at the prevailing real wage and loan interest rate. As a result, the level of employment and output will be constrained by credit availability. In addition, banks will be unable to obtain all the deposits that they would like to have at the prevailing ceiling deposit interest rate and the stock of real deposits will reflect the households' desired holdings of deposits.

1. Steady state relationships

Since the economy is subject to stochastic production shocks, the realized period-to-period outcomes for the economy will not converge to a steady state, but meaningful steady state solutions do exist for ex ante expectations of these outcomes and hence for the ex ante plans of economic agents. This section analyzes the long-run properties of our model in terms of the ex-ante plans formulated by the entrepreneurs, bank owners and households.

The economy's long-run behavior can be described in terms of four relationships (where all variables for period t+1 are specified in terms of their expected values at time t):

(40) $$L_t = \sum_h L^H_{h,t}(r_t, W_t/P_t, \rho_{t+1}) \leq \sum_j L^F_{j,t}(1+R_t, \frac{W_t}{P_{t+1}})$$

(41) (a) $$\frac{B_t}{P_{t+1}} = \sum_i \sum_j b_{i,j,t}(1+R_t, \frac{W_t}{P_{t+1}}, m_1, m_2, \rho_{t+1})$$

$$= \sum_h \frac{W_t}{P_{t+1}} L^H_{h,t}(r_t, W_t/P_t, \rho_{t+1})$$

(b) $$\frac{B_t}{P_{t+1}} = \sum_h \frac{(1-k)}{(1-s)} \frac{D^H_{h,t}}{P_{t+1}} \left[r_t, W_t/P_t, \rho_{t+1}\right]$$

$$= \sum_h \frac{W_t}{P_{t+1}} L^H_{h,t}(r_t, W_t/P_t, \rho_{t+1})$$

(42) (a) $$\sum_i \frac{S_{it}}{P_{t+1}} = \sum_h (1-k) \frac{D^H_{h,t}}{P_{t+1}} \left[r_t, W_t/P_t, \rho_{t+1}\right]$$

$$= \sum_i \sum_j b_{i,j,t}(1+R_t, \frac{W_t}{P_{t+1}}, m_1, m_2, \rho_{t+1})$$

(b) $$\sum_i \frac{S_{i,t}}{P_{t+1}} = s \sum_h \frac{(1-k)}{(1-s)} \frac{D^H_{h,t}}{P_{t+1}} \left(r_t, W_t/P_t, \rho_{t+1}\right)$$

(43) $$\frac{D_t}{P_t} = \sum_h \frac{D^H_{h,t}}{P_t} \left(r_t, W_t/P_t, \rho_{t+1}\right) = \frac{H_t/k}{P_t}$$

Equation (40) represents the relationship between the sum of the firms' <u>unconstrained</u> ex-ante demands (when they are not credit rationed) for labor and the sum of the households' desired supplies of labor. If the firms could obtain all the credit they desired at the prevailing interest rate, then $\sum_j L^F_{j,t}$ defines the amount of labor they would hire (given the existing real wage). However, when the banks credit ration the firms, firm j can hire only $L_{j,t} = B_{j,t}/W_t$ (equations (3) and (4)) where $B_{j,t}$ is the credit made available to it. The real wage will therefore adjust until the sum of the firms' credit-constrained demands for labor equals the households' desired supply of labor services $\sum_h L^H_{h,t}$.

Ex ante credit market equilibrium is achieved when the ex ante supply of bank credit is sufficient to support the anticipated real wage bill. However, as indicated by equations (41a) and (41b), this equilibrium could be achieved either when the banks' owners hold only the minimum officially required level of equity or when they hold more than the minimum required level of equity. Given the banks' cost structures and the ceiling deposit interest rate, there will be a range of low ceiling loan rates for which the bank owners will find it profitable to operate with only the minimum required level of equity (see Appendix V). In this situation, the banks' supply of credit will equal $\sum \frac{(1-k)}{(1-s)} D^H_{h,t}$, where k is the required reserve ratio and s is the minimum required ratio of bank equity to total lending. For the level of lending to support the real wage bill, equation (41b) must hold. There will also be a middle range of ceiling loan interest rates, however, for which bank owners will find it profitable to fund their lending activities by holding more than the minimum required level of equity. 1/ As a result, the banks will achieve their desired level of lending to each firm (equation (41a)) 2/ by substituting equity for deposits as a source

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1/ As will be discussed, there is also a third range of high ceiling loan interest rates for which the banks will again hold only the minimum amount of equity.

2/ The desired levels of lending by the banks to the firms in equation (41a) are defined in the situation where the banks credit ration the firms (see Appendix III).

of funding. For an ex ante credit market equilibrium, the sum of the banks' desired lending to firms must equal the economy's wage bill.

The level of equity in the banking industry is given in equations (42a) and (42b). Equation (42b) describes the level of equity when the banks hold only the minimum required level of equity; whereas equation (42a) indicates the level of equity when the banks hold more than the minimum required level of equity.

Equation (43) describes the conditions for ex ante money market equilibrium. Since households do not hold currency and banks do not hold excess reserves, the supply of deposits equals H_t/k, where H_t is the stock of base money. The household's real holdings of deposits are given by

$$\sum_h \frac{D^H_{h,t}}{P_t}.$$

Since we assume that the authorities, in issuing money to finance government spending, keep the stock of base money growing at a constant exogenous rate (which must equal the expected rate of inflation, ρ), the price level (P_t) will adjust to ensure that equation (43) is satisfied at each point in time.

The role that financial repression and creditworthiness considerations play in determining the economy's steady state position can be illustrated using Figure 1 (see Appendix V for derivations). In the northeast (NE) quadrant, curve 1 represents the combinations of the loan rate ($1+R_t$) and real wage (W_t/P_{t+1}) that would set the sum of the firms' <u>unconstrained</u> (not credit-rationed) demands for labor equal to the sum of the households' desired supplies of labor. This curve has a negative slope since a higher real wage, which would reduce the firms' demand for labor (and increase the households' supply), would have to be offset by a lower loan rate, which would increase the firms' demand for labor. Any point to the left of curve 1 represents a situation where the firms' unconstrained demand for labor exceeds the households' desired supply of labor.

Curve 2 in the NE quadrant represents the combinations of the loan rate and real wage that would lead banks--when firms are credit rationed--to provide a real supply of credit that equals the real wage bill. This curve has two segments: AB is relevant over the range of R in which banks would be induced to hold only the minimum required level of equity, and BC is relevant over the range in which banks would hold more than the minimum required level of equity. 2/

2/ Strictly speaking, the portion of BC that lies to the right of curve 1 is not relevant to the analysis, since curve 2 is drawn on the assumption that firms are credit rationed. As already noted, the firms' employment decisions would be affected by credit rationing only when they are operating in the region to the left of curve 1.

As shown in Appendix V, the banks' owners will hold the minimum required level of equity whenever the loan rate is between R_{**} and R_{*} (equation (41b) holds). 1/ For loan rates below R_{**} the banks' owners would not find it profitable to utilize all of the deposits the households would make available and the ceiling deposit rate would no longer be a binding constraint. 2/ Between R_{**} and R_{*}, the banks' owners would find it profitable to use all deposits made available by the households but would hold only the minimum amount of equity. 3/ In this situation, there is only one level of the real wage $(\frac{W}{P})$ that would ensure that the real supply of bank credit $\left(\sum_h \frac{(1-k)}{(1-s)} \frac{D^H_{h,t}}{P_{t+1}}\right)$ equals the real wage bill $\sum_h \left(\frac{W_t}{P_{t+1}}\right) L^H_{h,t}$. 4/

In the range BC of curve 2, the banks' owners would find it profitable both to use all deposits made available to them by the household sector and to hold more equity than the minimum level required by the authorities. This segment of curve 2 is positively sloped at relatively low values of R, since a higher real wage implies a large wage bill which the banks will fund only if the loan rate rises. 5/ In this range, the banks will see a higher expected profit from an addition dollar of lending since the firms will have relatively low debt servicing obligations (due to both a low loan interest rate and a small stock of loans). However, as the loan interest rate rises and the stock of loans held by firms expands, the probability that the firms will default on their debt servicing obligations will begin

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1/ R_{**} and R_{*} will vary from bank to bank unless the owners have identical internal rates of discount and marginal utilities of consumption.

2/ The region below R_{**} is therefore not relevant for the present analysis, since we are examining only the case where the authorities interest rate ceilings are binding.

3/ There is also a high interest rate R^* at which the banks would again want to hold only the minimum required level of equity. This reflects the fact that, as the loan interest rate rises, a point is reached at which the banks begin to see a negative expected return on lending an additional dollar even at a higher interest, since the firms would have higher probabilities of defaulting on their loans. R^* represents the loan rate at which these default probabilities are high enough so that the bank would no longer have an incentive to hold more than the minimum amount of equity.

4/ In Appendix V it is shown that (W/P) would increase with a higher ceiling deposit interest rate or minimum equity ratio, decline with a higher required reserve ratio, and potentially rise or fall with a higher rate of inflation. It can also be noted that point C lies vertically above point B, since the real wage rate that solves condition (41b) is unique for given levels of r_t and ρ_{t+1}.

5/ The stock of deposits made available to banks by households will also rise as the real wage increases, but not as rapidly as the real wage bill rises.

to rise; and, eventually, the banks' owners will see the expected return from lending an additional dollar, even at a higher interest rate, turn negative.

Curve 3 in the northwest (NW) quadrant of Figure 1 indicates the amount of real credit that banks would extend at different loan interest rates. The curve has two segments: one where the banks hold only the minimum required level of equity (DE) and another where they hold more than the minimum required level of equity (EF). As already discussed in the case of curve 2, there is only one real wage $(\bar{W}/P)$ at which the stock of real credit $\left[\sum_h \frac{(1-k)}{(1-s)} \frac{D_h^H}{P_{t+1}}\right]$ will equal the real wage bill when the banks' owners hold only the minimum required level of equity. Given the values of $(\bar{W}/P)$, k, s, ρ_{t+1}, and r_t, the real supply of credit will therefore be fixed until the loan rate rises to a level high enough to induce the banks' owners to hold more than the minimum required level of equity. Thus, just as in the case of curve 2, curve 3 has a vertical segment (DE) in the range of loan rates between R_{**} and R_*. 1/

Segment EF of curve 3 describes the supply of real credit at different loan interest rates when the banks' owners find it profitable to hold more than the minimum required level of equity. However, when banks have more than the minimum required level of equity, it has already been noted that increases in the loan interest rate (R) will change the real desired supply of bank credit and thereby the real wage bill that can be funded. As a result, the real wage would also change. Since curve 3 portrays the banks' desired real supply of lending solely as a function of the loan interest rate, the relationship between R_t and W_t/P_{t+1} implicit in curve 2 must be used to describe the W_t/P_{t+1} that would prevail at each value of R_t. This relationship can be substituted into the banks' desired supply of loans $(\sum_i \sum_j b_{i,j,t})$ in order to obtain a relationship between the supply of bank credit, R_t, and the other policy variables and cost parameters (see Appendix V). The real supply of credit initially will rise with a higher loan interest rate, since the expected profit on an additional dollar of lending will be positive when interest rates are relatively low and the firms have limited debt servicing obligation. As the loan interest rate

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1/ As already noted, R_{**} is the minimum loan interest rate at which the banks would be willing to use all available deposits and the minimum required level of equity. Below R_{**}, the banks' owners would not want to use all available deposits and the deposit interest rate ceiling would not longer be a binding constraint. R_* is the loan interest rate at which the banks would begin to find it profitable to hold more equity than the required minimum level of equity (see Appendix V).

rises, however, a point will eventually be reached where the expected profit on additional lending turns negative when firms face relatively large debt service obligations. As a result, the supply of bank credit will eventually be backward bending as loan interest rates rise.

Curve 4 in the southeast (SE) quadrant of Figure 1 portrays the level of deposits that would be provided by the household sector (net of required reserves $(\sum_h (1-k)\, D^H_{h,t}/P_{t+1}))$ to the banks at each level of the real wage (W_t/P_{t+1}). As noted in (37) and Appendix IV, the households' desired real holdings of deposits is a positive function of the real wage.

2. Financial policies and intermediation costs

The relationships in Figure 1 can be used to illustrate the macroeconomic effects of changes in financial policies and the costs of intermediation. If the authorities establish a ceiling loan rate of R_1, for example, then the banks' owners would find it profitable to supply a level of real credit equal to OG. Such a level of lending would support a real wage bill that would be consistent with the real wage $(W/P)_1$. Given the ceiling deposit interest rate and the expected rate of inflation, the supply of real deposits from the household sector, net of required reserves, would be equal to OM (=GI). This implies that the banks would hold equity equal to IJ, which would exceed the required amount GH. At this level of lending, the firms would have an excess demand for credit, which is indicated by the fact that, at R_1, a real wage equal to ON would be needed before the firms' excess demand for labor would be eliminated.

Since R_1 lies in the range where the banks' owners would see an expected profit from lending an additional dollar at a higher interest rate, increases in the ceiling loan rate would result in a larger stock of credit, reflecting the willingness of the banks' owners to expand their holdings of equity. The resulting increase in the supply of credit would allow firms to hire more labor which would in turn lead to the real wage being bid up. As the real wage rose, the households would expand their real holdings of deposits. However, if the ceiling loan rate was in the range where the bank owners would see a negative return on an additional dollar of lending even at a higher interest rate (such as R_2), 1/ then raising the loan interest rate would result in a lower stock of credit, a lower real wage, and a smaller real stock of deposits.

This result has important policy implications. Although financial reforms that involve raising the ceilings on loan interest rates are likely to be expansionary when firms' debt-servicing positions are relatively

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1/ This situation would reflect relatively high probabilities that the firms would default on their debt-servicing obligations.

strong, a higher ceiling loan interest can be contractionary if the debt-servicing position of the firms is relatively weak (e.g., there is a high probability that they will default on their debt-service obligations). This indicates the importance of dealing with the debt-servicing difficulties of firms at an early stage in any adjustment program that incorporates sharp increases in nominal and real interest rates in an economy with a repressed financial system.

As examples of the macroeconomic consequences of changes in the banks' cost structure, Figures 2 and 3 illustrate the effects of an increase in the banks monitoring costs and a higher required reserve ratio, respectively. With higher monitoring costs, the minimum loan rate at which the banks' owners would find it profitable to fully utilize the deposits made available by households and hold either the minimum required level of equity (R_{**}) or more than the minimum level of equity (R_{*}) would have to rise (Figure 2). 1/ The new minimum loan interest rates would now be R_{*}' and R_{*}'. However, the scale of the banks' lending in the range between R_{**}' and R_{*}' would be the same as in the range between R_{**} and R_{*}. As discussed in Appendix V, this reflects the fact that, when banks hold only the minimum required level of equity (as in the ranges R_{**}' to R_{*}' or R_{**} to R_{*}), the real supply of credit $\left(\sum_h \frac{(1-k)}{(1-s)} \frac{D^H_{h,t}}{P_{t+1}}\right)$ is independent of the banks' monitoring costs (equation (41b)).

However, in the range where the banks hold more equity than the minimum required level (between R_{*}' and $R^{*'}$), the amount of credit the banks would extend at any given loan interest rate will be smaller when monitoring costs increase. With higher monitoring costs, the banks' owners would attempt to ensure that they obtain a higher expected return on any loan. At a given loan interest rate, a higher expected return can be achieved only by restricting the amount of credit extended to a firm, which would lower the probability that the firm would default on its debt-servicing obligations. As a result, curves 2 and 3 both shift in toward the origin.

At the given ceiling loan interest rate (R_1), higher operating costs will lead the banks to reduce the real supply of credit from OG to OG'. This would reduce the firms' ability to hire labor which would result in a decline in the real wage (from $(W/P)_1$ to $(W/P)_2$). With a lower real wage, households would also reduce their real holdings of deposits (deposits net of required reserves would fall from OM to OM'). Thus, reduced financial efficiency can depress the economy's real wage and the stocks of real credit and deposits.

1/ R^* would have to decline.

A higher required reserve ratio would also result in a contraction of the stocks of real credit and real deposits, as well as a fall in the real wage (Figure 3). With a higher required reserve ratio, the banks' effective cost of using deposits would rise, since a small proportion of each dollar of deposits could be used to fund lending activities. As a result of these costs, the minimum loan interest rate (R_{**}) at which the banks' owners would find it profitable to fully utilize all deposits supplied by the households at the ceiling deposit interest rate and hold just the minimum amount of equity must rise (to R'_{**}). However, the loan interest rate (R_*) at which the banks' owners would find it profitable to use all available deposits and hold more than the minimum required equity would not be changed (see Appendix V). This reflects the fact that at R_* bank equity rather than deposits are the marginal source of bank funds, and the marginal cost that is relevant for lending decisions is the implicit cost of foregone consumption associated with adding an extra dollar of equity. This also means that, over the range where banks' owners hold more equity than the required minimum (between R_* and R^*), the supplies of bank credit would have the same slope (see Appendix V).

A higher required reserve ratio would also reduce the amount of credit that the banks would supply when they hold only the minimum required level of bank equity, which equals $\sum_h \frac{(1-k)}{(1-s)} \frac{D^H_{h,t}}{P_{t+1}}$. The smaller amount of credit can naturally support only a smaller real wage bill ($\sum_h (\frac{W_t}{P_{t+1}}) L^H_{h,t}$).

In terms of Figure 3, the value of $(\tilde{W}/P)$ that results in an equality between the real supply of credit and the real wage bill must fall from $(\bar{W}/P)_0$ to $(\bar{W}/P)_1$. A higher required reserve ratio thus shifts curve 2 in toward the origin from ABC to A'B'C', and curve 3 shifts from DEF to D'E'F'. In addition, the curve in the SE quadrant relating the real wage (W_t/P_{t+1}) to the amount of net deposits received by the banking system shifts up (to reflect a higher k).

At a given ceiling loan interest rate (such as R_1), a higher k would thus reduce the real supply of bank credit from OG to OG'. Since firms would have less credit, they would be able to hire less labor, and the real wage would fall from $(W/P)_0$ to $(W/P)_1$. The stock of real deposits net of required reserves would also fall from OM to OM'. Although not explicitly examined in Figure 3, a higher k would produce even sharper declines in the stock of real credit and the real wage if the ceiling loan interest rate was in the range where the supply of bank credit was backward bending.

IV. Financial Reform, Prudential Supervision, and the Cost of Official Safety Nets

During the past two decades, many developing countries have liberalized their financial systems in order to improve financial sector efficiency, to increase financial savings, and to achieve a more efficient mix of investments. These liberalizations often involved the removal or elevation of ceiling interest rates, reductions in required reserve ratios and freer entry into the financial system. As noted earlier, however, a number of these reform efforts ended in periods of financial instability that required extensive restructuring of both the corporate and financial sectors and often created large public sector funding obligations as the authorities provided emergency lending to a broad range of enterprises and financial institutions.

This experience has raised the issue of whether a financial reform is likely to expose authorities to new credit risks through the operations of any official safety net underpinning the domestic financial system. Since the authorities in most developing countries have implemented either explicit deposit insurance arrangements or have historically intervened to prevent widespread losses for depositors, there is often the perception that depositors, especially small depositors, will be fully protected in the case of an institutional failure. This can naturally make depositors indifferent regarding the lending activities of the financial institutions. Since depositors are the primary source of funding for banks in developing countries, this eliminates an important source of market discipline on the banks' managers and owners and places a correspondingly greater burden on the bank supervisors to monitor for fraud and mismanagement.

The potential funding obligations of the authorities that are associated with maintaining an official safety net can be linked to the scale of deposits in financial institutions and the probability that some of these institutions will fail. In general, it is difficult to characterize the probability that a financial institution will fail, especially if the authorities are considering a long time horizon. However, the analysis that we have developed in this paper can be used to formulate an explicit measure of the probability at the beginning of the period that a bank will default on its deposit payment obligations at the end of the period. Moreover, this formulation will allow us to gauge the effects of changes in financial policies during a reform period on the probability of institutional failure.

1. Deposit insurance obligations and the probability of institutional failure

Our analysis assumes that the authorities guarantee the repayment of both the principal and the interest payments that households are to receive on their deposits. 1/ For bank i, the authorities maximum potential deposit insurance payments (measured in terms of period t+1's price level) is $(1+r_t-k)D_{i,t}/P_{t+1}$, since the authorities can use the banks required reserves ($kD_{i,t}$) to help meet their deposit insurance obligations. Note that this upper bound will be realized only if the authorities cannot recover any of the bank's earnings when the bank fails. 2/

A bank will fail if the revenues it receives on the loans that it made at the beginning of the previous period are less than the sum of the payments it owes to depositors. 3/ In Section II, it was assumed that a bank's end of period revenues equal the sum of the interest and principal repayments of loans made by borrowers that do not default plus the entire value of the output (less ex post monitoring costs) of all the firms that do default on their loan obligations. In this section, our objective is to characterize the ex ante expected value of the authorities' deposit insurance payments (EDP), and to analyze how financial reforms influence the expected size of these safety net obligations. Accordingly, to make the analysis tractable, we will employ four simplifying assumptions: First, we will consider the case where the banks get full repayment from borrowers that do not default and nothing from firms that do default. 4/ Second, we will continue to assume that the bank supervisory authorities impose a prudent man standard on the bank's owners. As noted earlier, this implies that the bank's owners will incorporate the potential losses arising from bad loans (including those that would be sufficient to force the bank into bankruptcy) into their decisions (in the form of negative utility for the owners) regarding the optimal scale and direction of their lending activities. Third, all banks will be taken as holding only the minimum required level of equity. Finally, we will assume that each bank lends to a set of n identical borrowers. 5/

In this situation, let $Z_{i,j,t}$ be the real revenues that will be received by bank i as a result of a loan made to firm j in period t:

1/ As discussed earlier, this allows the households to assume that the nominal return on deposits is certain. If only the deposit principal was guaranteed by the authorities, then the return on deposits would become uncertain even if the nominal deposit rate was constrained by a ceiling deposit rate.

2/ This could represent a situation where the bank's owner, knowing that he will default on his deposit obligations, consumes all the available earnings prior to declaring bankruptcy.

3/ Monitoring costs that were paid for at the beginning of the period will not influence this default decision.

4/ This would correspond to the case where the firm's owner consumes all available revenues whenever it becomes clear that those revenues are insufficient to meet the firm's debt-service obligations. We will also assume that the bank does not incur any ex post monitoring costs.

5/ Each individual borrower will nevertheless be subject to an independent production shock.

$$(44)\quad Z_{i,j,t} = \begin{cases} (1+R_t)\ B_{i,j,t}/P_{t+1} & \text{with probability } 1-\lambda^*_{j,t+1} \\ 0 & \text{with probability } \lambda^*_{j,t+1} \end{cases}$$

The net profits of bank i in period t is thus:

$$(45)\qquad \Pi_{i,t} = \sum_{j}^{n} Z_{i,j,t} - (1+r_t-k)\ \frac{D_{i,t}}{P_{t+1}}$$

If $\Pi_{i,t} < 0$, the bank defaults. When all borrowers are identical, the minimum number of borrowers (n_s) that must successfully service their debt obligations to ensure that $\Pi_{i,t} \geq 0$ can be defined as the solution to

$$(46)\qquad \Pi_{i,t} = n_s(1+R_t)\ \frac{B_{i,j,t}}{P_{t+1}} - (1+r_t-k)\ \frac{D_{i,t}}{P_{t+1}} = 0$$

$$\text{where } \frac{B_{i,j,t}}{P_{t+1}} = \frac{1}{n}\,\frac{(1-k)}{(1-s)}\,\frac{D_{i,t}}{P_{t+1}}\ \underline{1}/$$

This implies that

$$(47)\qquad n_s = \frac{(1+r_t-k)(1-s)}{(1+R_t)(1-k)}\ n$$

Since the probability that any given firm will service its debt insurance is given by $1-\lambda^*_{j,t+1}$ and is independent of what happens to the other firms, the expected deposit insurance payments (EDP) faced by the authorities at the beginning of the period is given by

$$(48)\qquad EDP_t = \sum_{\upsilon=0}^{n_s} b(\upsilon;n,(1-\lambda^*_{j,t+1}))\left[(1+r_t-k)\ \frac{D_{i,t}}{P_{t+1}} - \upsilon(1+R_t)\ \frac{B_{i,j,t}}{P_{t+1}}\right]$$

1/ The assumption that banks hold only the minimum equity required implies $\sum_j B_{i,j,t}/P_{t+1} = \frac{(1-k)}{(1-s)}\ \frac{D_{i,t}}{P_{t+1}}$ (equation (29)). Moreover, since all borrowers are identical, each borrower receives 1/n of the total amount of credit.

where

$$(49) \qquad b(\upsilon;n,(1-\lambda^*_{j,t+1})) = \binom{n}{\upsilon} (1-\lambda^*_{j,t+1})^{\upsilon}\left(\lambda^*_{j,t+1}\right)^{n-\upsilon}$$

is the Binomial probability that exactly υ ($< n_s$) loans will be repaid and the bank will default. As already noted, the first term inside the square bracket represents the authorities' maximum deposit insurance payment obligation; whereas the second term inside the bracket represents the amount of loan repayments that the authorities collect when the bank enters bankruptcy. Since $b(\upsilon;\ n,1-\lambda^*))$ can be approximated by a Poisson distribution, 1/ we can write equation (48) as

$$(50) \qquad EDP_t = (1+r_t-k)\frac{D_{i,t}}{P_{t+1}} \sum_{\upsilon=0}^{n_s} \frac{\delta^{\upsilon} e^{-\delta}}{\upsilon!}$$

$$- (1+R_t)\frac{B_{i,j,t}}{P_{t+1}} \sum_{\upsilon=0}^{n_s} \frac{\delta^{\upsilon} e^{-\delta}}{\upsilon!}\, \upsilon$$

Equation (50) indicates the importance of closure rules in determining the extent of the authorities' potential losses from the deposit insurance system. For example, if the bank's owner can use the revenues from the successful loan repayments to finance consumption expenditures prior to a declaration of bankruptcy, the first term on the right hand side of equation (50) represents the authorities' anticipated loss. The second term represents the expected recovery of loan repayments receipts if the authorities can prevent the bank's owner from using these resources to fund his consumption. By combining

equations (47), (50), and $\frac{B_{i,j,t}}{P_{t+1}} = \frac{1}{n}\frac{(1-k)}{(1-s)}\frac{D_{i,t}}{P_{t+1}}$, we can write

$$(51) \qquad EDP_t = (1+r_t-k)\frac{D_{i,t}}{P_{t+1}} \sum_{\upsilon=0}^{n_s} \frac{\delta^{\upsilon} e^{-\delta}}{\upsilon!}\left(1-\frac{\upsilon}{n_s}\right)$$

where $\delta = n(1-\lambda^*_{j,t+1})$

1/ See Feller (1962), pp. 142-143.

To examine the conditions under which a financial reform can increase the authorities' potential funding obligations, we must consider how the expression in equation (51) responds to a change in financial policies. In particular, we will be concerned with the effects of increases in the ceiling interest rates on loans (R) and deposits (r), a reduction in the required reserve ratio (k), and an increase in the minimum equity ratio (s). If X represents the financial policy instrument being changed, then equation (51) implies

$$(52) \qquad \frac{\partial EDP_t}{\partial X} = \frac{EDP_t}{(1+r_t-k)} \frac{\partial(1+r_t-k)}{\partial X} + \frac{EDP_t}{(D_{i,t}/P_{t+1})} \frac{\partial(D_{i,t}/P_{t+1})}{\partial X}$$

$$+ (1+r_t-k) \frac{D_{i,t}}{P_{t+1}} \sum_{\upsilon=0}^{n_s} (1-\frac{\upsilon}{n_s})(\frac{\upsilon}{\delta} -1) \frac{\delta^{\upsilon} e^{-\delta}}{\upsilon!} \frac{\partial \delta}{\partial X}$$

$$+ (1+r_t-k) \frac{D_{i,t}}{P_{t+1}} \sum_{\upsilon=0}^{n_s} \frac{\delta^{\upsilon} e^{-\delta}}{\upsilon! \ (n_s)^2} \frac{\upsilon \partial n_s}{\partial X}$$

with $\frac{\upsilon}{\delta}$ -1 < 0 since $\delta = n(1-\lambda^*_{j,t+1}) > n_s \geq \upsilon$. 1/ Since

$\frac{\partial\delta}{\partial X} = -n\partial\lambda^*_{j,t+1}/\partial X$, we can write equation (52) as:

$$(53)\quad \frac{\partial EDP_t}{\partial X} = \frac{EDP_t}{(1+r_t-k)} \frac{\partial(1+r_t-k)}{\partial X} + \frac{EDP_t}{(D_{it}/P_{t+1})} \frac{\partial(D_{i,t}/P_{t+1})}{\partial X}$$

$$- (1+r_t-k) \frac{D_{i,t}}{P_{t+1}} \sum_{\upsilon=0}^{n_s} (1- \frac{\upsilon}{n_s}) (\frac{\upsilon}{\delta} -1) \frac{\delta^\upsilon e^{-\delta}}{\upsilon!} n \frac{\partial\lambda^*_{j,t+1}}{\partial X}$$

$$+ (1+r_t-k) \frac{D_{i,t}}{P_{t+1}} \sum_{\upsilon=0}^{n_s} \frac{\partial^\upsilon e^{-\delta}}{\upsilon!(n_s)^2} \frac{\partial n_s}{\partial X}$$

For the case in which banks hold only the minimum required level of equity, then (53) can be further simplified by noting that in the steady

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1/ This condition follows from the requirement that, in the steady state, banks will only make loans that are expected to yield a profit. This requires that

$$(1-\lambda^*_{j,t+1}) n \frac{B_{i,j,t}}{P_{t+1}} (1+R_t) - (1+r_t-k) \frac{D_{i,t}}{P_{t+1}} - \text{monitoring cost} > 0$$

or, using $\frac{B_{i,j,t}}{P_{t+1}} = \frac{1}{n} \frac{(1-k)}{(1-s)} \frac{D_{i,t}}{P_{t+1}}$,

$$(1-\lambda^*_{j,t+1}) \frac{(1-k)}{(1-s)} (1+R_t) - (1+r_t-k) > 0,$$

or $1-\lambda_{j,t+1} > \frac{(1+r_t-k)(1-s)}{(1+R_t)(1-k)} = \frac{n_s}{n}$,

or $n_s < (1-\lambda^*_{j,t+1}) n = \delta$

state $\frac{D_{i,t}}{P_{j+1}}$, $\lambda^{*}_{j,t+1}$, and n_s can be expressed as functions of

R_t, r_t, s, and k. In particular, holdings of the real deposits will be positively related to r_t and s, a negative function of k, and an ambiguous function of the expected rate of inflation (ρ_{t+1}). 1/

$$(54) \qquad \frac{D_{i,t}}{P_{t+1}} = \frac{D_{i,t}}{P_{t+1}} (\overset{(+)}{r_t}, \overset{(+)}{s}, \overset{(-)}{k}, \overset{(?)}{\rho_{t+1}})$$

The critical number of loan repayments (n_s) is a negative function of R_t and s and a positive function of r_t and k. 2/

$$(55) \qquad n_s = n_s (\overset{(-)}{1+R_t}, \overset{(+)}{r_t}, \overset{(+)}{k}, \overset{(-)}{s})$$

and the probability ($\lambda^{*}_{j,t+1}$) that the firms will default on their debt-service payments can be shown to be positively related to R_t and r_t but negatively related to k. 3/

$$(56) \qquad \lambda^{*}_{j,t+1} = \lambda^{*}_{j,t+1} (\overset{(+)}{1+R_t}, \overset{(+)}{r_t}, \overset{(-)}{k}, \overset{(+)}{s})$$

1/ This follows immediately from (37) and (V.14) in Appendix V.
2/ This can be verified from total differentiation of (47).
3/ Note from (6) that the derivative of λ^{*}_{jt+1} with respect to any financial policy parameter X (where X = R_t, r_t, s, or k) can be expressed as

$$\frac{\partial \lambda^{*}_{j,t+1}}{\partial X} = \lambda^{*}_{j,t+1} \left[\frac{1}{1+R_t} \frac{\partial(1+R_t)}{\partial X} + \frac{(1-\epsilon_{fB})}{B_{j,t}/P_{t+1}} \frac{\partial(B_{jt}/P_{t+1})}{\partial X} \right]$$

where ϵ_{fB} denotes the elasticity of $f_{j,t+1}$ with respect to $B_{j,t}/P_{t+1}$. Equation (56) follows from the assumption that $\epsilon_{fB} < 1$, and the condition $\frac{B_{j,t}}{P_{t+1}} = \frac{1-k}{n(1-s)} \frac{D_{i,t}}{P_{t+1}}$ when the minimum equity requirement is binding.

These relationships imply that changes in the ceiling loan and deposit interest rates would not have symmetric effects on the authorities' expected deposit insurance payments (EDP_t). If the ceiling deposit interest rate was increased in isolation, for example, the authorities' EDP_t would increase. A higher deposit interest rate would directly increase both the interest payments on each deposit and the stock of deposits that banks would be able to attract from the household sector. Even with an unchanged probability of bankruptcy, this would increase the payments the authorities would expect to make under the deposit insurance system. However, the probability of a bank failure would also increase for two reasons: (1) the minimum number of successful loan repayments (n_s) needed to ensure that a bank could service its deposit obligations would rise; and (2) since the larger stock of deposits would allow the bank to extend more loans to firms, the probability that the firms would default on their debt-service payments would also increase.

In contrast, a higher ceiling loan interest rate would have an ambiguous effect on the authorities' EDP_t. When banks hold only the minimum required level of equity, it was shown in Section III that a change in the loan rate would not lead to a change in the ex ante steady state levels of either the real stock of credit or the real wage. Thus, the real stock of deposits and the banks' deposit interest payments would remain unchanged (as long as the ceiling deposit interest rate remained unchanged). However, the probability of a bank defaulting on its deposit payments could either rise or fall. On the one hand, a higher loan rate reduces the minimum number of loan repayments (n_s) that are needed to enable a bank to successfully service its deposit obligations. On the other hand, a higher loan rate increases the probability that the bank's borrowers will default on their debt-service obligations. In this situation, the initial debt-servicing levels of the firms (and thereby their probabilities of default) will play a crucial role in determining whether the authorities' EDP increases. In particular, the larger the firms' initial debt-servicing obligations, the more likely it will be that an increase in the loan rate will increase the authorities' EDP. This indicates the importance of dealing with any debt-servicing difficulties of nonfinancial firms at an early stage, or (preferably) prior to undertaking a financial reform.

It has often been argued that increasing the minimum required level of equity in the banks (enhanced "capital" adequacy) is one means of reducing the authorities' EDP during a financial reform by creating a larger "buffer" between the deposit insurance system and a bank failure. 1/ In our analysis, it is indeed true that a higher minimum equity ratio (s) for the banks reduces the number of successful loan repayments that would be needed in order for a bank to avoid defaulting on its deposit payment obligations. However, when banks hold only the minimum level of equity, a higher s means that the banks in a repressed financial system would ultimately extend a larger stock of loans.

Why is this the case? With a ceiling deposit interest rate, the shadow price (or extra expected profit) of an additional dollar of deposits exceeds the deposit interest rate, and the banker has an incentive to use all the deposits he can obtain. However, the bank can stay in business and have access to those deposits only if it meets the minimum capital adequacy standards. When the bank's owner decides to hold only the minimum required level of bank equity, we have noted that this corresponds to the situation where the expected profit that can be made from creating an extra dollar of equity (which is the rate at which consumption can be transferred from t to t+1) is less than the owner's internal rate of discount. Holding equity (rather than relying exclusively on deposits) thus imposes an intertemporal cost on the bank's owner. In this situation, a higher capital adequacy requirement represents a higher operating cost for the banker. However, the banker can minimize the cost of a higher capital adequacy requirement by using his new equity to fund additional loans. The interest earnings on this additional lending provides at least a partial offset to the intertemporal costs imposed by the higher capital adequacy requirement.

Such additional lending would lead firms to attempt to hire additional labor, which would be forthcoming only at a higher real wage. A higher real wage would in turn increase EDP both directly, by increasing the stock of deposits in the banking system, and indirectly, by increasing the probability that firms will default on their debt-service payments and thereby the probability that the banks will default. Once again, a key issue is the scale of the firm's initial debt-servicing obligations and the

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1/ Furlang and Keeley (1987 and 1991) have argued that, for a given supervisory effort, increasing the minimum capital requirement will reduce the probability that a bank will enter bankruptcy. Our analysis differs from theirs in two key respects. First, Furlong and Keeley assume that the expected returns on the banks assets are independent of the scale of the bank's lending; whereas our analysis allows the expected returns to start to decline (due to a higher probability of default) once the firms' debts become high enough. Second, they view prudential supervision as limiting the risky assets that the bank can acquire (relative to capital); whereas our analysis assures that the banks' owners fully incorporate (in the form of negative utility) into their decisions the losses that could be incurred by the deposit insurance system.

probability that they will default on those obligations. 1/ The higher the initial probability that the firms will default on their debt-service obligations, the more likely it will be that a higher s will not reduce the authorities' expected deposit insurance payments.

Finally, many financial reforms have encompassed a lowering of required reserve ratios. A decline in k can reduce the authorities' EDP by reducing the minimum number of successful loan repayments (n_s) that are needed if the bank is to avoid defaulting on its deposit payments. However, a lower k will also allow banks to extend more loans to firms, which will drive up both the real wage (and thereby the level of bank deposits) and the probability that the firms will default on their debt-service obligations.

These results suggest that a financial reform encompassing increases in ceiling interest rates and the lowering of required reserve ratios can potentially increase the authorities' expected deposit insurance payments even if the reform is accompanied by higher minimum equity requirements for banks and strong prudential supervision. In particular, an increase in the authorities' EDP is most likely when the firms' debt-servicing positions are relatively weak (as reflected in a high probability that they will default on their debt-service obligations). This implies that, if the authorities do not want to face a large EDP, a financial reform should be preceded by steps to deal with any debt-service difficulties in the nonfinancial sector.

V. Conclusions

This paper has focused on developing a framework for the analysis of the macroeconomic effects of financial reform and the effects of such reforms on the cost of maintaining an official safety net. The analysis considered a multiperiod general equilibrium model of an economy with a repressed financial system which emphasized the interdependence between production shocks, firm creditworthiness, credit rationing, bank failures, and the cost of maintaining an official deposit insurance system. It was argued that any financial difficulties of nonfinancial firms should be

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1/ This reflects the fact that

$$\frac{\partial \lambda^*_{j,t+1}}{\partial s} = \frac{\partial \lambda^*_{j,t+1}}{\partial (\frac{W_t}{P_{t+1}})} \frac{\partial (\frac{W_t}{P_{t+1}})}{\partial s} + \frac{\partial \lambda^*_{j,t+1}}{\partial (B_{i,j,t}/P_{t+1})} \frac{\partial (B_{i,j,t}/P_{t+1})}{\partial s}$$

As shown in Appendix II, the sizes of $\frac{\partial \lambda^*_{j,t+1}}{\partial (W_t/P_{t+1})}$ and $\frac{\partial \lambda^*_{j,t+1}}{\partial (B_{i,j,t}/P_{t+1})}$ both depend on the initial level of $\lambda^*_{j,t+1}$.

addressed at an early stage, or a financial reform could have a contractionary effect on output. In systems with either explicit or implicit deposit insurance, a financial reform may increase the authorities' potential funding obligations even if the authorities put in place strong prudential supervision and enhanced capital adequacy standards. Indeed, the authorities may be able to attain the efficiency gains associated with a financial reform only if they are willing to accept the risk of greater funding obligations in the deposit insurance system. However, this tradeoff between financial efficiency and funding risk can be improved by strengthening the financial position of nonfinancial firms and the system of prudential supervision at an early stage in the reform process.

Notation

General subscripts and superscripts

t	an index of time periods
h,H	an individual household
i,I	an individual bank or bank owner
j,F	an individual firm or entrepreneur

Real quantities and related variables

Y	output
K	physical capital input
L	labor input
f	a production function describing maximum potential output
λ	a stochastic productivity factor multiplying f
λ^*	the critical value of λ at which the value of a firm's output is just sufficient to meet its debt-servicing obligations
C	consumption
M	the real resource costs incurred by banks in evaluating lending decisions ex ante and monitoring the behavior of firms ex post
m_k	parameters that characterize the costs of evaluation and monitoring ($k=0,1,2$).

Financial quantities and related variables

B	the nominal value of loans
D	the nominal value of deposits
S	the nominal value of the capital or equity of bank owners
H	the nominal stock of high powered money or bank reserves
X	the nominal stock of excess reserves
$\bar{B}$	an upper bound on B
$\bar{D}$	an upper bound on D
$\theta_{i,j}$	the share of bank i's total lending that is allocated to firm j

Prices, wages, and interest rates

P	the price of goods
W	the wage per unit of labor
R	the nominal interest rate on loans
r	the nominal interest rate on deposits
ρ	the expected rate of inflation of goods prices

Policy variables

k	the required minimum reserve ratio on deposits
s	the minimum equity requirement as a ratio of loans outstanding
$\bar{R}$	the ceiling level of R
$\bar{r}$	the ceiling level of r

Preferences and shadow prices

- U a utility function
- V a value function (indirect utility function)
- γ the marginal utility of consumption
- β a factor for discounting time
- ϕ the firm's shadow price of credit
- ϕ^D the bank's shadow price of deposits
- ϕ^S the bank's shadow price of relaxing the minimum equity requirement

Other notation

- E the expectations operator
- Π the real net profits of the bank
- EDP the ex ante expected value of the deposit insurance payment obligations of the authorities
- N_k the set of firms that default (k=1) and do not default (k=2) on their obligations to banks
- n_k the number of firms in N_k (k=1,2)
- n the total number of firms (n_1+n_2)
- n_s the minimum number of firms that must meet their loan obligations to enable the bank to meet its obligations to depositors without assistance from the authorities
- $b(\upsilon;n,1-\lambda^*)$ the binomial probability that exactly υ loans will be fully paid given n total loans and a repayment probability of $1-\lambda^*$ on each loan
- $\delta=n(1-\lambda^*)$ the expected number of repayments

The Firm

This Appendix derives conditions (9)-(12) in the text, which describe the behavior of the firm. The first order conditions for the firm's optimal utilization of capital and labor are derived by differentiating the firm's objective function, as described by equation (7), to obtain:

$$\text{(II.1)}\quad \frac{\partial V_j(\cdot_t)}{\partial K_{j,t}} = -\gamma_j^F + \beta_j^F \int_{\lambda^*_{j,t+1}}^{1} \frac{\partial V_j(\cdot_{t+1})}{\partial(\cdot_{t+1})} \frac{\partial(\cdot_{t+1})}{\partial K_{j,t}} d\lambda_{j,t+1}$$

$$- \beta_j^F V_j(\cdot_{t+1} | \lambda_{j,t+1} = \lambda^*_{j,t+1}) \frac{\partial \lambda^*_{j,t+1}}{\partial K_{j,t}}$$

$$\text{(II.2)}\quad \frac{\partial V_j(\cdot_t)}{\partial L_{j,t}} = \beta_j^F \int_{\lambda^*_{j,t+1}}^{1} \frac{\partial V_j(\cdot_{t+1})}{\partial(\cdot_{t+1})} \frac{\partial(\cdot_{t+1})}{\partial L_{j,t}} d\lambda_{j,t+1}$$

$$- \beta_j^F V_j(\cdot_{t+1} | \lambda_{j,t+1} = \lambda^*_{j,t+1}) \frac{\partial \lambda^*_{j,t+1}}{\partial L_{j,t}} - \phi_j \frac{W_t}{P_{t+1}}$$

Note that:

$$\text{(II.3)}\quad V_j(\cdot_{t+1} | \lambda_{j,t+1} = \lambda^*_{j,t+1}) = V_j(0) = 0$$

$$\text{(II.4)}\quad \frac{\partial V_j(\cdot_{t+1})}{\partial(\cdot_{t+1})} = \gamma_j^F$$ (the Benveniste-Scheinkman condition, as discussed by Sargent (1987), pp. 21-22).

$$\text{(II.5)}\quad \frac{\partial(\cdot_{t+1})}{\partial K_{j,t}} = 1+\lambda_{j,t+1} \frac{\partial f_{j,t+1}}{\partial K_{j,t}}$$

and

$$\text{(II.6)}\quad \frac{\partial(\cdot_{t+1})}{\partial L_{j,t}} = \lambda_{j,t+1} \frac{\partial f_{j,t+1}}{\partial L_{j,t}} - (1+R_t) \frac{W_t}{P_{t+1}}$$

Hence,

$$\text{(II.7)}\quad \frac{\partial V_j(\cdot_t)}{\partial K_{j,t}} = -\gamma_j^F + \gamma_j^F\beta_j^F \int_{\lambda^*_{j,t+1}}^{1} \left[1 + \lambda_{j,t+1} \frac{\partial f_{j,t+1}}{\partial K_{j,t}}\right] d\lambda_{j,t+1}$$

and

$$\text{(II.8)}\quad \frac{\partial V_j(\cdot_t)}{\partial L_{j,t}} = \gamma_j^F\beta_j^F \int_{\lambda^*_{j,t+1}}^{1} \left[-(1+R_t)\frac{W_t}{P_{t+1}} + \lambda_{j,t+1} \frac{\partial f_{j,t+1}}{\partial L_{j,t}}\right] d\lambda_{j,t+1} - \phi_j \frac{W_t}{P_{t+1}}$$

Equation (9) follows immediately from (II.7) when $\frac{\partial V_j(\cdot_t)}{\partial K_{j,t}} = 0$.

Now consider the case in which <u>the credit-rationing constraint is not binding</u>, so that $\phi_j = 0$ in (II.8). To characterize the optimal joint choices of $K_{j,t}$ and $L_{j,t}$ in terms of variables exogenous to the firm (namely, $1+R_t$ and W_t/P_{t+1}), consider the system of identities

$$\text{(II.9)}\quad V_{KK}dK_{j,t} + V_{KL}dL_{j,t} = -V_{KR}d(1+R_t) - V_{KW}d\left(\frac{W_t}{P_{t+1}}\right)$$

$$\text{(II.10)}\quad V_{LK}dK_{j,t} + V_{LL}dL_{j,t} = -V_{LR}d(1+R_t) - V_{LW}d\left(\frac{W_t}{P_{t+1}}\right)$$

where the $V_{K.}$ and $V_{L.}$ terms--for . = K,L,R,W--denote the partial derivatives of $\frac{\partial V_j(\cdot_t)}{\partial K_{j,t}}$ and $\frac{\partial V_j(\cdot_t)}{\partial L_{j,t}}$ with respect to $K_{j,t}$, $L_{j,t}$, $1+R_t$ and $\frac{W_t}{P_{t+1}}$.

From the conditions for a maximum we know that

$$V_{KK} < 0,\ V_{LL} < 0,\ \text{and}\ \Delta = \begin{vmatrix} V_{KK} & V_{KL} \\ V_{LK} & V_{LL} \end{vmatrix} > 0.$$

Accordingly, to establish (11) and (12) from the system (II.9) and (II.10), we need to examine the signs of V_{KL}, V_{KR}, V_{KW}, V_{LR}, and V_{LW}.

By differentiating (II.7) and using (6), it can be seen, to begin with, that:

$$\text{(II.11)} \quad V_{KR} = - \gamma_j^F \beta_j^F \left[1+\lambda^*_{j,t+1} \frac{\partial f_{j,t+1}}{\partial K_{j,t}} \right] \frac{\partial \lambda^*_{j,t+1}}{\partial (1+R_t)} < 0$$

since

$$\text{(II.12)} \quad \frac{\partial \lambda^*_{j,t+1}}{\partial (1+R_t)} = \frac{W_t}{P_{t+1}} \frac{L_{j,t}}{f_{j,t+1}} > 0$$

and

$$\text{(II.13)} \quad V_{KW} = - \gamma_j^F \beta_j^F \left(1 + \lambda^*_{j,t+1} \frac{\partial f_{j,t+1}}{\partial K_{j,t}} \right) \frac{\partial \lambda^*_{j,t+1}}{\partial (W_t/P_{t+1})} < 0$$

since

$$\text{(II.14)} \quad \frac{\partial \lambda^*_{j,t+1}}{\partial (W_t/P_{t+1})} = \frac{(1+R_t)L_{j,t}}{f_{j,t+1}} > 0.$$

Similarly, by differentiating (II.8), it can be seen that:

$$\text{(II.15)} \quad V_{LR} = - \gamma_j^F \beta_j^F \frac{W_t}{P_{t+1}} \left[1 - \lambda^*_{j,t+1} \left(\frac{L_{j,t}}{\lambda^*_{j,t+1}} \frac{\partial \lambda^*_{j,t+1}}{\partial L_{j,t}} \right)\right]$$

and

$$\text{(II.16)} \quad V_{LW} = - \gamma_j^F \beta_j^F (1+R_j) \left[1 - \lambda^*_{j,t+1} \left(\frac{L_{j,t}}{\lambda^*_{j,t+1}} \frac{\partial \lambda^*_{j,t+1}}{\partial L_{j,t}} \right)\right]$$

where

$$\text{(II.17)} \quad \frac{\partial \lambda^*_{j,t+1}}{\partial L_{j,t}} = \frac{1}{f_{j,t+1}} \left[\frac{(1+R_t)W_t}{P_{t+1}} - \lambda^*_{j,t+1} \frac{\partial f_{j,t+1}}{\partial L_{j,t}} \right]$$

This implies $V_{LR} < 0$ and $V_{LW} < 0$ whenever $\frac{L^*_{j,t}}{\lambda^*_{j,t+1}} \frac{\partial \lambda^*_{j,t+1}}{\partial L_{j,t}} < \frac{1}{\lambda^*_{j,t+1}}$,

which will presumably hold for sufficiently small $\lambda^*_{j,t+1}$. Finally, by differentiating (II.7), it can be seen that:

$$\text{(II.18)} \quad V_{KL} = \gamma^F_j \beta^F_j \left[\frac{1 - (\lambda^*_{j,t+1})^2}{2} \frac{\partial^2 f_{j,t+1}}{\partial K_{j,t} \partial L_{j,t}} - \left(1 + \lambda^*_{j,t+1} \frac{\partial f_{j,t+1}}{\partial K_{j,t}} \right) \frac{\partial \lambda^*_{j,t+1}}{\partial L_{j,t}} \right]$$

So $V_{KL} = V_{LK} > 0$ whenever

$$\text{(II.19)} \quad \frac{L_{j,t}}{\lambda^*_{j,t+1}} \frac{\partial \lambda^*_{j,t+1}}{\partial L_{j,t}} < \frac{1-\lambda^*_{j,t+1}}{\lambda^*_{j,t+1}} \left[\frac{(1+\lambda^*_{j,t+1}) L_{j,t} \frac{\partial^2 f_{j,t+1}}{\partial K_{j,t} \partial L_{j,t}}}{2(1+\lambda^*_{j,t+1} \frac{\partial f_{j,t+1}}{\partial K_{j,t}})} \right]$$

which will presumably hold for sufficiently small $\lambda^*_{j,t+1}$.

Conditions (11) and (12) describe the "normal case" for which the probability of default ($\lambda^*_{j,t+1}$) is sufficiently small to make both V_{LR} and V_{LW} negative and V_{KL} positive. For this case, application of Cramer's Rule to the system (II.9) and (II.10) indicates that the optimal levels of $K_{j,t}$ and $L_{j,t}$ have unambiguously negative partial derivatives with respect to both $(1+R_t)$ and W_t/P_{t+1}.

Next consider the case in which <u>the credit rationing constraint is binding</u>, so that $L_{j,t}$ is defined by (3) with $B_{j,t} = \bar{B}_{j,t}$. To derive (10) we examine the signs of the second derivatives in the identity:

$$\text{(II.20)} \quad V_{KK} dk = - V_{KR}\, d(1+R_t) - V_{KW}\, d\left(\frac{W_t}{P_{t+1}} \right) - V_{K\bar{B}}\, d\left(\frac{\bar{B}_{j,t}}{P_{t+1}} \right)$$

where $V_{K\bar{B}} = \frac{\partial^2 V_j(\cdot_t)}{\partial K_{j,t} \partial(\bar{B}_{j,t}/P_{t+1})}$. We again differentiate (II.7) using both equation (6) and

$$(II.21)\quad dL_{j,t} = \frac{P_{t+1}}{W_t}\, d\left(\frac{\bar{B}_{j,t}}{P_{t+1}}\right) - \left(\frac{P_{t+1}}{W_t}\right)^2 \frac{\bar{B}_{j,t}}{P_{t+1}}\, d\left(\frac{W_t}{P_{t+1}}\right)$$

The terms V_{KK} and V_{KR} are unchanged from the previous case. Moreover:

$$(II.22)\quad V_{KW} = \gamma_j^F \beta_j^F \int_{\lambda^*_{j,t+1}}^{1} \left[- \lambda_{j,t+1} \frac{\partial^2 f_{j,t+1}}{\partial K_{j,t} \partial L_{j,t}} \frac{\bar{B}_{j,t}}{P_{t+1}} \left(\frac{P_{t+1}}{W_t}\right)^2 \right] d\lambda_{j,t+1}$$

$$- \beta_j^F \gamma_j^F \left[1 + \lambda^*_{j,t+1} \frac{\partial f_{j,t+1}}{\partial K_{j,t}} \right] \frac{\partial \lambda^*_{j,t+1}}{\partial (\frac{W_t}{P_{t+1}})} < 0$$

since

$$(II.23)\quad \frac{\partial \lambda^*_{j,t+1}}{\partial (\frac{W_t}{P_{t+1}})} = \frac{\lambda^*_{j,t+1}}{f_{j,t+1}} \frac{\partial f_{j,t+1}}{\partial L_{j,t}} \frac{\bar{B}_{j,t}}{P_{t+1}} \left(\frac{P_{t+1}}{W_t}\right)^2 > 0$$

By contrast

$$(II.24)\quad V_{K\bar{B}} = \gamma_j^F \beta_j^F \int_{\lambda^*_{j,t+1}}^{1} \lambda_{j,t+1} \frac{\partial^2 f_{j,t+1}}{\partial K_{j,t} \partial L_{j,t}} \left(\frac{P_{t+1}}{W_t}\right) d\lambda_{j,t+1}$$

$$- \beta_j^F \gamma_j^F \left[1 + \lambda^*_{j,t+1} \frac{\partial f_{j,t+1}}{\partial K_{j,t}} \right] \frac{\partial \lambda^*_{j,t+1}}{\partial (\bar{B}_{j,t}/P_{t+1})}$$

is ambiguous in sign, with

$$(II.25)\quad \frac{\partial \lambda^*_{j,t+1}}{\partial \left(\frac{\bar{B}_{j,t}}{P_{t+1}}\right)} = \frac{(1+R_t)}{f_{j,t+1}} - \frac{\lambda^*_{j,t+1}}{f_{j,t+1}} \frac{\partial f_{j,t+1}}{\partial L_{j,t}} \left(\frac{P_{t+1}}{W_t}\right)$$

Equation (10), which follows immediately from equation (II.20) and the signs we have established for the second derivations, describes the "normal" case in which $V_{K\bar{B}} > 0$. It is straightforward to show that this is the case for which the probability of default ($\lambda^*_{j,t+1}$) is sufficiently small to satisfy equation (II.19). As indicated by equation (II.24), this amounts to the case in which the rise in the marginal "value" of capital associated with an increase in the amount of labor the firm can employ (the first term on the right hand side of equation (II.24)) outweighs the decrease in the marginal value of capital associated with a higher probability of default (the second term on the right hand side of equation (II.24)).

The Bank

This Appendix derives conditions (27)-(30) in the text, which describe the behavior of the bank. It also establishes that it is not optimal for the risk-neutral bank to hold excess reserves.

The first order conditions for the bank's optimal lending, equity and excess reserve levels are derived by differentiating the value function described by equation (22) in the text. To establish that it is optimal not to hold any excess reserves, note that, with nominal excess reserve holdings denoted by $X_{i,t}$, the bank's balance sheet constraint would be altered from (13) to

$$(\text{III}.1) \qquad \sum_{j=1}^{n} B_{i,j,t} + X_{i,t} = (1-k)D_{i,t} + S_{i,t}$$

and the value function would be modified by subtracting $\frac{X_{i,t}}{(1-k)P_{t+1}}$ from the bracketed expression that multiplies ϕ_i^D in equation (22). Accordingly, using (26), (20), and (III.1), we would have

$$(\text{III}.2) \qquad \frac{\partial V_i(\cdot_t)}{\partial\left[\frac{X_{i,t}}{P_{i,t+1}}\right]} = -\beta_i^I \gamma_i^I \left(\frac{1+r_t-k}{1-k}\right) - \frac{\phi_i^D}{1-k}$$

This derivative is negative since ϕ_i^D is positive when $\bar{D}_{i,t}$ is binding and zero otherwise. This implies that $X_{i,t} = 0$ is always optimal.

To obtain the derivatives of the value function with respect to the choice variables $B_{j,t}/P_{t+1}$ and $S_{i,t}/P_{t+1}$ (recalling that $B_{j,t} = B_{i,j,t}$), we can use conditions (24)-(26) and (20). By differentiating equation (20) with respect to the choice variables, after using (13) to substitute out $D_{i,t}$, it can be seen from (24)-(26) that

$$(\text{III}.3) \qquad \frac{\partial V_i(\cdot_t)}{\partial\left[\frac{B_{j,t}}{P_{t+1}}\right]} = \beta_i^I \gamma_i^I \left\{ (1-\lambda_{j,t+1}^*)(1+R_t) + \frac{(\lambda_{j,t+1}^*)^2}{2} \frac{\partial f_{j,t+1}}{\partial L_{j,t}} \frac{P_{t+1}}{W_t}\right.$$

$$+ \frac{(\lambda^*_{j,t+1})^2}{2} \frac{\partial f_{j,t+1}}{\partial K_{j,t}} \frac{\partial K_{j,t}}{\partial (B_{j,t}/P_{t+1})} - \frac{1-r_t+k}{1-k}$$

$$- m_1 - m_2 \frac{\partial \lambda^*_{j,t+1}}{\partial (B_{j,t}/P_{t+1})} \Bigg\} - \frac{\phi^D_i}{1-k} - \phi^S_i\ s$$

$$\text{(III.4)} \qquad \frac{\partial V_i(\cdot_t)}{\partial \left[\frac{S_{i,t}}{P_{t+1}} \right]} = -\gamma^I_i(1+\rho_{t+1}) + \beta^I_i\gamma^I_i \left[\frac{1+r_t-k}{1-k} \right] + \frac{\phi^D_i}{1-k} + \phi^S_i$$

Accordingly, conditions (27) and (28) follow immediately at an optimum point where both of these derivatives vanish.

For the case in which the bank's operations are constrained by both the ceiling on deposits and the minimum capital requirement, it is straightforward to show that (29) follows from (13). For the case in which the deposit ceiling is binding ($\phi^D_i>0$) but the minimum capital requirement is not binding ($\phi^S_i=0$), we can substitute (28) into (III.3) to obtain

$$\text{(III.5)} \qquad \frac{\partial V_i(\cdot_t)}{\partial \left[\frac{B_{j,t}}{P_{t+1}} \right]} = \beta^I_i\gamma^I_i \Bigg\{ (1-\lambda^*_{j,t+1})(1+R_t)$$

$$+ \frac{(\lambda^*_{j,t+1})^2}{2} \frac{\partial f_{j,t+1}}{\partial L_{j,t}} \frac{P_{t+1}}{W_t}$$

$$+ \frac{(\lambda^*_{j,t+1})^2}{2} \frac{\partial f_{j,t+1}}{\partial K_{j,t}} \frac{\partial K_{j,t}}{\partial (B_{j,t}/P_{t+1})}$$

$$- m_1 - m_2 \frac{\partial \lambda^*_{j,t+1}}{(\partial B_{j,t}/P_{t+1})} \Bigg\} -\gamma^I_i\ (1+\rho_{t+1})$$

Letting $V_{B\bullet}$--for $\bullet$ = B, R, W, m_1, m_2, ρ--respectively denote the partial derivatives of $\frac{\partial V_i(\cdot_t)}{\partial(B_{j,t}/P_{t+1})}$ with respect to $B_{j,t}/P_{t+1}$, $1+R_t$, W_t/P_{t+1} m_1, m_2, and $1+\rho_{t+1}$, we can characterize the optimal choice of $B_{j,t}/P_{t+1}$ by considering the identity

$$\text{(III.6)} \quad - V_{BB}\, d(B_{j,t}/P_{t+1}) = V_{BR}\, d(1+R_t) + V_{BW}\, d(W_t/P_{t+1}) + V_{Bm_1}\, dm_1 + V_{Bm_2}\, dm_2 + V_{B\rho}\, d(1+\rho_{t+1})$$

From the second-order conditions for a maximum, we know $V_{BB}<0$. The signs of V_{BR}, V_{BW}, V_{Bm}, V_{Bm_1}, and $V_{B\rho_2}$ can be established by differentiating (III.5) and using the information that $V_{BB}<0$. In general, it is straightforward to show that V_{Bm_1} and $V_{B\rho}$ are negative, and that V_{Bm_2} is also negative whenever the elasticity of the optimizing firm's output with respect to credit is less than unity. This establishes (30).

Under the assumptions:

$$\text{(III.7)} \quad \frac{\partial}{\partial(1+R_t)} \frac{\partial K_{j,t}}{\partial(B_{1,t}/P_{t+1})} = \frac{\partial}{\partial(W_t/P_{t+1})} \frac{\partial K_{j,t}}{\partial(B_{j,t}/P_{t+1}} = 0$$

it can be shown that

$$\text{(III.8)} \quad V_{BW} = \beta_i^I \gamma_i^I \left\{\left[1+R_t + \frac{m_2}{(B_{j,t}/P_{t+1})}\right] \frac{\lambda^*_{j,t+1}}{(W_t/P_{t+1})}(1-\epsilon_{fB})(\epsilon_{fK}\epsilon_{KB}-\epsilon_{fL}) - \frac{1+R_t}{2(W_t/P_{t+1})}\epsilon_{fL}\right\}$$

$$\text{(III.9)} \quad V_{BR} = \beta_i^I \gamma_i^I \left\{1 - \frac{m_2}{f_{j,t+1}}(1-\epsilon_{fB})(1-\epsilon_{fR}) - \lambda^*_{j,t+1}\left[1 + (1-\epsilon_{fB})(1-\epsilon_{fR})\right]\right\}$$

where $\epsilon_{f.}$ and $\epsilon_{K.}$--for . = B, K, L, R--represent the elasticities of $f_{j,t+1}$ and $K_{j,t}$ with respect to $B_{j,t}/P_{t+1}$, $K_{j,t}$, $L_{j,t}$, and $1+R_t$. Note that for small $\lambda^*_{j,t+1}$, $V_{BW} < 0$, and (for $m_2/f_{j,t+1}$ also small) $V_{BR} < 0$. Note also that $\epsilon_{fK} > 0$, $\epsilon_{fL} > 0$, $\epsilon_{fR} < 0$ (from (10)), and under normal conditions $\epsilon_{KB} > 0$ (from (10)). Accordingly, when there are decreasing returns to the scale of credit (i.e., when $\epsilon_{fB} < 1$), it can be seen from (III.8) that $V_{BW} < 0$ will hold in general if $\epsilon_{fL} > \epsilon_{fK}\,\epsilon_{KB}$. Moreover, it can be seen from (III.9) that $V_{BR} > 0$ will

hold for $\lambda^*_{j,t+1} > \left[1 - \frac{m_2}{f_{j,t+1}} (1-\epsilon_{fB})(1-\epsilon_{fR})\right] / \left[1+(1-\epsilon_{fB})(1-\epsilon_{fR})\right]$.

The Household

This appendix derives conditions (35)-(39) in the text, which describe the behavior of the household. The first order conditions for the household's supply of labor and real deposit holdings are derived by differentiating the household's objective function, as described by condition (33) in the text, to obtain the general form

$$\text{(IV.1)} \quad \frac{\partial V_h(\cdot_t)}{\partial Z_t} = \frac{\partial U_{h,t}}{\partial Z_t} + \frac{\partial U_{h,t}}{\partial C_{h,t}} \frac{\partial C_{h,t}}{\partial Z_t} + \beta_h^H E_t \left\{ \frac{\partial V_h(\cdot_{t+1})}{\partial(\cdot_{t+1})} \frac{\partial(\cdot_{t+1})}{\partial Z_t} \right\}$$

where Z_t represents the relevant choice variable for period t (i.e., $L_{h,t}$ or $D_{h,t}/P_{t+1}$). Note from (34) that $\frac{\partial C_{h,t}}{\partial L_{h,t}} = \frac{W_t}{P_t}$ and $\frac{\partial C_{h,t}}{\partial(D_{h,t}/P_{t+1})} = -(1+\rho_{t+1})$;

also note from updating (34) that $\frac{\partial(\cdot_{t+1})}{\partial L_{h,t}} = 0$ and $\frac{\partial(\cdot_{t+1})}{\partial(D_{h,t}/P_{t+1})} = 1+r_t$.

Finally note, as the Benveniste-Scheinkman condition, that

$\frac{\partial V_h(\cdot_{t+1})}{\partial(\cdot_{t+1})} = \frac{\partial U_{h,t+1}}{\partial C_{h,t+1}}$. 1/ Thus, it is readily seen that (35) and (36) must hold at an optimum.

Consider next the system of steady-state equations obtained by totally differentiating the first order conditions (equations (35) and (36)) and the budget constraint (equation (34)) under the steady-state conditions

$C_{h,t} = C_{h.t+1}$ and $\frac{D_{h,t-1}}{P_t} = \frac{D_{h,t}}{P_{t+1}}$. This system of equations can be written as:

$$\text{(IV.2)} \quad \frac{W_t}{P_t} U_{CC} dC_{h,t} + U_{LL} dL_{h,t} = - U_C d(W_t/P_t)$$

$$\text{(IV.3)} \quad [\beta_h^H(1+r_t) - (1+\rho_{t+1})] U_{CC} dC_{h,t} + U_{DD} d(D_{h,t}/P_{t+1}) =$$

$$- \beta_h^H U_C \, dr_t + U_C \, d\rho_{t+1}$$

1/ See the discussion in Sargent (1987), pp. 21-22.

$$\text{(IV.4)}\quad dC_{h,t} + (\rho_{t+1} - r_t)d(D_{h,t}/P_{t+1}) - \frac{W_t}{P_t}\, dL_{h,t} = L_{h,t}d(W_t/P_t)$$
$$+ (D_{h,t}/P_{t+1})dr_t - (D_{h,t}/P_{t+1})d\rho_{t+1}$$

where $U_C = \dfrac{\partial U_{h,t}}{\partial C_{h,t}}$, $U_{CC} = \dfrac{\partial^2 U_{h,t}}{\partial C^2_{h,t}}$, $U_{DD} = \dfrac{\partial^2 U_{h,t}}{\partial (D_{h,t}/P_{t+1})^2}$,

$U_{LL} = \dfrac{\partial^2 U_{h,t}}{\partial L^2_{h,t}}$, and all second-order cross derivatives vanish (recall assumption (31)). The left-hand side of this system has the matrix form

$$\begin{bmatrix} (W_t/P_t)U_{CC} & 0 & U_{LL} \\ [\beta^H_h(1+r_t) - (1+\rho_{t+1})]U_{CC} & U_{DD} & 0 \\ 1 & -(r_t-\rho_{t+1}) & -(W_t/P_t) \end{bmatrix} \begin{bmatrix} dC_{h,t} \\ d(D_{h,t}/P_{t+1}) \\ dL_{h,t} \end{bmatrix}$$

and the determinant of the square matrix is

$$\text{(IV.5)}\quad \Delta = -(W_t/P_t)^2\, U_{CC}U_{DD} - (r_t-\rho_{t+1})\,[\beta^H_h\,(1+r_t) - (1+\rho_{t+1})U_{CC}U_{LL}$$
$$- U_{DD}U_{LL}$$

Notice that, by differentiating (35) and (36) subject to (34) and the steady-state conditions,

$$\text{(IV.6)}\quad \frac{\partial^2 V_h(\cdot_t)}{\partial L^2_{h,t}} = U_{LL}$$

$$\text{(IV.7)}\quad \frac{\partial^2 V_h(\cdot_t)}{\partial L_{h,t}\partial(D_{h,t}/P_{t+1})} = \frac{W_t}{P_t}\, U_{CC}\, \frac{\partial C_{h,t}}{\partial(D_{h,t}/P_{t+1})} = (r_t-\rho_{t+1})\, \frac{W_t}{P_t}\, U_{CC}$$

$$\text{(IV.8)} \quad \frac{\partial^2 V_h(\cdot_t)}{\partial (D_{h,t}/P_{t+1})^2} = [\beta_h^H (1+r_t) - (1+\rho_{t+1})](r_t - \rho_{t+1})U_{CC} + U_{DD}$$

The second order condition for a maximum is:

$$\text{(IV.9)} \quad \frac{\partial^2 V_h(\cdot_t)}{\partial L_{h,t}^2} \frac{\partial^2 V_h(\cdot_t)}{\partial (D_{h,t}/P_{t+1})^2} - \left[\frac{\partial^2 V_h(\cdot_t)}{\partial L_{h,t} \partial (D_{h,t}/P_{t+1})} \right]^2 > 0$$

Accordingly, from (IV.6)-(IV.9)

$$\text{(IV.10)} \quad -U_{DD} U_{LL} < (r_t - \rho_{t+1})[\beta_h^H(1+r_t) - (1+\rho_{t+1})] U_{CC} U_{LL}$$

$$-(r_t - \rho_{t+1})^2 \left(\frac{W_t}{P_t} \right)^2 (U_{CC})^2$$

Thus, by combining (IV.5) and (IV.10), it can be seen that

$$\text{(IV.11)} \quad \Delta < -\left(\frac{W_t}{P_t} \right)^2 U_{CC} U_{DD} - (r_t - \rho_{t+1})^2 \left(\frac{W_t}{P_t} \right)^2 (U_{CC})^2 < 0$$

By applying Cramer's Law, the system (IV.2)-(IV.4) can be solved to yield:

$$\text{(IV.12)} \quad dC_{h,t} = \frac{1}{\Delta} \left\{ \left(\frac{W_t}{P_t} U_C U_{DD} - L_{h,t} U_{LL} U_{DD} \right) d \left[\frac{W_t}{P_t} \right] \right.$$

$$+ \left[\beta_h^H (r_t - \rho_{t+1}) U_C U_{LL} - (D_t/P_{t+1}) U_{LL} U_{DD} \right] dr_t$$

$$\left. - \left[(r_t - \rho_{t+1}) U_C U_{LL} - (D_t/P_{t+1}) U_{LL} U_{DD} \right] d\rho_{t+1} \right\}$$

$$\text{(IV.13)} \quad d\left(\frac{D_{h,t}}{P_{t+1}} \right) = \frac{1}{\Delta} \left\{ \left[\beta_h^H (1+r_t) - (1-\rho_{t+1}) \right] \left(L_{h,t} U_{LL} - \frac{W_t}{P_t} U_{CC} \right) U_{CC} d \left[\frac{W_t}{P_t} \right] \right.$$

$$+\left[\beta_h^H\left(\frac{W_t}{P_t}\right)^2 U_C U_{CC}+\left[\beta_h^H(1+r_t)-(1+\rho_{t+1})\right]\frac{D_{h,t}}{P_{t+1}}U_{LL}U_{CC}+\beta_h^H U_C U_{LL}\right]dr_t$$

$$-\left[\left(\frac{W_t}{P_t}\right)^2 U_C U_{CC}+\left[\beta_h^H(1+r_t)-(1+\rho_{t+1})\right]\frac{D_{h,t}}{P_{t+1}}U_{LL}U_{CC}+U_C U_{LL}\right]d\rho_{t+1}\Bigg\}$$

$$\text{(IV.14)}\quad dL_{h,t}=\frac{1}{\Delta}\left\{\left[\frac{W_t L_{h,t}}{P_t}U_{CC}U_{DD}+\left((r_t-\rho_{t+1})\left[\beta_h^H(1+r_t)-(1+\rho_{t+1})\right]U_{CC}+U_{DD}\right)U_C\right]d\left[\frac{W_t}{P_t}\right]\right.$$

$$+\left[\frac{W_t}{P_t}\frac{D_{h,t}}{P_{t+1}}U_{CC}U_{DD}-\beta_h^H\frac{W_t}{P_t}(r_t-\rho_{t+1})U_C U_{CC}\right]dr_t$$

$$\left.-\left[\frac{W_t}{P_t}\frac{D_{h,t}}{P_{t+1}}U_{CC}U_{DD}-\frac{W_t}{P_t}(r_t-\rho_{t+1})U_C U_{CC}\right]d\rho_{t+1}\right\}$$

Note also, from (36), that

$$\text{(IV.15)}\quad [\beta_h^H(1+r_t)-(1+\rho_{t+1})]<0$$

since $\frac{\partial U_{h,t}}{\partial C_{h,t}}>0$ and $\frac{\partial U_{h,t}}{\partial(D_{h,t}/\rho_{t+1})}>0$.

Accordingly, it can be seen that, in general:

$\frac{\partial C_{h,t}}{\partial(W_t/P_t)}>0$, $\frac{\partial D_{h,t}}{\partial(W_t/P_t)}>0$, $\frac{\partial D_{h,t}}{\partial r_t}>0$, and $\frac{\partial D_{h,t}}{\partial \rho_{t+1}}<0$. It can also be seen from (IV.14) that $\frac{\partial L_{h,t}}{\partial(W_t/P_t)}>0$ whenever $r_t-\rho_{t+1}<0$ and $U_c-\frac{W_t L_{h,t}}{P_t}U_{cc}>0$, or since $\frac{\partial^2 V_h(\cdot_t)}{\partial(D_{h,t}/P_{t+1})^2}<0$ must hold as a second order condition (recall (IV.8)), whenever $r_t-\rho_{t+1}>0$ and U_{DD} is negligible. This explains the "unambiguous" signs shown in (37)-(39). Note further that, when the expected real interest rate $(r_t - \rho_{t+1})$ is positive: $\frac{\partial C_{h,t}}{\partial r_t} > 0$ and $\frac{\partial C_{h,t}}{\partial \rho_{t+1}} < 0$ (i.e., the income effect on consumption dominates the substitution effect), $\frac{\partial L_{h,t}}{\partial r_t} < 0$, and $\frac{\partial L_{h,t}}{\partial \rho_{t+1}} > 0$.

Steady State Solutions

As noted in the text, equations (40)-(43) describe a financially repressed economy's long-run position. Using these relationships, this appendix first derives the slopes of the curves in Figure 1 and then considers the effects on the economy of alternative financial policies.

1. Slopes of curves in Figure 1

Curve 1 in the northeast quadrant of Figure 1 represents the combinations of $(1+R_t)$ and W_t/P_{t+1} under which condition (40) holds as an equality. The slope of curve 1 equals 1/

$$\text{(V.1)}\qquad \frac{d(1+R_t)}{d(W_t/P_{t+1})} = \frac{\sum\limits_h \frac{\partial L^H_{h,t}}{\partial(W_t/P_t)}(1+\rho_{t+1}) - \sum\limits_j \frac{\partial L^F_{j,t}}{\partial(W_t/P_{t+1})}}{\sum\limits_j \frac{\partial L^F_{j,t}}{\partial(1+R_t)}}$$

In curve 2, the segment AB, corresponding to condition (41b), represents the situation when

$$\text{(V.2)}\qquad \sum_h \frac{(1-k)}{(1-s)} \frac{D^H_{h,t}}{P_t}\left[r_t,\ (W_t/P_{t+1})(1+\rho_{t+1}),\ 1+\rho_{t+1}\right] =$$

$$\sum_h \left(\frac{W_t}{P_{t+1}}\right)(1+\rho_{t+1})\ L^H_{h,t}\left(r_t, \left(\frac{W_t}{P_{t+1}}\right)(1+\rho_{t+1}),\ 1+\rho_{t+1}\right)$$

where $\frac{W_t}{P_{t+1}}(1+\rho_{t+1})$ has been substituted for W_t/P_t. Since r_t, ρ_{t+1}, k and s are given in the steady state by the authorities monetary and financial policies, there is only one value of W_t/P_{t+1} $(\bar{W}/P)$ that satisfies (V.2). Changes in $(\bar{W}/P)$ will be related to the other variables by

---

1/ These derivatives use the fact that $W_t/P_t = (W_t/P_{t+1})(P_{t+1}/P_t) = (W_t/P_{t+1})(1+\rho_{t+1})$. Thus, $L^H_{h,t}(r_t,\ W_t/P_t,\ 1+\rho_{t+1}) = L^H_{h,t}(r_t,\ (W_t/P_{t+1})(1+\rho_{t+1}),\ 1+\rho_{t+1})$.

$$\text{(V.3)} \quad \sum_h \left[\frac{(1-k)}{(1-s)} \frac{\partial (D^H_{h,t}/P_t)}{\partial (W_t/P_t)} - \left(\frac{W_t}{P_{t+1}}\right) (1+\rho_{t+1}) \frac{\partial L^H_{h,t}}{\partial (W_t/P_t)} - L^H_{h,t}(1+\rho_{t+1}) \right] d\left(\frac{\bar{W}}{P}\right) \quad (-)$$

$$= \sum_h \left[\left(\frac{W_t}{P_{t+1}}\right)(1+\rho_{t+1}) \frac{\partial L^H_{h,t}}{\partial r_t} - \frac{(1-k)}{(1-s)} \frac{\partial (D^H_{h,t}/P_t)}{\partial r_t} \right] dr_t \quad (-)$$

$$+ \sum_h \left[- \frac{(1-k)}{(1-s)} \frac{\partial (D^H_{h,t}/P_t)}{\partial \rho_{t+1}} - \frac{(1-k)}{(1-s)} \frac{\partial (D^H_{h,t}/P_t)}{\partial (W_t/P_t)} \left(\frac{W_t}{P_{t+1}}\right) \right. \quad (?)$$

$$\left. + \left(\frac{W_t}{P_{t+1}}\right) (1+\rho_{t+1}) \frac{\partial L^H_{h,t}}{\partial (1+\rho_{t+1})} + \left(\frac{W_t}{P_{t+1}}\right)^2 (1+\rho_{t+1}) \frac{\partial L^H_{h,t}}{\partial (W_t/P_t)} \right] d\rho_{t+1}$$

$$+ \sum_h \frac{D^H_h/P_t}{(1-s)} dk \; (+) - \sum_h \frac{(1-k)}{(1-s)^2} (D^H_{h,t}/P_t)\, ds \; (-)$$

where the signs reflect the assumptions that the households' labor supply is more sensitive to changes in the real wage than their demand for deposits, and that the demand for deposits is more sensitive to changes in r than the supply of labor. Under these assumptions, $\frac{\bar{W}}{P}$ is an increasing function of r_t and s and a decreasing function of k.

The segment BC of curve 2, corresponding to condition (41a) represents the situation where the banks hold more than the minimum required level of equity and

$$\text{(V.4)} \quad \sum_j \sum_j \frac{B_{i,j,t}}{P_{t+1}} = \sum_i \sum_j b_{i,j,t} \left(1+R_t, \frac{W_t}{P_{t+1}}, m_1, m_2, 1+\rho_{t+1}\right)$$

$$= \sum_h \left(\frac{W_t}{P_{t+1}}\right) L^H_{h,t} \left(r_t, W_t/P_t, 1+\rho_{t+1}\right)$$

This implies that

$$
\begin{aligned}
\text{(V.5)}\qquad 0 = {} & \overset{(?)}{\sum_i \sum_j \frac{\partial b_{i,j,t}}{\partial(1+R_t)}}\, d(1+R_t) \\
& + \overset{(-)}{\left[\sum_i \sum_j \frac{\partial b_{i,j,t}}{\partial\left(\frac{W_t}{P_{t+1}}\right)} - \sum_h L^H_{h,t} - \sum_h \frac{W_t}{P_{t+1}} \frac{\partial L^H_{h,t}}{\partial(W_t/P_t)} (1+\rho_{t+1})\right]} d\left(\frac{W_t}{P_{t+1}}\right) \\
& + \overset{(-)}{\sum_i \sum_j \frac{\partial b_{i,j,t,}}{\partial m_1}}\, dm_1 + \overset{(-)}{\sum_i \sum_j \frac{\partial b_{i,j,t}}{\partial m_2}}\, dm_2 \\
& - \overset{(?)}{\sum_h \left(\frac{W_t}{P_{t+1}}\right) \frac{\partial L^H_{h,t}}{\partial r_t}}\, dr_t \\
& - \overset{(?)}{\left[\sum_h \left(\frac{W_t}{P_{t+1}}\right) \frac{\partial L^H_{h,t}}{\partial(1+\rho_{t+1})} + \sum_h \left(\frac{W_t}{P_{t+1}}\right)^2 \frac{\partial L^H_{h,t}}{\partial(W_t/P_t)} - \sum_i \sum_j \frac{\partial b_{i,j,t}}{\partial(1+\rho_{t+1})}\right]} d\rho_{t+1}
\end{aligned}
$$

The slope of curve (2) along segments BC therefore equals

$$
\text{(V.6)}\qquad \left.\frac{d(1+R_t)}{d\left(\frac{W_t}{P_{t+1}}\right)}\right|_{(2)} = \frac{-\left[\sum_i \sum_j \frac{\partial b_{i,j,t}}{\partial(W_t/P_{t+1})} - \sum_h L^H_{h,t} - \sum_h \left(\frac{W_t}{P_{t+1}}\right) \frac{\partial L^H_{h,t}}{\partial(W_t/P_{t+1})} (1+\rho_{t+1})\right]}{\sum_i \sum_j \frac{\partial b_{i,j,t}}{\partial(1+R_t)}}
$$

$$
\gtrless 0 \quad \text{as} \quad \frac{\partial b_{i,j,t}}{\partial(1+R_t)} \gtrless 0
$$

Curve 3 (in the northwest quadrant of Figure 1) also has two segments: DE and EF. As discussed in the main text, the segment DE is defined only in the range of loan interest rates between R_{**} and R_{*}, which corresponds to the range in which ϕ_1^D and ϕ_1^S are both greater than zero. Using the banks' first order conditions (equations (27) and (28)) this implies that

$$(V.7) \quad - s\gamma_i^I \left[\frac{(1-r_t-k)}{(1-k)} \beta_i^I - (1+\rho_{t+1}) \right] <$$

$$\gamma_i^I \beta_i^I \left\{ (1-\lambda^*_{j,t,1}) (1+R_t) + \frac{\lambda^{*2}_{j,t+1}}{2} \frac{\partial f_{j,t+1}}{\partial L_{j,t}} (P_{t+1}/W_t) \right.$$

$$+ \frac{\lambda^{*\,2}_{j,t+1}}{2} \frac{\partial f_{j,t+1}}{\partial K_{j,t}} \frac{\partial K_{j,t}}{\partial (B_{i,j,t}/P_{t+1})} - \frac{(1-r_t-k)}{(1-k)}$$

$$\left. - m_1 - m_2 \frac{\partial \lambda^*_{j,t+1}}{\partial (B_{i,j,t}/P_{t+1})} \right\} < - \gamma_i^I \left[\frac{(1+r_t-k)}{1-k} \beta_i^I - (1+\rho_{t+1}) \right]$$

At R_*, the expression in the center of the above inequality becomes equal to the upper bound value in the inequality (corresponding to $\phi_1^S = 0$).

If all bank owners are identical ($\beta_i^I = \beta^I$ and $\gamma_i^I = \gamma^I$), then

$$(V.8) \quad \gamma^I \beta^I \left\{ (1-\lambda^*_{j,t+1}) (1+R_t) + \frac{\lambda^{*\,2}_{j,t+1}}{2} \frac{\partial f_{j,t+1}}{\partial L_{j,t}} (P_{t+1}/W_t) \right.$$

$$+ \frac{\lambda^{*\,2}_{j,t+1}}{2} \frac{\partial f_{j,t+1}}{\partial K_{j,t}} \frac{\partial K_{j,t}}{\partial (B_{i,j,t}/P_{t+1})} - \frac{(1+r_t-k)}{(1-k)}$$

$$\left. - m_1 - m_2 \frac{\partial \lambda^*_{j,t+1}}{\partial (B_{i,j,t}/P_{t+1})} \right\} = - \gamma^I \left[\frac{(1+r_t-k)}{(1-k)} \beta^I - (1+\rho_{t+1}) \right]$$

and $B_{i,j,t}/P_{t+1} = \theta_{i,j} \frac{(1-k)}{(1-s)} \sum_h \frac{D^H_{i,h,t}}{P_{t+1}}$ where $\theta_{i,j}$ is the share of bank i's total funds that it is optimal to make available to firm j when the bank holds only the minimum required amount of equity.

Equation (V.8) can be solved for the R's at which the bank would find it optimal to hold the minimum required level of equity. Since the left hand side of (V.8) will be a positive function of R when $\lambda^*_{j,t+1}$ is low and a negative function of R when $\lambda^*_{j,t+1}$ becomes high enough, there is both a low value (R_*) and a high value (R^*) at which the bank would find it optimal to hold only the minimum required level of equity. Above R^* the bank would not want to fully utilize all of the deposits made available to it by the households at the ceiling deposit interest rate, and the official ceiling interest rate would therefore no longer be a binding constraint.

As discussed in the text, the derivation of segment EF requires use of segment BC on Curve 2. Using equation (V.4) we will have

$$\text{(V.9)} \quad \sum_i \sum_j d\left(\frac{B_{i,j,t}}{P_{t+1}}\right) = \sum_i \sum_j \frac{\partial b_{i,j,t}}{(1+R_t)} d(1+R_t) + \sum_i \sum_j \frac{\partial b_{i,j,t}}{\partial\left(\frac{W_t}{P_{t+1}}\right)} d\left(\frac{W_t}{P_{t+1}}\right)$$

$$+ \sum_i \sum_j \frac{\partial b_{i,j,t}}{\partial m_1} dm_1 + \sum_i \sum_j \frac{\partial b_{i,j,t}}{\partial m_2} dm_2 + \sum_i \sum_j \frac{\partial b_{i,j,t}}{\partial(1+\rho_{t+1})} d\rho_{t+1}$$

with $d\left(\frac{W_t}{P_{t+1}}\right)$ given by solving equation (V.5).

Letting

$$\text{(V.10)} \quad z = \frac{\overset{(+)}{\left[\sum_h L^H_{h,t} + \sum_h (W_t/P_{t+1})\, \partial L^H_{h,t}/\partial(W_t/P_t)\, (1+\rho_{t+1})\right]}}{\underset{(-)}{\left[\sum_i \sum_j \frac{\partial b_{i,j,t}}{\partial\left(\frac{W_t}{P_{t+1}}\right)} - \sum_h L^H_{h,t} - \sum_h \left(\frac{W_t}{P_{t+1}}\right) \partial L^H_{h,t}/\partial(W_t/P_t)(1+\rho_{t+1})\right]}}$$

and noting that $-1 < z < 0$, it can be shown that substitution of (V.5) into (V.9) yields:

$$\text{(V.11)}\qquad \sum_i \sum_j d\left(\frac{B_{i,j,t}}{P_{t+1}}\right) = \overset{(-)}{-\sum_i \sum_j} \overset{(?)}{\frac{\partial b_{i,j,t}}{\partial(1+R_t)}} \overset{(-)}{zd(1+R_t)} \overset{(-)}{-\sum_i \sum_j} \overset{(-)}{\frac{\partial b_{i,j,t}}{\partial m_1}} \overset{(-)}{zdm_1} \overset{(-)}{-\sum_i \sum_j} \overset{(-)}{\frac{\partial b_{i,j,t}}{\partial m_2}} \overset{(-)}{zdm}$$

$$+ \overset{(+)}{(1+z)} \sum_h \left[\frac{W_t}{P_{t+1}}\right] \overset{(?)}{\frac{\partial L^H_{h,t}}{\partial r_t}} dr_t + \left[\overset{(+)}{(1+z)} \left(\sum_h \left[\frac{W_t}{P_{t+1}}\right] \overset{(?)}{\frac{\partial L^H_{h,t}}{\partial(1+\rho_{t+1})}}\right.\right.$$

$$\left.\left. + \sum_h \left(\frac{W_t}{P_{t+1}}\right)^2 \overset{(+)}{\frac{\partial L^H_{h,t}}{\partial(W_t/P_t)}}\right) \overset{(-)}{- z\sum_i \sum_j} \overset{(-)}{\frac{\partial b_{i,j,t}}{\partial(1+\rho_{t+1})}} \right] d\rho_{t+1}$$

Thus, in curve (3), the real supply of credit can be portrayed as:

$$\text{(V.12)}\qquad \sum_i \sum_j \frac{B_{i,j,t}}{P_{t+1}} = \tilde{\ell}\,((1+R_t),\ m_1,\ m_2,\ r_t,\ 1+\rho_{t+1})$$

with $\frac{\partial \tilde{\ell}}{\partial r_t} < 0$ if and only if $\frac{\partial L^H_{h,t}}{\partial r_t} < 0$. For low values of R, $\frac{\partial \tilde{\ell}}{\partial(1+R_t)} > 0$,

but above some level of R, $\frac{\partial \tilde{\ell}}{\partial(1+R_t)} \leq 0$.

2. Effects of financial policies

a. Banks hold only the minimum required level of equity

Equation (V.2) can be rewritten as:

$$\text{(V.2')}\qquad \sum_h (w_t/P_t)\, L^H_{h,t}(r_t,\ W_t/P_t,\ 1+\rho_{t+1}) = \sum_h \frac{(1-k)}{(1-s)} \frac{D^h_{h,t}}{P_t} \left[\ r_t,\ W_t/P_t,\ 1+\rho_{t+1}\ \right]$$

By differentiating and rearranging terms, we obtain:

$$\text{(V.13)}\qquad \sum_h \left[L^H_{h,t} + (W_t/P_t)\overset{(+)}{\frac{\partial L^H_{h,t}}{\partial(W_t/P_t)}} - \frac{(1-k)}{(1-s)}\frac{\partial\left(\frac{D^H_{h,t}}{P_t}\right)}{\partial(W_t/P_t)}\right] d\left(\frac{W_t}{P_t}\right) =$$

$$\sum_h \left[\frac{(1-k)}{(1-s)}\overset{(+)}{\frac{\partial(D^H_{h,t}/P_t)}{\partial r_t}} - (W_t/P_t)\frac{\partial L^H_{h,t}}{\partial r_t}\right] dr_t$$

$$+ \sum_h \left[\frac{(1-k)}{(1-s)}\overset{(-)}{\frac{\partial(D^H_{h,t}/P_t)}{\partial(1+\rho_{t+1})}} - \overset{(-)}{\left(\frac{W_t}{P_t}\right)}\overset{(?)}{\frac{\partial L^H_{h,t}}{\partial(1+\rho_{t+1})}}\right] d\rho_{t+1}$$

$$\overset{(-)}{-}\sum_h \overset{(+)}{\left(\frac{D^H_{h,t}}{P_t}\right)}\frac{dk}{1-s} + \sum_h \overset{(+)}{\frac{(1-k)}{(1-s)^2}}\frac{D^H_{h,t}}{P_t}\, ds.$$

The above signs assume that the households' desired deposit holdings is more sensitive to changes in r_t than is their desired supply of labor; and the households' desired supply of labor is taken as more sensitive to changes in the real wage than their desired holdings of deposits. As a result,

$$\text{(V.14)}\qquad d\left(\frac{W_t}{P_t}\right) > 0 \text{ if } dr_t > 0$$

$$< 0 \text{ if } dk < 0$$

$$> 0 \text{ if } ds > 0$$

$$< 0 \text{ if } d\rho_{t+1} > \text{ and } \frac{\partial L^H_{h,t}}{\partial(1+\rho_{t+1})} > 0$$

b. <u>Banks hold more than minimum amount of equity</u>

Using equation (V.5)

$$\text{(V.15)}\qquad \left[\sum_i \frac{\partial b_{i,j,t}}{\left(\frac{W_t}{P_{t+1}}\right)} - \sum_h L^H_{h,t} - \overset{(-)}{\sum_h \left(\frac{W_t}{P_{t+1}}\right) \frac{\partial L^H_{h,t}}{W_t/P_t} (1+\rho_{t+1})}\right] d\left(\frac{W_t}{P_{t+1}}\right) =$$

$$- \overset{(-)}{\sum_i \sum_j} \overset{(?)}{\frac{\partial b_{i,j,t}}{\partial (1+R_t)}} d(1+R_t)\ \overset{(-)}{dm_1} - \sum_i \sum_j \overset{(-)}{\frac{\partial b_{i,j,t}}{\partial m_1}} dm_1 - \overset{(-)}{\sum_i \sum_j} \overset{(-)}{\frac{\partial b_{i,j,t}}{\partial m_2}} dm_2$$

$$+ \sum_h \overset{(?)}{\left(\frac{W_t}{P_{t+1}}\right) \frac{\partial L^H_{h,t}}{\partial r_t}} dr_t$$

$$+ \left[\sum_h \overset{(?)}{\left(\frac{W_t}{P_{t+1}}\right) \frac{\partial L^H_{h,t}}{\partial (1+\rho_{t+1})}} + \overset{(+)}{\sum_h \left(\frac{W_t}{P_{t+1}}\right)^2} \overset{(-)}{\frac{\partial L^H_{h,t}}{\partial (W_t/P_t)}} - \sum_i \sum_j \overset{(-)}{\frac{\partial b_{i,j,t}}{\partial (1+\rho_{t+1})}}\right] d\rho_{t+1}$$

Thus:

$$\text{(V.16)}\qquad d\left(\frac{W_t}{P_{t+1}}\right) \gtrless 0 \text{ for } d(1+R_t) > 0 \text{ as } \frac{\partial b_{i,j,t}}{\partial (1+R_t)} \gtrless 0$$

$$< 0 \text{ if } dm_1 > 0$$

$$< 0 \text{ if } dm_2 > 0$$

$$> 0 \text{ if } dr_t > 0 \text{ and } \frac{\partial L^H_{h,t}}{\partial r_t} < 0$$

$$< 0 \text{ if } d\rho_{t+1} > 0 \text{ and either } \frac{\partial L^H_{h,t}}{\partial (1+\rho_{t+1})} > 0 \text{ or the}$$

last two terms in the coefficient on $d\rho_{t+1}$ dominate the first term.

Also, using $\sum\limits_{t} \frac{S_{i,t}}{P_{t+1}} = \sum\limits_{i}\sum\limits_{j} \frac{B_{i,j,t}}{P_{t+1}} - \sum\limits_{h} (1-k) \frac{D^{H}_{h,t}}{P_{t+1}}$, and (V.4): 1/

$$
\begin{aligned}
\text{(V.17)}\qquad \sum_{i} d\left(\frac{S_{i,t}}{P_{t+1}}\right) = {} & \overset{(?)}{\sum_{i}\sum_{j} \frac{\partial b_{i,j,t}}{\partial(1+R_t)}}\, d(1+R_t) \\
& + \left[\sum_{i}\sum_{j} \frac{\partial b_{i,j,t}}{\partial\left(\frac{W_t}{P_{t+1}}\right)} - \sum_{h} (1-k)\, \overset{(-)}{\frac{\partial\left(\frac{D^{H}_{h,t}}{P_{t+1}}\right)}{\partial(W_t/P_t)}}\, (1+\rho_{t+1})\right] d\left(\frac{W_t}{P_{t+1}}\right) \\
& \overset{(-)}{-} (1-k) \sum_{h} \overset{(+)}{\frac{\partial(D^{H}_{h,t}/P_{t+1})}{\partial r_t}}\, dr_t + \overset{(+)}{\sum_{h} \left(\frac{D^{H}_{h,t}}{P_{t+1}}\right)} dk \\
& + \left[\sum_{i}\sum_{j} \frac{\partial b_{i,j,t}}{\partial(1+\rho_{t+1})} - (1-k) \sum_{h} \frac{\partial(D^{H}_{h,t}/P_{t+1})}{\partial(1+\rho_{t+1})}\right. \\
& \left. + (1-k) \sum_{h} \frac{(D^{H}_{h,t}/P_t)}{(1+\rho_{t+1})^2} - \sum_{h} (1-k) \frac{\partial\left(\frac{D^{H}_{h,t}}{P_{t+1}}\right)}{\partial(W_t/P_t)} \frac{W_t}{P_{t+1}}\right] d\rho_{t+1}
\end{aligned}
$$

1/ The coefficient on $d\rho_{t+1}$ reflects

$$\frac{D^{H}_{h}}{P_{t+1}} = \frac{D^{H}_{h}}{P_t} \frac{P_t}{P_{t+1}} = \frac{(D^{H}_{h}/P_t)}{(1+\rho_{t+1})}$$