

# The Return to Foreign Aid

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## Abstract

This paper investigates the marginal productivity of investment in the world's poorest economies. The aim is to estimate the return on investments financed by foreign aid as well as by domestic resource mobilization, using cross-country aggregate data. In practice the return on both investment categories can be expected to vary considerably across countries and time. As a consequence we develop a correlated random coefficients approach to the issue at hand, which allows us to estimate the average aggregate rate of return on "aid investments" and "domestic investments". Across a wide array of estimators our principal finding is remarkably robust; the average aggregate gross return on "aid investments" falls in a 20-30 percent range, roughly the same as the return on investments funded by other sources than aid. This finding is well in accord with micro estimates of the economic return to aid.

**Keywords:** Productivity, foreign aid, random coefficients, panel data.

**JEL classification:** O47, F35, C23

## 1. Introduction

The productivity impact from investments in physical capital is a natural starting point for any analysis of the sources of growth. Indeed, according to some theories, physical capital investments are about the only thing that matters (e.g. Jones and Manuelli, 1990). And according to some empirical studies, the rate of investment is about the only robust determinant of productivity growth (Levine and Renelt,

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1992). While debate persists as to whether physical capital is as important to development as the work cited above could be taken to suggest, it remains uncontroversial that physical capital accumulation is an important part of any country's struggle for economic development. From a practical perspective, investments in physical capital can derive from fundamentally two sources in poor places. Either the economy mobilizes the resources itself (perhaps in cooperation with foreign investors), or capital can be accumulated through foreign aid financed investments. In this paper we seek to provide estimates of the gross rate of return to investments funded by either source, measured at the aggregate level. However, we are particularly preoccupied with the question of how productive foreign aid investments are, since pinning down the productivity of aid financed investments will give an indication of how much international aid transfer's may contribute to the economic development of the third world.

One way to begin thinking about the potential of aid financed investments is to examine the microeconomic evidence obtained at the project level. At this level of aggregation aid investments have long been found to yield sizable economic returns. Nearly two decades ago, Paul Mosley observed that:

The microeconomic data from evaluations are encouraging: all donors who calculated *ex-post* rates of return on their projects reveal a large preponderance of successful projects. The World Bank, the largest development agency, reports average *ex-post* rates of return of over 10 percent in every continent and every sector over the 20 year period 1961-81. [Mosely; 1986, p. 22]

Contemporary micro evidence does not shatter the above image of relative success. On the contrary, the returns cited by Mosely in the mid eighties are still representative. As Tables 1 and 2 document, whether looking across sectors or regions median returns are quite respectable; ranging from 10 to 30 percent.<sup>1</sup>

Against this background a natural next step is to compare macro estimates of the return on aid investments to these micro estimates. If the macro return is larger this is consistent with externalities, perhaps associated with aid investments in roads, telecommunication, irrigation and so forth. Conversely, if the macro return is lower than micro estimates then this is consistent with macro theories that build on ideas involving misallocation of funds, Dutch disease, rent-seeking or the like.

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<sup>1</sup>All the projects summarized in the tables are funded by the World Bank, who is also responsible for the *ex post* evaluations.

Table 1: Median economic rates of return, by sector; 1996-2001

Sector	Number of projects	Real rate of return <sup>a</sup> (percent)
Agriculture	108	20
Electric Power & Other energy	66	15
Transportation	102	29
Oil and Gas	20	30
PSD/Industry	7	17
Telecommunication	16	23
Urban development	27	19
Water supply and sanitation	36	10

<sup>a</sup>This refers to the revised rate of return at evaluation.

Source: Operations Evaluation Department, World Bank, 2001.

Table 2: Median economic rates of return, by region; 1996-2001

Region	Number of projects	Real rate of return <sup>a</sup> (percent)
Africa	78	20
East Asia and Pacific	97	20
Europe and Central Asia	44	28
Latin America and Caribbean	64	20
Middle East and North Africa	32	17
South Asia	79	20

<sup>a</sup>This refers to the revised rate of return at evaluation

Source: Operations Evaluation Department, World Bank, 2001.

Regrettably the empirical literature of the macro impact of aid has not produced estimates of the economic return on aid investments. Macro studies from the last decade have typically run (panel) growth regressions where foreign aid is added to a list of other controls, i.e., the “Barro-regression” approach.<sup>2</sup> Consequently, the estimated impact of aid will in theory depend on both elasticities of the production function as well as preference parameters (Barro and Sala-i-Martin, 1992). As a result, the estimated coefficients are not comparable to the micro evidence cited above.

The present paper aims to fill this gap in the literature. Our approach builds on a set of assumptions, most of which are familiar from the growth accounting literature (Solow, 1957). First, we adopt an aggregate production function, exhibiting constant returns to rival factors of production: physical and human capital. Second, we assume that factor shares reflect the marginal productivity of individual factors

<sup>2</sup>E.g., Burnside and Dollar, 2000; Hansen and Tarp, 2001; Dalgaard, Hansen and Tarp, 2004, and Easterly, Levine and Roodman, 2004. See Roodman (2004) for an overview.

of production. Third we assume aid inflows stimulate the build-up of physical capital.<sup>3</sup>

On this basis we derive an equation that allows us to identify the aggregate real rate of return on aid financed investments using data for a panel of developing countries. From an econometric perspective a number of difficulties arise, which we confront below: it is unlikely that returns are constant over time and space; total factor productivity is unavoidably left in the residuals, and is likely to be correlated with the regressors; aid inflows are endogenous and so on. These complications forces us to examine our data using a number of different estimators. Nevertheless, our principal finding is remarkably robust: overall the average *gross* rate of return on foreign aid appears to be in the range 20-30 percent. This finding conforms well with the micro returns cited above.

It should be emphasized at the outset that the present study does not attempt to address the question of whether aid, as such, increases productivity in the long run. We are only interested in how productive aid financed investments are in their own right. This distinction is important. For example, it may be the case that aid inflows crowds out, say, domestic investments in physical capital. In this case the *net* result from aid transfers could be a productivity decline albeit ‘aid investments’ themselves are productive. Of course, it could also be the case that aid investments stimulate domestic investment efforts. Either way, in order to obtain estimates for the return on aid investments we condition on various production inputs. Consequently it is not possible to assess such claims directly. In this respect the present paper differs fundamentally in scope from the existing literature on aid effectiveness.

The paper proceeds as follows. Section 2 develops a framework suitable for estimating the aggregate return to foreign aid investments. Section 3 discusses our estimation strategy, while section 4 presents our principal results. Finally, section 5 provides concluding remarks.

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<sup>3</sup>While we believe the present paper represents a first attempt at estimating the return on aid financed investments, the fundamental approach is similar to the one adopted in the work aimed at estimating the return on R&D investments or public investments, respectively. The pioneering paper in the former literature is Griliches (1979), in the latter it is Aschauer (1989).

## 2. Theory

Assume output in a developing country is produced using a neoclassical production technology

$$Y(t) = A(t)F(K(t), H(t)), \quad (1)$$

where  $A$  represents total factor productivity,  $H$  human capital, while  $K$  is a composite index of physical capital. Specifically let

$$K(t) \equiv G(K^d(t), K^f(t)), \quad (2)$$

where  $K^d$  is “domestically generated physical capital” (or “domestic capital” for short), and  $K^f$  is aid-financed capital equipment – or simply “aid capital”.<sup>4</sup> We impose constant returns to scale in the three (non-rival) inputs taken together

$$\lambda Y(t) = A(t)F(G(\lambda K^d(t), \lambda K^f(t)), \lambda H(t)).$$

In addition we assume that aid capital is a non-essential production input, that is

$$G(K^d, 0) = \pi K^d,$$

where  $\pi$  is a parameter, which is greater than zero. An example of a function that fulfills this requirement is a standard CES function exhibiting homogeneity of degree one in  $K^d$  and  $K^f$ . Taken together these two assumptions imply that in the event the stock of aid capital is zero, constant returns to human input and (domestic) capital input prevail. As a result, regardless of whether aid is present or not, the production technology is consistent with the national accounts identity which states that total capital and labor compensation equals total value added.

In theory there is good reason to believe that the two types of investment efforts may have different impacts on economic activity. For example, a large fraction of total aid flows comes in the shape of investments in infrastructure (Hjertholm and White, 2000). From this perspective, foreign aid investments may have an economic return above private (equipment) investments. On the other hand, if the government (and donors) are less effective at identifying productive investment projects than the private agents, the impact of aid capital on output may be considerably smaller than that of domestic capital. Moreover, one could easily imagine scenarios where aid capital and domestic capital are either complements or substitutes in generating the aggregate total stock of productive capital  $K$ .

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<sup>4</sup>Needless to say, in practice it is difficult to dichotomize “domestically generated inputs”, and “aid financed inputs” based on national accounts data. We return to this issue below. For now we will simply assume that this distinction is feasible.

These uncertainties represent the reason why we adopt a general specification, rather than, say, a Cobb-Douglas solution. In particular it allows us to avoid making any priors regarding the impact and substitutability of the two kinds of capital. As a result, in the remaining we do not impose *any* conditions on the relative size of the partial derivatives,  $G'_1$  and  $G'_2$ , nor on the cross-partial  $G''_{12}$ . In general the latter could be either positive, negative or zero (perfect substitutes).<sup>5</sup>

Following the recent empirical growth literature (e.g., Bils and Klenow, 2000) we proxy human capital by

$$H(t) = e^{\psi u(t)} L(t), \quad (3)$$

where  $L$  is the (raw) labor force and  $u$  is years of schooling. The parameter  $\psi$  has the interpretation of a Mincerian return to schooling.

Inserting equations (2) and (3) into the production function and differentiating the resulting equation with respect to time yields

$$\hat{Y}(t) = \hat{A}(t) + \frac{F_K G'_1 K^d(t)}{F} \hat{K}^d(t) + \frac{F_K G'_2 K^f(t)}{F} \hat{K}^f(t) + \alpha_L(t) \hat{H}(t), \quad (4)$$

The hat-notation indicates growth rates, e.g.  $\hat{Y}(t) = \dot{Y}/Y$ . Note that  $\hat{H}(t) = \psi \dot{u}(t) + n(t)$ , where  $n$  is the growth rate of the labor force, while  $\alpha_L(t) = (\partial Y / \partial H) H / Y$  represents the share of labor in value added.

Next suppose capital is accumulated according to

$$\dot{K}^i(t) = I^i(t) - \delta^i(t) K^i(t), \quad i = d, f \quad (5)$$

where  $I^i(t), i = d, f$  represents the flow of investments based on domestic savings and foreign aid, respectively. Equation (5) can be restated to yield

$$\hat{K}^i(t) = \frac{Y(t) I^i(t)}{K^i(t) Y(t)} - \delta^i(t), \quad i = d, f.$$

Substituting this expression into equation (4) then leaves us with

$$\begin{aligned} \hat{Y}(t) = & [A(t) F_K G'_1] \frac{I^d(t)}{Y(t)} + [A(t) F_K G'_2] \frac{I^f(t)}{Y(t)} + \alpha_L \hat{H}(t) \\ & + \hat{A}(t) - \left[ \frac{A(t) F_K G'_1 K^d(t)}{Y(t)} \delta^d(t) + \frac{A(t) F_K G'_2 K^f(t)}{Y(t)} \delta^f(t) \right] \end{aligned} \quad (6)$$

where we have used that  $Y(t) = A(t) F(\cdot)$ . Finally we use the definitions

$$\rho^d(t) \equiv \frac{\partial Y(t)}{\partial K^d(t)} = A F_K G'_1, \quad \rho^f(t) \equiv \frac{\partial Y(t)}{\partial K^f(t)} = A F_K G'_2$$

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<sup>5</sup>Of course, in the CES example the cross partial always either positive or zero.

and

$$\alpha_K(t) \equiv \frac{A(t)F_K G}{Y(t)}, \quad \gamma(t) \equiv \frac{G'_1 K^d(t)}{G},$$

which leaves us with the following expression for the growth rate of output

$$\hat{Y}(t) = \rho^d(t) \frac{I^d(t)}{Y(t)} + \rho^f(t) \frac{I^f(t)}{Y(t)} + \alpha_L(t) \hat{H}(t) + \hat{A}(t) - \{\alpha_K(t)[\gamma(t)\delta^d(t) + (1 - \gamma(t))\delta^f(t)]\}. \quad (7)$$

Accordingly  $\rho^i(t)$  has the interpretation of the (aggregate) return on the two types of capital.<sup>6</sup> Hence, from an accounting perspective, the contribution of e.g. aid capital to output growth is simply the product of the aid to GDP ratio multiplied by the relevant economic return.

### 3. Econometric issues

Assuming equation (7) holds for all countries, it is clear that nothing guarantees that the returns are the same across countries and time. So fundamentally the objective is to try to identify the average values of  $\rho^d(t)$  and  $\rho^f(t)$  across time and countries. In this section we discuss some of the econometric issues related to the estimation of the average aggregate returns.

#### 3.1. An observable growth accounting equation

First, observable measures for domestic investment and aid investment must be defined. As not all aid is used for investment it is not possible to extract primary data from any database. Yet, the sum of the two types of investment is known as it equals gross capital formation ( $I$ ). In order to identify the two investment components we assume that aid investment is linearly related to the foreign aid inflows ( $F(t)$ ):

$$\frac{I^f(t)}{Y(t)} = \frac{\beta F(t)}{Y(t)} + \phi(t), \quad 0 \leq \beta < 1, \quad (8)$$

where  $\phi(t)$  is a country and time specific component, which is treated as random in the following.

The important assumption in (8) is that the expected marginal share of aid flows, which are invested, is constant. But, notice that the average share may vary across countries and time.

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<sup>6</sup>Capitals' share of total income in this economy is  $(\rho^f K^f + \rho^d K^d)/Y = F_K G/Y = \gamma\alpha + \alpha(1 - \gamma) = \alpha$ .

Combining (8) and the adding-up constraint ( $I(t) = I^d(t) + I^f(t)$ ), domestically funded investments can be found as the residual

$$\frac{I^d(t)}{Y(t)} = \frac{I(t) - \beta F(t)}{Y(t)} - \phi(t). \quad (9)$$

It is worth noticing that  $\beta$  and  $\phi$  are not (only) related to the standard notion of fungibility of foreign aid. Donor preferences towards specific projects or programmes (i.e., the composition of aid from each donor) also play a prominent role in determining the size of  $\beta$  and  $\phi(t)$ .

Inserting equations (8) and (9) into equation (7) yields

$$\begin{aligned} \hat{Y}(t) = \rho^d(t) \left[ \frac{I(t) - \beta F(t)}{Y(t)} - \phi(t) \right] + \rho^f(t) \left[ \frac{\beta F(t)}{Y(t)} + \phi(t) \right] + \alpha_L(t) \hat{H}(t) \\ + \hat{A}(t) - \{ \alpha_K(t) [\gamma(t) \delta^d(t) + (1 - \gamma(t)) \delta^f(t)] \}. \end{aligned} \quad (10)$$

Finally, using a convex combination of the returns to domestic investment and aid investment,  $\tilde{\rho}(t) = (1 - \beta)\rho^d(t) + \beta\rho^f(t)$ , and the definition of human capital (equation (3)), we can rearrange (10) to an observable growth accounting equation:

$$\begin{aligned} \hat{Y}(t) = \rho^d(t) \left[ \frac{I(t) - F(t)}{Y(t)} \right] + \tilde{\rho}(t) \left[ \frac{F(t)}{Y(t)} \right] + \alpha_L(t) \psi \dot{u}(t) + \alpha_L(t) n(t) \\ + \hat{A}(t) - \{ \alpha_K(t) [\gamma(t) \delta^d(t) + (1 - \gamma(t)) \delta^f(t)] \} + \phi(t) \frac{1}{\beta} (\tilde{\rho}(t) - \rho^d(t)). \end{aligned} \quad (11)$$

In this equation there is a measurement error,  $\phi(t) \frac{1}{\beta} (\tilde{\rho}(t) - \rho^d(t))$ , which is zero if the returns on the two types of investments are equal, but in general it is correlated both with the returns and the regressors. As should be clear, when estimating the parameters of equation (11) neither  $\rho^f(t)$  nor  $\beta$  are identified. However, for given values of  $\rho^d(t)$ ,  $\tilde{\rho}(t)$ , and  $\beta$  the return on aid investment is given as

$$\rho^f(t) = \rho^d(t) + \frac{1}{\beta} (\tilde{\rho}(t) - \rho^d(t)), \quad (12)$$

and it follows that  $\rho^f(t) \leq \rho^d(t)$  as  $\tilde{\rho}(t) \leq \rho^d(t)$  for all values of  $\beta \in (0, 1]$ .

### 3.2. The econometric model

Since the returns to investments in physical and human capital varies not only over time but also across countries, our primary goal is to estimate the average return to investments across aid receiving countries and across time. Hence, we specify (11) as a random coefficients model and seek to estimate the mean of the coefficients.

Specifically, let the returns and the growth rate of total factor productivity be random vectors with a constant mean and covariance matrix. Then the growth equation can be written as a structural regression model such that for any randomly drawn country at time  $t$  we may think of (11) as a conditional expectation

$$E(y_{it} | \mathbf{x}_{it}, \mu_{it}, \rho_{it}, \phi_{it}) = \mathbf{x}_{it} \rho_{it} + \mu_{it} + \phi_{it} \iota \rho_{it} \quad (13)$$

where  $y_{it}$  is the growth rate of GDP in country  $i$  at time  $t$ ,  $\mathbf{x}_{it}$  is the vector of regressors, i.e.,  $\mathbf{x}_{it} = [\{(I_i(t) - F_i(t))/Y_i(t)\}, (F_i(t)/Y_i(t)), \dot{u}_i(t), n_i(t)]$ ;  $\rho_{it}$  is the corresponding vector of returns and parameters, while  $\mu_{it}$  captures the growth rate of total factor productivity and the depreciation rates, suitably scaled. Finally,  $\phi_{it}$  is the aid investment measurement error and  $\iota = \frac{1}{\bar{\beta}}[-1, 1, 0, 0]$ .

Following the panel data literature we assume the random coefficients have an additive error-component structure, which may be specified as follows

$$\rho_{it} = \bar{\rho} + \Theta_{it} = \bar{\rho} + \Upsilon_i + \Lambda_t + \xi_{it} \quad (14)$$

$$\mu_{it} = \bar{\mu} + \theta_{it}^\mu = \bar{\mu} + v_i^\mu + \lambda_t^\mu + \varepsilon_{it}^\mu \quad (15)$$

$$\phi_{it} = \bar{\phi} + \theta_{it}^\phi = \bar{\phi} + v_i^\phi + \lambda_t^\phi + \varepsilon_{it}^\phi. \quad (16)$$

$\bar{\rho}$ ,  $\bar{\mu}$ , and  $\bar{\phi}$  are the unconditional expectations,  $E(\rho_{it}) = \bar{\rho}$ ,  $E(\mu_{it}) = \bar{\mu}$ ,  $E(\phi_{it}) = \bar{\phi}$ , and the error components  $\Upsilon_i, \Lambda_t, \xi_{it}, v_i^\mu, \lambda_t^\mu, \varepsilon_{it}^\mu, v_i^\phi, \lambda_t^\phi$ , and  $\varepsilon_{it}^\phi$  are assumed to be mean zero random variables with a standard panel data error-components covariance structure

$$\begin{aligned} E(\Upsilon_i \Upsilon_j') &= 0, & E(v_i^\mu v_j^\mu) &= 0, & E(v_i^\phi v_j^\phi) &= 0 & \text{for } i \neq j \\ E(\Lambda_t \Lambda_s') &= 0, & E(\lambda_t^\mu \lambda_s^\mu) &= 0, & E(\lambda_t^\phi \lambda_s^\phi) &= 0 & \text{for } t \neq s \\ E(\xi_{it} \xi_{js}') &= 0, & E(\varepsilon_{it}^\mu \varepsilon_{js}^\mu) &= 0, & E(\varepsilon_{it}^\phi \varepsilon_{js}^\phi) &= 0 & \text{for } i \neq j, \text{ and } t \neq s. \end{aligned}$$

The covariances between the relevant components of  $\rho_{it}$ ,  $\mu_{it}$ , and  $\phi_{it}$ , say,  $\Upsilon_i, v_i^\mu$ , and  $v_i^\phi$  are left unrestricted, as these are obviously related, being the random components of returns and TFP growth.

By including a time constant variation ( $\Upsilon_i, v_i^\mu, v_i^\phi$ ) we allow for the possibility that some countries have higher returns and TFP growth rates than others throughout the whole period in question and that these countries invest more (or less) of the aid inflow than the average country. Furthermore, the common variation across countries ( $\Lambda_t, \lambda_t^\mu$ ) captures world wide movements in the returns. The common time varying measurement error,  $\lambda_t^\phi$ , may reflect changes in donor policies regarding aid modalities, i.e., changes from projects (investment in physical capital) to

programmes (with higher fractions of expenditures on government consumption such as road maintenance or teacher salaries).

Even though the covariances between the measurement error and the coefficients may be non-zero we do assume the covariance structure is constant:

$$E(\Theta_{it}\theta_{js}^\phi) = E(\Upsilon_i v_j^\phi) + E(\Lambda_t \lambda_s^\phi) + E(\xi_{it} \varepsilon_{js}^\phi) = \Sigma_{\Upsilon v} \delta_{ij} + \Sigma_{\Lambda \lambda} \delta_{ts} + \Sigma_{\xi \varepsilon} \delta_{ij} \delta_{ts} \quad (17)$$

for all  $i, j$  and  $t, s$  where  $\delta_{ab}$  is Kronecker's delta.

Turning to the regressors, we consider a fairly general error-component model

$$\mathbf{x}_{it} = f_i + g_t + r_{it} \quad (18)$$

where the specific components,  $r_{it}$ , are assumed to follow a general covariance stationary process, independent of the common effects,  $g_t$ , and the time invariant effects  $f_i$ .<sup>7</sup>

Given the specification of the coefficients and the regressors, the possible association between the returns and the regressors can also be specified:

$$E(\Theta_{it} \mathbf{x}_{js}) = E(\Upsilon_i f_j) + E(\Lambda_t g_s) + E(\xi_{it} r_{js}) = \Sigma_{\Upsilon f} \delta_{ij} + \Sigma_{\Lambda g} \delta_{ts} + \Sigma_{\xi r} \delta_{ij} \delta_{ts}. \quad (19)$$

Each of the covariance-components,  $\Sigma_{\Upsilon f}$ ,  $\Sigma_{\Lambda g}$  or,  $\Sigma_{\xi r}$  may be non-zero, in which case the model is a correlated random coefficient model. The correlated random coefficient model has been studied fairly recently by Heckman and Vytlacil (1998) and Wooldridge (1997, 2003 and 2005). In the present analysis we follow Wooldridge (2003, 2005).

Inserting equations (14)-(16) in (13) and using the error form of the model it may be formulated as

$$y_{it} = \mathbf{x}_{it} \bar{\rho} + c + u_{it} \quad (20)$$

$$c = \bar{\mu} + \sigma_{\mathbf{x}\Theta} + \sigma_{\Theta\phi} + \bar{\phi} \bar{\mathbf{l}} \bar{\rho}, \quad (21)$$

$$u_{it} = (\mathbf{x}_{it} \Theta_{it} - \sigma_{\mathbf{x}\Theta}) + (\theta_{it}^\phi \mathbf{l} \Theta_{it} - \sigma_{\Theta\phi}) + \theta_{it}^\mu + \theta_{it}^\phi \bar{\mathbf{l}} \bar{\rho} + \bar{\phi} \mathbf{l} \Theta_{it} + e_{it} \quad (22)$$

where

$$\sigma_{\mathbf{x}\Theta} = E(\mathbf{x}_{it} \Theta_{it}) \equiv tr(\Sigma_{\Upsilon f} + \Sigma_{\Lambda g} + \Sigma_{\xi r}),$$

$$\sigma_{\Theta\phi} = E(\theta_{it}^\phi \mathbf{l} \Theta_{it}) \equiv tr[(\Sigma_{\Upsilon v} + \Sigma_{\Lambda \lambda} + \Sigma_{\xi \varepsilon}) \mathbf{l}],$$

and  $e_{it}$  is the expectation error derived from the structural model (13).

<sup>7</sup>Assuming independence of the three components is stronger than needed. However, as we require more than mean independence in the following the assumption is convenient.

In this system  $E(u_{it}) = 0$  (by construction) and, hence,  $\bar{\rho}$  can be consistently estimated if there exist a set of instruments such that  $E(u_{it}|\mathbf{z}_{it}) = 0$  where  $\mathbf{z}_{it}$  is the vector of instruments. In addition, equation (21) makes clear that the intercept in the equation is of little interest, being a sum of mean and covariance components.

### 3.3. Identification

Wooldridge (2003) considers estimation of average effects in the correlated random coefficients model in a cross-section and shows that standard instrumental variables estimators are consistent under fairly weak conditions. In the following we state these assumptions and show how standard panel data transformations of the regressors yield valid instruments under reasonable assumptions.

It follows from (20) and (22) that a vector of instrumental variables,  $\mathbf{z}_{it}$ , must satisfy the following exogeneity conditions:<sup>8</sup>

$$E(y_{it}|\mathbf{x}_{it}, \mu_{it}, \rho_{it}, \phi_{it}, \mathbf{z}_{it}) = E(y_{it}|\mathbf{x}_{it}, \mu_{it}, \rho_{it}, \phi_{it}). \quad (\text{A1})$$

$$E(\mu_{it}|\mathbf{z}_{it}) = E(\mu_{it}) = \bar{\mu}, \quad E(\rho_{it}|\mathbf{z}_{it}) = E(\rho_{it}) = \bar{\rho} \quad (\text{A2})$$

$$E(\Theta_{it}\mathbf{x}_{it}|\mathbf{z}_{it}) = E(\Theta_{it}\mathbf{x}_{it}) \equiv \Sigma_{Yf} + \Sigma_{\Lambda g} + \Sigma_{\xi r}. \quad (\text{A3})$$

$$E(\theta_{it}^{\phi}|\mathbf{z}_{it}) = 0 \quad (\text{A4})$$

$$E(\Theta_{it}\theta_{it}^{\phi}|\mathbf{z}_{it}) = E(\Theta_{it}\theta_{it}^{\phi}) \equiv \Sigma_{Yv} + \Sigma_{\Lambda\lambda} + \Sigma_{\xi\varepsilon}. \quad (\text{A5})$$

Assumption (A1) is the order condition, stating that the instrumental variables are redundant in the structural equation (13). Assumption (A2) adds the condition that the instrumental variables are ignorable for the random coefficients, while assumption (A3) specifies that the instruments are also ignorable for the covariance between the regressors and the random coefficients. Assumption (A3) is stronger than needed, as the necessary condition is that the trace of the conditional covariance matrix should not depend on (functions of) the instrument. However, it is hard to imagine cases in which this distinction is important.<sup>9</sup> Finally, it should be noted that independence of the coefficients and the instruments is a sufficient condition for (A2) and (A3).

Because of the measurement error in aid investments, two additional conditions are added. The first of these, (A4), is the standard condition, stating that the

<sup>8</sup>Assumptions (A1)-(A3) are given in Wooldridge (2003).

<sup>9</sup>Wooldridge (2003) specifies the independence condition for each of the diagonal elements in the conditional covariance matrix. Needless to say, this intermediate assumption is also sufficient but not necessary.

instruments are ignorable for the measurement error. The second, (A5), adds a conditional independence assumptions for the covariance between the random return coefficients and the measurement error.

Assumptions (A2), (A4) and (A5) can be gathered by considering the vector of random components in the model, say,  $\chi_{it} = [\Theta_{it}', \theta_{it}^\mu, \theta_{it}^\phi]'$ . A sufficient condition, encompassing the three conditions above, is second order independence of  $\chi_{it}$  with respect to the instruments

$$\begin{aligned} E(\chi_{it} | \mathbf{z}_{it}) &= 0 \\ \text{Var}(\chi_{it} | \mathbf{z}_{it}) &= \text{Var}(\chi_{it}). \end{aligned} \tag{24}$$

This shows that the crucial new assumption in the correlated random coefficient model, compared to standard models, is (A3).

From (A1)-(A5) it follows that the conditional expectation of the regression error given the instruments is zero,  $E(u_{it} | \mathbf{z}_{it}) = 0$ .  $E(\theta_{it}^\mu | \mathbf{z}_{it}) = 0$  and  $E(\bar{\phi} \mathbf{1} \Theta_{it} | \mathbf{z}_{it}) = 0$  by (A2),  $E(\theta_{it}^\phi \mathbf{1} \bar{\rho} | \mathbf{z}_{it}) = 0$  by (A4), while  $E(\mathbf{x}_{it} \Theta_{it} | \mathbf{z}_{it}) = \sigma_{\mathbf{x}\Theta}$  and  $E(\phi_{it} \mathbf{1} \Theta_{it} | \mathbf{z}_{it}) = \sigma_{\Theta\phi}$  follows from (A3) and (A5). Therefore  $\mathbf{z}_{it}$  is a valid instrument in equation (20) and given the existence of such an instrument and the usual rank condition, we can consistently estimate the average returns,  $\bar{\rho}$ . Subsequently, using (12), consistent estimates of the average of  $\rho^f$  can be obtained for given values of  $\beta$ .

The structure of the model shows that various panel data transformations of the regressors may be valid instruments, depending on the specific assumptions about the covariance between the returns and the regressors. Below we consider each variance component in turn.

First, suppose the association between the random components and the regressors is solely via a common variation across time. That is, one of the following expressions hold,  $E(\Lambda_t | x_{it}) \neq 0$ ,  $E(\lambda_t^\mu | x_{it}) \neq 0$ ,  $E(\lambda_t^\phi | x_{it}) \neq 0$ ,  $\Sigma_{\Lambda g} \neq 0$ , or  $\Sigma_{\Lambda \lambda} \neq 0$ . Let  $\mathbf{z}_{it}$  be the residuals from a regression of  $\mathbf{x}_{it}$  on time dummies, denoted  $\check{\mathbf{x}}_{it}$ . As the regression on time dummies eliminates  $g_t$ , and we assume independence between the components in  $\mathbf{x}_{it}$  the transformation leads to  $E(\theta_{it}^\mu + \theta_{it}^\phi \mathbf{1} \bar{\rho} + \bar{\phi} \mathbf{1} \Theta_{it} + e_{it} | \check{\mathbf{x}}_{it}) = 0$  and  $E(\mathbf{x}_{it} \Theta_{it} + \theta_{it}^\phi \mathbf{1} \Theta_{it} | \check{\mathbf{x}}_{it}) = \sigma_{\mathbf{x}\Theta} + \sigma_{\Theta\phi}$  such that  $\check{\mathbf{x}}_{it}$  is a valid instrument. By the partialling out interpretation of the projection on time dummies it follows that a standard pooled OLS regression of (20) augmented by time dummies yields consistent estimates of the average returns given the assumption.

Second, assume the association between the random components and the regressors is only via co-movements across countries. Here, the specific association is from one of the assumptions:  $E(\Upsilon_i | x_{it}) \neq 0$ ,  $E(v_i^\mu | x_{it}) \neq 0$ ,  $E(v_i^\phi | x_{it}) \neq 0$ ,  $\Sigma_{f\Upsilon} \neq 0$ , or  $\Sigma_{\Upsilon v} \neq 0$ . This case is considered by Wooldridge (2005) who shows

that the standard fixed effects estimator is consistent. The point to note is that regression of  $\mathbf{x}_{it}$  on country dummies, or alternatively the first differences of  $\mathbf{x}_{it}$ , removes  $f_i$ , which is the source of association between the regressors and the regression error. Hence, there are several consistent estimators: the fixed effects estimator, OLS after first differencing, or IV-estimation of the levels using the first differences of the regressors as instruments.

Third, a contemporaneous association between the idiosyncratic random components and the regressors may be present. This is specified as  $E(\xi_{it}|x_{it}) \neq 0$ ,  $E(\epsilon_{it}^{\mu}|x_{it}) \neq 0$ ,  $E(\epsilon_{it}^{\phi}|x_{it}) \neq 0$ ,  $\Sigma_{\xi r} \neq 0$ , or  $\Sigma_{\xi \epsilon} \neq 0$ . If this is the only association, a standard IV-regression using the lagged regressors as instruments ( $\mathbf{z}_{it} = \mathbf{x}_{it-s}$ ,  $s > 0$ ) is consistent, given the assumption that the idiosyncratic components are uncorrelated over time.

Finally, if all covariance components are allowed to be non-zero each of the estimators given above are inconsistent but we can combine the transformations to obtain valid instruments. Specifically, the lagged differences of the regressors, conditional on time dummies are valid instruments ( $\mathbf{z}_{it} = \Delta \mathbf{x}_{it-s}$ ,  $s > 0$ ). Needless to say, while this transformation produces valid instruments, the instruments may be weak. We address this issue in the empirical section.

## 4. Empirical Results

In the empirical analysis we use data for 78 countries covering the 40 years 1960-1999. Data on GDP (constant 1995 US\$), investment (gross capital formation), and the labor force are from the World Development Indicators 2002 CD-rom. Aid is gross ODA disbursements from the DAC 2002 CD-rom while education is measured by total years of education in the population (tyr15) from the updated Barro-Lee data set (Barro and Lee, 2000). Investment and aid are transformed to percentages of GDP, and the annual data is subsequently divided into eight, non-overlapping, five-year epochs of averages. The countries in the sample and summary statistics are listed in Appendix A.

### 4.1. Main Results

Table 3 reports the main regression results.<sup>10</sup> The dependent variable is the average annual growth rate of real GDP. The four, essential, regressors are total investment

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<sup>10</sup>All regression results are obtained using “dpd for Ox” (Doornik, Arellano and Bond, 2001) and a modified version of Jarociński’s Ox package for instrumental variable regression (Jarociński, 2003).

less aid, aid, the average annual change in years of schooling and the average annual growth rate of the labor force. As shown in Section 3.1 the average returns and the other structural parameters can be recovered from the estimated parameters. The average return to domestic investment,  $\bar{\rho}^d$ , is the coefficient upon investment less aid, while the average elasticity of output with respect to (raw) labor input,  $\bar{\alpha}_L$ , is the coefficient upon the growth rate of the labor force. The average return to education,  $\bar{\psi}$ , can be estimated as the ratio of the coefficient upon education to the coefficient upon labor force growth. Finally, using equation (12), the return to aid investments can be derived for given values of the expected marginal share of aid invested,  $\beta$ .

The columns in Table 3 report the estimated parameters based on different estimators. All regressions include time dummies to take account of the possible association between the regressors and the common variation over time in the random components. Regression (1) is a standard least squares regression. As described in the previous section, if the association between the regressors and the random components is *only* through the common variation over time, this estimator is consistent. Regression (2) takes account of the time invariant association between the regressors and the random components by using least squares after a first difference transformation of the data.

Regressions (3)-(5) are instrumental variable regressions—TSLS, LIML, and GMM with sequential moment restrictions, respectively—using (lags of) the first differenced regressors as instruments. We use two lags of the differences of investments and aid flows, while the differences of the annual average changes in education and the average labor force growth rate are included using lags 0 and 1. Hence, the model has eight instruments for the four endogenous regressors. The validity of the instruments are tested using the Sargan/Hansen test. The  $p$ -values of these tests statistics are reported in Table 3 and, as seen, we cannot reject the assumption of valid instruments.

The inclusion of the differences of the annual average changes in education and the average labor force growth rate in the instrument set implies that we assume no contemporaneous association between the return parameters and these differences. We test this specific assumption using the difference in criterion function test (the ‘Difference Sargan test’ in Table 3) and cannot reject the hypothesis that these two instruments are valid.

While all of our instruments appear to be valid, this does not ensure unbiased estimators as the instruments may be weak. In testing for weak instruments we

follow Stock and Yogo (2005). Under Stock and Yogo's approach, weakness of the instruments is defined in terms of the squared bias of the IV-estimator relative to the squared bias of the least squares estimator. The details of the test are given in Appendix B. The result is that we reject the hypothesis of weak instruments for the TSLS estimator. Hence, the instruments are strong, in the sense of leading to relatively small biases in the TSLS estimator.

Even though the instruments are strong we also present results using the LIML estimator, as this estimator is (partially) robust to weak instruments and hence more reliable than the TSLS estimator when the instruments are weak. (See Stock, Wright and Yogo (2002) for a discussion of weak instrument problems and solutions). As seen from Table 3 the point estimates, as well as the standard errors and test statistics are very similar using either of the two estimators, TSLS or LIML, strengthening the assumption of valid and strong instruments.

However, the TSLS and LIML estimators are not efficient in the presence of conditional heteroskedasticity in the errors. Therefore, in Regression (6), we present results of a GMM estimator using sequential moment restrictions. The estimator is the panel data GMM-estimator proposed by Arellano and Bover (1995).<sup>11</sup> The moment restrictions are given from the condition:  $E(u_{it}|\Delta\ddot{\mathbf{x}}_{it-s}) = 0$  for  $s > 0$ .<sup>12</sup> As the restrictions hold for all  $t - s < t$ , they imply that

$$E(u_{it}|\Delta\ddot{\mathbf{x}}_{it-2}) = 0 \text{ and } E(u_{it-1}|\Delta\ddot{\mathbf{x}}_{it-2}) = 0$$

leading to

$$E(u_{it}|\Delta\ddot{\mathbf{x}}_{it-2}) - E(u_{it-1}|\Delta\ddot{\mathbf{x}}_{it-2}) = E(\Delta u_{it}|\Delta\ddot{\mathbf{x}}_{it-2}) = 0.$$

This suggests a set of moment conditions resembling the conditions in Arellano and Bover (1995):

$$E(\Delta\ddot{\mathbf{x}}_{it-1}u_{it}) = 0, \quad t = 3, \dots, T, \quad (25)$$

$$E(\Delta\ddot{\mathbf{x}}_{is}\Delta u_{it}) = 0, \quad s < t - 1 \text{ and } t = 4, \dots, T. \quad (26)$$

To avoid finite sample bias in the GMM regression we do not use all moment conditions implied by (26) but restrict the model to include at most two lags of the instruments. Hence, Regression (6) is based on 54 over identifying moment restrictions. In addition, to avoid downward bias in the estimated standard errors, we use the small sample correction of the variance estimates proposed by Windmeijer (2005).

<sup>11</sup>See also Arellano (2003, Chapter 8)

<sup>12</sup>For the annual average change in education and the labor force growth rate we use  $s \geq 0$ .

Table 3: Estimates of average growth accounting parameters for 77 countries across 8 five year epochs, 1960-1999. Using gross ODA from DAC as aid regressor

Dependent variable	Average annual growth rate in real GDP				
	OLS	FD	TOLS	LIML	GMM
Estimator	(1)	(2)	(3)	(4)	(5)
Investment less aid	16.995 (3.04)	16.848 (3.91)	20.971 (5.62)	21.590 (5.07)	18.396 (6.28)
Aid	18.291 (5.30)	17.669 (8.24)	19.730 (11.0)	20.363 (9.83)	19.696 (7.83)
Education, change	2.074 (1.48)	1.095 (1.32)	0.311 (2.55)	0.331 (2.64)	0.539 (1.84)
Labor force growth	0.321 (0.25)	1.030 (0.43)	1.194 (0.46)	1.251 (0.49)	0.280 (0.36)
SEE	2.824	3.438	2.905	2.926	2.916
Sargan/Hansen test			0.355	0.360	0.360
Difference Sargan test			0.689	0.406	0.555
Observations	527	450	311	311	426

Estimates of the return to education and the average aggregate return on aid investments

$\bar{\psi}$	6.456 (6.52)	1.062 (1.33)	0.261 (2.12)	0.264 (2.13)	1.929 (7.25)
$\bar{\rho}^f, \beta = 0.5$	19.587 (8.48)	18.490 (13.8)	18.489 (18.9)	19.136 (17.2)	20.996 (10.8)
$\bar{\rho}^f, \beta = 0.7$	18.847 (6.62)	18.021 (10.6)	19.198 (14.3)	19.837 (12.9)	20.253 (8.99)
$\bar{\rho}^f, \beta = 0.9$	18.435 (5.63)	17.760 (8.84)	19.592 (11.9)	20.227 (10.6)	19.840 (8.11)

Robust standard errors in parentheses. The standard errors in (6) are estimated using the finite sample correction proposed in Windmeijer (2005). All regressions include time dummies. The instruments in regressions (3)-(5) are differences of investments and aid flows; lagged once and twice, and differences of changes in education and labor force growth; contemporaneous and lagged once. The GMM regression (6) combines equations in levels and first differences as explained in the text. The difference Sargan test is testing the validity of using the contemporaneous differences of changes in education and labor force growth as instruments. For the Sargan/Hansen tests the  $p$ -values of the test statistics are reported.

Turning to the results, the estimated average return to domestic investment is remarkably constant across estimators. The two Least Squares-based regressions ((1) and (2)) both result in point estimates of the return just below 17 percent whereas the three IV-based estimators have higher point estimates (18-21 percent). Overall, based on the three IV-based estimators, the estimated average aggregate return to domestic investment appear to be of a reasonable order of magnitude compared to, say, anticipated rates of return in the US.

The estimates of the composite average return ( $\bar{\rho}$ ) show the same pattern as the return on domestic investment. In Regressions (1) and (2) the average, composite, return is about 18 percent, while the point estimates are 20 percent in the IV-regressions (3)-(5).

In order to estimate the average aggregate return on aid investments, we need to specify values of the marginal rate of investments out of aid flows ( $\beta$ ). It is difficult to pinpoint an exact interval for  $\beta$ , but we assume a lower limit of 0.5 is not unreasonable.<sup>13</sup> In the bottom part of Table 3 we report estimated returns to aid investments for three different values of the marginal share of aid investments ( $\beta = 0.5, 0.7, 0.9$ ). The returns are estimated using equation (12) and the standard errors are calculated using the Delta method.

Even though the three IV-regressions results differ in that the estimated return on domestic investments is larger than the composite return in the TSLS and LIML regressions, while reverse result is obtained in the GMM regression, the estimated average aggregate return on aid investments is remarkably constant across the three estimators. If only half of the aid flows are invested, we find the average aggregate return to be between 18.5 and 21 percent. If almost all of the aid flows are invested ( $\beta = 0.9$ ) the return about 20 percent. Hence, overall, the analysis suggests that the average aggregate return on aid investments is close to 20 percent, roughly as the median returns for World Bank projects reported in Tables 1 and 2.

Finally, as seen from Table 3, the impact of education and labor force growth are both highly imprecisely determined, leading to a poorly determined estimate of the return to schooling. The lack of precision of the estimate is not surprising in light of other empirical analyses of the return to schooling (e.g., Temple, 2001). Taking the lack of precision into account, the results for labor force growth are not extreme, as values around 2/3 is always within a one standard error bound.

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<sup>13</sup>A rough guideline can be obtained by looking at the allocation of ODA commitments across sectors 1973-1997 (Hjertholm and White, 2000). In that period roughly 60 percent of the ODA commitments were allocated to either "Social infrastructure and services", "Economic infrastructure and services", or "Production sectors".

## 4.2. Robustness

In this section we look into the robustness of the empirical results. The first problem we address is related to the data. As it is impossible to get data on aid investments and, hence, to avoid the measurement error problem, we test the model by using an alternative measure of the aid flows. A second problem is the relatively short time-series dimension in the data set, which may be too small to permit a stochastic modelling of the covariance structure. In particular, the convergence of  $\frac{1}{NT} \sum_i \sum_t \mathbf{x}_{it} \Lambda_t$  towards the trace of  $\Sigma_{\Lambda g}$  requires “large”  $T$ . We address this problem by an alternative parametrisation of the model in which period specific returns are treated as unknown parameters to be estimated.

### 4.2.1. Alternative aid data

Because of the uncertainty in the share of aid investments we test the model by using an alternative measure of the aid flows. Specifically, we use the aid variable given in WDI 2002 from which we subtract technical assistance and action related to debt relief.<sup>14</sup> We refer to the resulting series of aid flows as “net aid”. Although the two measures of aid are highly correlated, the distributions are different. The median of net aid is less than the median of gross ODA, while the interquartile range is larger.<sup>15</sup> Since the investment share is expected to be higher for net aid compared to gross ODA, the model predicts a higher estimate of  $\bar{\rho}$  while the average return to domestic investment should be unchanged.

Table 4 shows results in accordance with this prediction. Compared to the results in Table 3 there is almost no change in the point estimates of the average aggregate return on domestic investment, while the point estimate of the composite return is consistently higher in Table 4. Hence, using net aid, we get higher estimated returns on aid investments for any given level of the marginal rate of investments out of aid flows, as expected.

Moreover, even taking the higher marginal investment rate into account, the estimated average aggregate return on aid investments appear to be close to 25 percent rather than the 20 percent, we obtained using gross aid. Thus, “cleaning the data” by removing non-invested aid such as technical assistance and debt rescheduling shows that the 20 percent return may be a low estimate.

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<sup>14</sup>Data on aid flows for technical assistance and action related to debt relief are from the DAC 2002 CD-rom.

<sup>15</sup>The regression coefficient of gross ODA on net aid is 0.79, and significantly less than one.

Table 4: Estimates of average growth accounting parameters for 78 countries across 8 five year epochs, 1960-1999. Using net aid.

Dependent variable	Average annual growth rate in real GDP				
	OLS	FD	TOLS	LIML	GMM
Estimator	(1)	(2)	(3)	(4)	(5)
Investment less aid	16.990 (3.06)	16.744 (3.81)	20.748 (5.73)	20.537 (5.18)	19.031 (5.17)
Aid	19.409 (4.66)	18.840 (7.84)	24.934 (10.2)	23.971 (13.2)	23.163 (6.34)
Education, change	2.137 (1.45)	1.138 (1.30)	0.467 (2.58)	0.318 (2.67)	0.538 (1.86)
Labor force growth	0.343 (0.24)	1.035 (0.43)	1.139 (0.44)	1.182 (0.46)	0.306 (0.34)
SEE	2.818	3.434	2.913	2.917	2.909
Sargan/Hansen test			0.390	0.261	0.425
Difference Sargan test			0.509	0.188	0.670
Observations	529	451	310	310	430

Estimates of the return to education and the average aggregate return on aid investments

$\bar{\psi}$	6.234 (5.94)	1.099 (1.32)	0.410 (2.26)	0.269 (2.27)	1.757 (6.44)
$\bar{\rho}^f, \beta = 0.5$	21.829 (7.05)	20.936 (13.5)	29.119 (16.3)	27.405 (23.0)	27.294 (8.99)
$\bar{\rho}^f, \beta = 0.7$	20.447 (5.64)	19.738 (10.2)	26.727 (12.7)	25.443 (17.4)	24.933 (7.37)
$\bar{\rho}^f, \beta = 0.9$	19.678 (4.91)	19.073 (8.44)	25.399 (10.8)	24.353 (14.3)	23.622 (6.59)

Robust standard errors in parentheses. The standard errors in (6) are estimated using the finite sample correction proposed in Windmeijer (2005). All regressions include time dummies. The instruments in regressions (3)-(6) are differences of investments and aid flows; lagged once and twice, and differences of changes in education and labor force growth; contemporaneous and lagged once. The GMM regression (6) combines equations in levels and first differences as explained in the text. The difference Sargan test is testing the validity of using the contemporaneous differences of changes in education and labor force growth as instruments. For the Sargan/Hansen tests the  $p$ -values of the test statistics are reported.

Table 5: Weighted average estimates of average growth accounting parameters for 77 countries across 8 five year epochs, 1960-1999. Using gross aid.

Dependent variable	Average annual growth rate in real GDP			
	OLS	FD	LIML	GMM
Estimator	(1)	(2)	(3)	(4)
Investment less aid	15.742 (3.07)	14.299 (3.15)	25.534 (5.68)	18.376 (7.10)
Aid	20.736 (5.01)	36.500 (5.16)	33.267 (12.2)	25.410 (7.47)
Education, change	2.755 (2.14)	1.301 (2.60)	1.752 (4.97)	2.329 (2.83)
Labor force growth	0.357 (0.23)	0.710 (0.23)	1.103 (0.59)	0.261 (0.33)
SEE	2.774	3.339	3.443	3.115
Observations	527	450	311	426
Estimates of the return to education and the average aggregate return on aid investments				
$\bar{\psi}$	7.721 (6.72)	1.832 (3.78)	1.588 (4.26)	8.917 (14.5)
$\bar{\rho}^f, \beta = 0.5$	25.729 (8.48)	58.702 (9.68)	41.000 (18.4)	32.444 (12.5)
$\bar{\rho}^f, \beta = 0.7$	22.876 (6.43)	46.015 (7.01)	36.581 (14.9)	28.425 (9.32)
$\bar{\rho}^f, \beta = 0.9$	21.291 (5.36)	38.967 (5.62)	34.126 (12.9)	26.192 (7.89)

Robust standard errors in parentheses. The standard errors in (4) are based on the finite sample correction proposed in Windmeijer (2005). The standard error of estimation (SEE) is computed across all periods.

#### 4.2.2. Alternative model specification

As pointed out above, it might be problematic that the time series dimension in the data set is limited to (at most) 8 periods. This means that simple pooling of cross sections over time may not be adequate as  $\frac{1}{NT} \sum_i \sum_t (\mathbf{x}_{it} \Lambda_t) = \frac{1}{T} \sum_i [(\frac{1}{N} \sum_{it} \mathbf{x}_{it}) \Lambda_t]$  only converge to  $tr(\Sigma_{\Lambda g})$  as  $T \rightarrow \infty$ .

In order to assess the impact of pooling we consider a parametric specification of the time variation using a time-varying parameter design. Specifically, we

estimate period specific slope parameters<sup>16</sup>

$$\rho_{it} = \rho_t + \Upsilon_i + \xi_{it} \quad (27)$$

Subsequently, the average parameters are estimated using a weighted average of the period specific returns. Inspired by Swamy (1970) the weighted average is computed by

$$\hat{\rho}_w = \sum_{t=1}^T W_t \hat{\rho}_t, \quad (28)$$

where the weights are

$$W_t = \{\sum_{t=1}^T [\widehat{V}(\bar{\rho}) + \text{Var}(\hat{\rho}_t)]^{-1}\}^{-1} [\widehat{V}(\bar{\rho}) + \text{Var}(\hat{\rho}_t)]^{-1} \quad (29)$$

in which  $\text{Var}(\hat{\rho}_t)$  are the estimated covariance matrices for each  $\hat{\rho}_t$ ,  $t = 1, \dots, T$ , and the variance of the mean return is estimated by

$$\widehat{V}(\bar{\rho}) = \frac{1}{T-1} \sum_{t=1}^T (\hat{\rho}_t - \bar{\rho})(\hat{\rho}_t - \bar{\rho})', \quad (30)$$

using the arithmetic mean,  $\bar{\rho} = \frac{1}{T} \sum_{t=1}^T \hat{\rho}_t$ , as the first step estimate.<sup>17</sup> The variance of  $\hat{\rho}_w$  is estimated by

$$\text{Var}(\hat{\rho}_w) = \{\sum_{t=1}^T [\widehat{V}(\bar{\rho}) + \text{Var}(\hat{\rho}_t)]^{-1}\}^{-1}. \quad (31)$$

Table 5 lists the results of the time-varying specification of the model using gross aid as in Table 3. As the TSLS estimator in this setting is very similar to the GMM estimator, we do not report the TSLS results.

The consequence of the changed model specification is an increase in the estimated aggregate average return on aid investments. In particular the LIML estimation now results in considerably higher returns on both domestic and aid investments. The changes are less pronounced for the GMM estimates, because this estimator is using time-specific moment conditions in all specifications, such that the only change is the time variation in the parameters. Even so, the estimated return on aid investments increases compared to Table 3.

In sum, the alternative measure of aid and the use of an alternative model specification results in higher point estimates of the average aggregate return on aid investments. Yet, taking the parameter uncertainty into account, the changes are relatively small, and they support an overall result that the average aggregate return on aid investments is in the range of 20-30 percent.

<sup>16</sup>The specification of  $\mu_{it}$  and  $\phi_{it}$  is also changed, but this has already been taken into account by the inclusion of time dummies in the regressions in Tables 3 and 4.

<sup>17</sup>Hsiao, Peseran and Tahmiscioglu (1999) show that the estimator is an "empirical Bayes estimator", and by Monte Carlo simulations they illustrate that the estimator has good properties in dynamic panels (averaging across individuals instead of across time, though).

## 5. Conclusion

Over the last several decades researchers have scrutinized the effectiveness of aid as a tool to reduce poverty in the third world. No doubt much is yet to be learned on this issue. We believe the present paper contributes to this research agenda by providing an estimate of the *average* gross real rate of return on aid financed investments in physical capital.

The return on aid investments can be identified on the basis of a standard growth accounting framework. The advantage of this line of attack is the comparative simplicity of the structural model. Another advantage is the theoretical separation of production function parameters from preferences parameters, which is not feasible in Barro-type growth regressions. This separation is what allows us to identify the gross real rates of return.

The transparency of the economic model comes at the cost of added econometric complexity as returns are likely to vary across countries and time. Moreover, the returns are in all likelihood correlated with the unobserved TFP growth rates and, hence, the investment ratios. A feasible, and fairly simple, solution to the econometric problem lies in formulating the structural model as a correlated random coefficient model in which the average returns can be identified and consistently estimated using instrumental variable estimators.

Our principal finding is that the average aggregate gross rate of return on aid investments lies in the range 20-30 percent. Intriguingly, this is in accord with median World Bank project level estimates. Moreover, aid investments are roughly as productive as domestically funded investments in physical capital.

In many ways this is an encouraging finding. It is certainly broadly consistent with the empirical work on aid effectiveness invoking *ad hoc* growth specifications, which tend to find that aid, on average, stimulates productivity. At the same time it is a sobering finding, since our estimates provide a sense of the limitations of aid in stimulating economic activity in poor economies.

Needless to say our return estimates do not have direct bearing on the long-run growth impact from aid. Another limitation is that our analysis does not address the return on aid financed investments in human capital.

Our approach fundamentally recognizes that the return on aid financed investments is likely to vary considerably across countries and time. Exploring this heterogeneity, both at the aggregate level and – data permitting – in a more disaggregated context, is likely to be a revealing avenue for future research. For example, previous research have suggested that factors like the policy environment,

the institutional setting in general, or perhaps geographic circumstances, matter for the aggregate marginal productivity of aid financed investments. Our approach is capable of turning these propositions into testable hypotheses. We are currently following this track in ongoing research.

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## Appendix A. The sample and summary statistics

Table A: The sample of countries

Algeria	Egypt, Arab Rep.	Kuwait	Senegal
Argentina	El Salvador	Lesotho	Sierra Leone
Bahrain	Fiji	Malawi	Singapore
Bangladesh	Gambia, The	Malaysia	South Africa
Barbados	Ghana	Mali	Sri Lanka
Benin	Greece*	Malta	Sudan
Bolivia	Guatemala	Mauritius	Swaziland
Botswana	Guyana	Mexico	Syrian Arab Rep.
Brazil	Haiti	Mozambique	Thailand
Cameroon	Honduras	Nepal	Togo
Central African Rep.	Hong Kong, China	Nicaragua	Trinidad and Tobago
Chile	Hungary	Niger	Tunisia
China	India	Pakistan	Turkey
Colombia	Indonesia	Panama	Uganda
Congo, Dem. Rep.	Iran, Islamic Rep.	Papua New Guinea	Uruguay
Congo, Rep.	Israel	Paraguay	Venezuela, RB
Costa Rica	Jamaica	Peru	Zambia
Cyprus	Jordan	Philippines	Zimbabwe
Dominican Republic	Kenya	Poland	
Ecuador	Korea, Rep.	Rwanda	

\*Greece not in the base sample using aid data from DAC.

Table B: Means and standard deviations of the five central variables

Period	Average annual growth rate of GDP	Investment less aid, percent of GDP	Aid, percent of GDP	Average annual change in years of education	Average annual growth rate of labor force
1	4.85	14.49	2.87	0.009	2.43
(N = 47)	(2.55)	(7.69)	(3.36)	(0.053)	(0.98)
2	5.02	14.90	3.51	0.075	2.30
(N = 54)	(2.67)	(7.4)	(4.83)	(0.113)	(1.03)
3	4.83	17.90	2.84	0.060	2.77
(N = 61)	(3.5)	(7.77)	(3.41)	(0.06)	(1.05)
4	4.81	20.93	2.94	0.093	2.72
(N = 69)	(3.67)	(8.3)	(3.18)	(0.108)	(0.98)
5	2.29	20.06	3.07	0.086	2.76
(N = 73)	(3.38)	(9.42)	(3.28)	(0.069)	(1.02)
6	3.32	16.37	4.27	0.104	2.54
(N = 73)	(3.01)	(9.11)	(4.7)	(0.077)	(0.84)
7	3.40	16.61	5.65	0.096	2.55
(N = 75)	(3.52)	(12.46)	(7.38)	(0.098)	(1.06)
8	3.51	18.63	3.67	0.064	2.53
(N = 75)	(2.16)	(9.26)	(4.94)	(0.04)	(1.01)
All	3.90	17.69	3.67	0.077	2.59
(N = 527)	(3.24)	(9.4)	(4.73)	(0.085)	(1.00)

The means and standard deviations cover the OLS regression sample with a total of 77 different countries. (Greece is not included). The standard deviations are in parentheses.

## Appendix B. Test of weak instruments

In testing for weak instruments we follow the suggestion in Stock and Yogo (2005) and use their bias definition of weak instruments. That is, a weak instrument set is defined as those instruments that potentially lead to asymptotic squared bias relative to the OLS bias greater than some prespecified value  $b$ . The squared relative bias is defined as

$$B_T^2 = \frac{(\mathbb{E}\hat{\rho}^{IV} - \rho)' \Omega (\mathbb{E}\hat{\rho}^{IV} - \rho)}{(\mathbb{E}\hat{\rho}^{OLS} - \rho)' \Omega (\mathbb{E}\hat{\rho}^{OLS} - \rho)}$$

where  $\Omega = \text{plim}(X'M_T X/NT)$  is the covariance of the endogenous regressors conditional on the time dummies. ( $M_T$  is the projection off the  $K_1$  time dummies, such that  $M_T X$  are the regressors when the time dummies are partialled out).

As explained in Stock and Yogo, for the TSLS estimator, the relative bias can be interpreted as the maximal bias based on the standardized units  $\sigma_u \Omega^{-1/2}$ , where  $\sigma^2$  is the variance of the errors in the structural equation (equation (20) in the text). Hence, if, say  $b = 0.1$  then the maximal squared bias of the TSLS estimator is ten percent of the bias of the OLS estimator, and the maximal bias is ten percent in absolute terms measured in the metric  $\sigma_u \Omega^{-1/2}$ .

In the test for weak instruments Stock and Yogo use the Cragg and Donald (1993) multivariate analog of the  $F$ -statistic. With 4 endogenous regressors (stacked as a  $NT \times K_0$  matrix,  $X$ ) and the 8 instruments (stacked as a  $NT \times K_2$  matrix,  $Z$ ) the test statistic is the minimum eigenvalue of

$$G = \hat{\Sigma}_{vv}^{-1/2'} X' M_T P_Z M_T X \hat{\Sigma}_{vv}^{-1/2} / K_2$$

where  $P_Z = Z(Z'Z)^{-1}Z'$  is the projection on the eight instruments and

$$\hat{\Sigma}_{vv} = X' M_T (I - P_Z) M_T X / (NT - K_1 - K_2)$$

is the estimated covariance matrix of the first stage residuals.

Stock and Yogo compute the boundary of the weak instrument set by Monte Carlo simulation of the minimal eigenvalue of  $G$ . As Stock and Yogo only consider up to 3 endogenous variables we have simulated the the boundary for 4 endogenous variables.<sup>18</sup> Based on the simulated boundary values a conservative  $\alpha\%$  test can be obtained by considering the  $1 - \alpha$  percentile of the  $\chi_{K_2}^2(K_2 \ell) / K_2$ -distribution, where  $\ell$  is the simulated minimal eigenvalue for the boundary of the weak instrument set.

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<sup>18</sup>In the Monte Carlo simulations we used the Gauss program written by Motohiro Yogo (<http://finance.wharton.upenn.edu/~yogo/>), hence we use the same set-up as Stock and Yogo.

Table C: Boundary values and critical values for the weak instrument test based on relative squared bias of TSLS relative to OLS. The model has 4 endogenous regressors and 8 instruments.

Maximal relative bias, $b$	0.05	0.10	0.20	0.30
Boundary value	11.59	5.70	2.79	1.74
95% critical value	16.84	9.79	6.08	4.66
90% critical value	15.81	9.02	5.49	4.15

Table C reports the boundary minimal eigenvalues and critical values for the TSLS estimator for various maximal bias values ( $b$ ) in the model with 4 endogenous regressors and 8 instruments.

The critical values in Table C can be used to test if the instruments in the TSLS regressions in Tables 3 and 4 are weak ( $H_0$ ) or strong ( $H_1$ ). In the model using gross aid the smallest eigenvalue is 14.65, while it is 15.08 in the model using net aid. Hence, as the null-hypothesis is rejected at the 5 percent level with  $b = 0.10$ , we conclude that the instruments are strong in the sense that the maximal bias of the estimators is at most ten percent relative to the OLS bias.

As the LIML estimator is more robust to weak instruments compared to the TSLS estimator, we may conclude that the LIML results also have a low relative bias. For the sequential moments GMM estimator we do not have a well specified test as the first stage reduced form is non-standard.