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Devaluation and Monetary Policy in Developing Countries: A General Equilibrium Model for Economies Facing Financial Constraints

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Abstract

This paper investigates the short-run response of output to devaluations and monetary policies in economies where firms are constrained to finance in advance their working capital by borrowing solely from domestic banks. It is shown that in contrast with traditional theories, but in accordance with recent evidence for developing countries, an anticipated devaluation or an anticipated increase in the rate of growth of the exogenous component of money might result in a short-run output contraction. In addition, the paper analyses how output responds when a devaluation is unanticipated or when agents cannot distinguish between temporary and permanent monetary shocks.

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Summary

Traditional models that attempt to explain the short-run behavior of output and prices in small open economies facing a fixed exchange rate usually conclude that a devaluation or an increase in the money supply will either be expansionary or have no effect on the level of output in the short run. However, recent empirical analysis of the short-run behavior of output in less developed countries (LDCs) does not seem to support the traditional conclusions. In particular, some evidence suggests that devaluations have generated short-run declines in output and that anticipated increases in rates of money growth may have had negative effects on output in some LDCs.

The purpose of this paper is to contribute to the literature by showing that financial constraints faced by LDCs play a crucial role in the observed short-run behavior of output, prices, and the level of foreign reserves. Specifically, it is shown that in economies where firms are constrained to finance their working capital by borrowing from a domestic banking sector, either an anticipated devaluation or an anticipated increase in the rate of growth of the exogenous component of the money supply might result in a contraction in the short-run level of aggregate output. This is demonstrated by constructing a two goods (tradable and nontradable goods) model in which an expected increase in domestic inflation leads to a reduction in the equilibrium level of the real monetary base and in the real amount of domestic credit available to finance output production. Thus, any variation in an exogenous variable that leads to an increase in expected inflation causes a "tightening up" of the financial constraint, with negative effects on the output levels of both tradable and nontradble goods. Therefore, even if a change in an exogenous variable generates a change in relative prices favoring an increase in the output of a particular good, the output of that good might decrease if the relative price effect is outweighed by a negative financial constraint effect.

In addition, the paper analyzes the responses of output to unanticipated shocks. It is shown that the contractionary effect of a devaluation will not be eliminated, but only postponed to a subsequent period, if the devaluation is unanticipated. Moreover, it is shown that the composition of output between tradable and nontradable goods will be affected if a permanent monetary shock is mistakenly viewed as temporary.

I. Introduction

The central theme of this paper is that financial constraints faced by less developed countries (LDCs) play a crucial role in the short-run responses of output, prices, and the level of foreign reserves to devaluations and other monetary and real shocks. Specifically, this paper shows that in an economy where firms are constrained to finance in advance their working capital by borrowing solely from a domestic banking sector, the outputs of tradable and nontradable goods depend positively on the equilibrium real monetary base (the financial constraint effect). In addition, the supply of nontradables (tradables) depends negatively (positively) on the price of tradable goods relative to the price of nontradables goods. Therefore, the response of aggregate output to exogenous shocks (devaluations and other monetary and real shocks) depends on the relative strengths of the financial constraint effect) and the relative price effects.

Traditional models that attempt to explain the short-run behavior of output and prices in small open economies facing a fixed exchange rate usually conclude that a devaluation or a monetary increase will either be expansionary or have no effect on the short-run level of output. 1/ However, recent empirical analysis of the short-run behavior of output in LDCs does not seem to support these traditional results. In particular, the studies by Edwards (1985) and Gomez-Oliver (1985) found that in recent years devaluations have generated a short-run decline in

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1/ For example, the traditional analysis of a devaluation in models that incorporate both tradable and nontradable goods concludes that in the short run, a devaluation will raise the real exchange rate (the price of tradable goods relative to the price of nontradable goods) and hence will lead to both an increase in the domestic demand for nontradables and an increase in the production of tradables. If unutilized capacity is an assumption of the model (as in Mundell (1962) and Fleming (1962)), a devaluation will result in a higher level of aggregate output. If, instead, the model assumes full employment (as in Dornbusch (1975)), a devaluation will temporarily shift resources from the production of nontradables to that of tradables with no change in the aggregate level of output and employment.

Regarding the analysis of expansions in the exogenous component of the money supply, models that assume some kind of price stickiness in the short run conclude that the initial increase in aggregate demand will stimulate economic activity, while those models that assume price flexibility conclude that the resulting increase in the relative price of nontradables will temporarily shift resources from the production of tradables towards the production of nontradables with no change in the aggregate level of output.

aggregate output in some LDCs. 1/ In addition, the evidence with respect to the effects of changes in the actual rate of growth of money on the level of output is mixed. While Barro (1979) found that the actual rate of growth of money had significant and positive explanatory power with respect to the level of the Mexican output, 2/ some of the evidence reported in Hanson (1980) seems to indicate the presence of a negative effect of money growth on the output levels of Brazil and Chile. 3/ 4/

In contrast with the traditional theories, but in accordance with recent evidence for LDCs, this paper establishes the plausibility that either an anticipated devaluation or an anticipated increase in the rate of growth of the exogenous component of the money supply might result in a contraction in the short-run level of output. This is demonstrated by constructing a model in which an expected increase in domestic inflation leads to a reduction in the equilibrium level of the real monetary base and in the real amount of domestic credit available to finance output production. 5/ Thus, any variation in an exogenous variable which leads

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1/ The study by Edwards covers the period 1965-80 and focuses on the following countries: India, Malaysia, Philippines, Sri Lanka, Thailand, Greece, Israel, Brazil, Colombia, El Salvador, South Africa, and Yugoslavia.

The study by Gomez-Oliver includes: Bolivia, Costa Rica, Ecuador, and Mexico. Gomez-Oliver found that devaluations during the 1970s were accompanied by a slowdown in real GDP for one or two years and that devaluations in the early 1980s were accompanied by actual contractions in real GDP.

2/ The study by Barro covers the period 1959-74. During most of that period (1959-70), the Mexican exchange rate was fixed at 12.50 pesos per U.S. dollar.

3/ The data for Chile covers the period 1952-70 and the data for Brazil covers the period 1952-74.

4/ Hanson disregarded this result since he interpreted it as inconsistent with rational expectations. His argument was that in the context of a Lucas (1973) aggregate supply function, the actual rate of change of the money supply will affect output only if money is a random walk. Since Hanson found that the Brazilean and Chilean money supply processes are significantly different from a random walk, he concluded that a regression of output on actual money growth "imposes an incorrect money supply process and therefore irrational expectations" (p. 980). However, in this paper we show that such a relation between output and money growth is perfectly consistent with the rational expectations hypothesis in the context of an output supply equation alternative to that of Lucas.

5/ It is important to note that the positive relationship between output and real money in this model does not arise from introducing money as a wealth variable. Instead, it is due to the financial constraint assumption which in turn is closely related to the standard cash-in-advance assumption used in the open economy models of Stockman (1980), Helpman (1981), and Aschauer and Greenwood (1983).

to an increase in expected inflation causes a "tightening up" of the financial constraint, with negative effects on the output levels of both tradable and nontradable goods. Even in the presence of a favorable relative price effect, therefore, the supply of a component of output might decrease if the relative price effect is outweighed by a negative financial constraint effect.

The idea that a devaluation might be contractionary in the short run is not new. There are several theoretical arguments that attempt to explain such a result. However, most arguments 1/ are based on demand-related effects 2/ generated by a devaluation, ranging from negative real balance effects on aggregate demand to income distributions towards economic agents with high marginal propensity to save. This paper advances the literature by showing that the introduction of a financial constraint in an economy where agents are fully rational and prices adjust to clear markets, adds an additional channel through which a devaluation might be contractionary. By demonstrating in addition that the channels leading to contractionary devaluations might also lead to output contraction in response to an increase in the rate of growth of the exogenous component of the money supply, the paper provides insights on a topic on which little theoretical research has been contributed. 3/

This paper follows a choice-theoretic approach. A macromodel is fully derived from the decision rules followed by rational agents who maximize utility or profits subject to the constraints imposed by the environment in which they live (which will be defined below). The model is then solved in a general equilibrium, rational expectations, stochastic framework.

The rest of the paper is organized as follows: Section II describes the basic characteristics of the model, with special emphasis on the assumptions regarding the organization of markets and the timing of transactions. Section III derives the optimal decision rules implied by the maximizing behavior of economic agents. Section IV derives the domestic aggregate supply and demand functions for tradable and nontradable commodities, and the aggregate demand for money. Section V sets out the complete macromodel to be solved in this fixed exchange rate economy. Under the assumption of complete current information,

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1/ For a review of this literature see Edwards (1985).

2/ There are also some models that advance supply-side arguments to explain the contractionary effects of a devaluation. The most common argument is that a devaluation increases the cost of imported intermediate inputs and hence results in a decrease in aggregate supply.

3/ There are, however, several models that produce an upward slowing Phillips curve (a negative correlation between output and inflation) in the long run. See for example Fry (1978), Mathieson (1980), and Stockman (1980).

Section VI then solves for the equilibrium levels of the outputs of tradable and nontradable goods, the stock of foreign exchange reserves, and the price of the nontradable good. This section also discusses the implications for the economy of a devaluation, a change in the foreign price of tradables, and a change in the exogenous component of the monetary base. In addition, it briefly analyzes how the economy responds to real shocks. Section VII then briefly discusses the implications of relaxing the assumption of full current information. Finally, Section VIII summarizes the main conclusions.

II. The Model: Basics

Consider an economy where all transactions are carried out with money. Households need money in advance to purchase commodities and firms need money in advance to finance their production process. That is, the transaction's role of money is recognized in this paper, and money enters the model in the same way as in the models of Clower (1967), McKinnon (1973), Kapur (1976a), Mathieson (1979), Lucas (1980), Stockman (1980, 1981), and Helpman (1981).

This economy nonetheless is assumed to face a severely limited capital market. The chief limitations are the absence of a market for primary securities together with an absence of any type of financial intermediation other than commercial banking. Moreover, the only source of finance for firms is assumed to be loans from domestic commercial banks. Sales of equities or bonds to domestic households or foreigners are ruled out. In this context, firms face the following "financial constraint": they need to finance their advances by demanding credit solely from the banking sector. Thus, the availability of real credit places a constraint on the level of output. Since the financial structure of this economy consists of only a banking sector and since money is the only tradable asset, the total supply of money in the economy coincides with the total supply of credit (to the government and to firms).

The monetary transactions involve four markets: tradable commodities, nontradable commodities (domestic) labor, and (domestic) credit. The economy is inhabited by four kinds of agents: firms, households, banks, and the government.

Firms supply output in a competitive environment using domestic labor services (working capital) provided by households as the only variable factor of production.

Households maximize expected utility over their lifetime. We assume the existence of overlapping generations at each moment, as in Samuelson (1958). Each generation lives for two periods. When young, households supply labor and demand commodities and money. Money is held to buy commodities at the end of this first period or to be carried over into the second period as a store of value with which to pay for consumption

during retirement. When old, households receive profits from the firms and commercial banks (they own both kinds of businesses) and net lump-sum transfers from the government and consume these and their previous savings. They hold money during the second period, but not at the end of it. There is no market for property rights and the only alternative left to the old generation is to pass them over to the next generation before dying. 1/ Inheritances of money, however, are ruled out; and, hence, it is in the best interest of the old generation to spend all their money balances during the period.

The credit market involves the banking system as the only supplier of credit and firms as the only agents demanding it. 2/ The banking system is made up of a central bank and commercial banks. The central bank has three functions: it finances the government (whose only function is to provide transfers to households) through the issuance of high-powered money, imposes a required reserve ratio on the commercial banks, and acts as the foreign exchange authority. In this fixed exchange rate regime, the monetary base (H_t) has two asset counterparts: the government credit (GC_t) and the foreign exchange reserves (F_t).

Commercial banks hold reserves, supply credit to firms, and issue only one kind of deposit: demand deposits that do not pay interest. 3/ It is assumed that banks incur no labor costs in providing the services of intermediation. Also, it is assumed that bank loans have a maturity length of one period.

It is assumed that households and firms keep all their money in the form of demand deposits issued by the commercial banks; that is, that their currency-deposit ratio is equal to zero. Hence, the entire monetary base is held as reserves by the banks. Domestic residents do not hold foreign exchange. It is assumed that they buy foreign money at the end of the period when they decide to import the tradable good. Finally, only domestic residents hold domestic demand deposits.

1/ It is assumed that, in the initial period of the life of this economy, an old generation owned all relevant property rights. Since every individual is assumed to maximize utility over only his own lifetime, it would have been in the best interest of the old generation to sell these property rights at the beginning of this period, because they were to die at the end of it. However, in this economy, there is no demand for property rights at the beginning of any period. Other old people obviously do not want to buy them, and young people are not able to buy them. They begin their life with no money and are unable to borrow. Hence, there is no market for property rights.

2/ It is also assumed that imperfections in the capital market prevent households from obtaining bank loans.

3/ It is assumed here that government regulations prevent perfect competition in the banking system. Fry (1982) argues that this is a mechanism through which the government can expropriate a large seigniorage.

Time is divided into discrete uniform intervals. To gain insight into the nature of the model, let us follow the activities of the economy during any given period t.

At the beginning of period t, firms and banks distribute profits realized at the end of period t-1 to households (the old generation), firms pay back past loans to the commercial banks, demand labor from households (the young generation), and loans from the banking system. Given the level of banks' reserves, at the beginning of the period, the supply of credit by the commercial banks is also determined at the beginning of the period. Finally, the old generation receive a lump sum net transfer from the government. The net governmental transfer (Tr_t) is assumed to be equal to the increase in government credit (ΔGC_t).

In this system of fixed exchange rates, the beginning of period monetary base (${}_aH_t$) differs from the end-of-period monetary base (${}_bH_t$) if there is any net change in the economy's stock of foreign reserves (F_t). While the component of the monetary base corresponding to the government credit (GC_t) is determined at the beginning of the period, changes in foreign exchange reserves occur at the end of the period when transactions in tradable commodities take place.

Hence, the beginning of period monetary base equals: 1/

$$(1) \quad {}_aH_t = {}_bH_{t-1} + \Delta GC_t = GC_t + F_{t-1}$$

while the monetary base at the end of the period is equal to:

$$(2) \quad {}_bH_t = {}_aH_t + \Delta F_t = GC_t + F_t$$

Once the credit and labor markets clear, production takes place and no further transactions occur until the end of the period when the markets for tradable and nontradable commodities open. At this moment, the young generation decides on its current and future consumption plan and thus, on its demand for money. Both generations demand commodities, and the price of the nontradable good and the equilibrium level of foreign reserves are determined. Firms also carry over money in an amount equal to the value of the total output (P_tY_t) in order to pay back banking loans and distribute profits. The old generation dies between periods and it is assumed that it has depleted its money balances on consumption before dying. At the start of t+1, a new generation is born and the whole process just described begins again.

1/ In general, the subscript "a" refers to "beginning of period" and the subscript "b" refers to "end of period."

III. The Maximizing Behavior of the Economic Agents

The problems to be solved by a representative bank, firm, and household are now considered. Agents are rational and maximizing behavior implies that they use all the available information relevant for their decisions. However, they do not have perfect foresight. Specifically, in this section it is assumed that agents do not observe the current period price levels of the commodities at the beginning of the period, nor do they know the future period prices of the commodities at the end of the period. Hence, they have to form expectations about these variables. In addition, agents have to form beginning-of-period expectations about the exchange rate.

1. The representative bank

As has already been said, the assumptions regarding imperfections in the capital market and barriers of entry imposed by government's regulations imply that the banking sector holds a monopolistic position such that demand deposits are noninterest bearing but bank loans to firms are interest bearing one period loans. Thus, the representative bank chooses its supply of credit to maximize:

$$(3) \qquad \underset{\{C_t^s\}}{\text{Max}} \; {}_{at}E\left[\frac{{}^{b}\pi_t}{PI_{t+1}}\right] = {}_{at}E\left[\frac{i_t}{PI_{t+1}} C_t^s\right]$$

where: E = the expectations operator; ${}_{at}E(X)$ refers to "beginning-of-period t expectations" of a variable X; ${}_{bt}E(X)$ will refer to "end-of-period t expectations" of a variable X;

${}^{b}\pi_t$ = nominal profits realized by the bank at the beginning of period t;

C_t^s = nominal amount of credit (loans) supplied by the representative bank;

i_t = nominal interest rate; and

PI_{t+1} = *domestic price index during period t+1.*

The bank maximizes "expected real profits" because it belongs to households who will only receive bank's profits at the beginning of the subsequent period.

It is assumed here that the central bank imposes a beginning-of-period reserve requirement ratio and hence, every period the commercial bank faces the following set of constraints:

(4) $_aR_t \geq K\ _aD_t$

(5) $_aR_t + C_t^s = {}_aD_t$

(6) $_aR_t \equiv RR_t + RE_t$

(7) $_aR_t = GC_t + F_{t-1} = {}_aH_t$

where: $_aR_t$ = beginning-of-period reserves in the vault of the commercial bank = required reserves (RR) plus excess reserves (RE);

$_aD_t$ = beginning-of-period level of demand deposits; and

K = reserve requirement ratio.

Equation (4) is the reserve requirement constraint imposed by the central bank. Equation (5) is the bank's balance sheet constraint, which requires that the total assets of the bank equal the total liabilities. Identity (6) defines the composition of the level of reserves, and equation (7) follows from the assumption that the nonbanking sector holds its money only in the form of demand deposits.

The maximization procedure will lead to the following commercial bank's decision rule to supply credit:

(8) $C_t^s = \frac{1-K}{K}\ _aH_t$

The commercial bank's supply of credit implies that the bank finds it optimal to supply as much credit as possible. The bound is, of course, given by the reserve requirement ratio and hence the optimal decision rule for the bank is to hold zero excess reserves. This result follows obviously from the assumption that the bank pays no interest on demand deposits, but it will also and equally obviously hold as long as the interest payments on deposits is lower than the interest charge on credit.

2. The representative firm

The objective of the firm is to maximize profits. The representative domestic firm is assumed to produce both tradable (Y_t^*) and nontradable (Y_t) commodities. The firm engages in a production process that yields joint products. In addition, a single input, domestic labor supply (L_t) is used in the production of the two outputs. Specifically, in order to obtain explicit results, the following product transformation curve is assumed:

$$(9) \quad L_t = \frac{1}{2}(Y_t^2 + (Y^*)_t^2)\frac{1}{\phi_t}$$

Equation (9) states costs of production in terms of labor units. ϕ_t is a productivity shock and is assumed to affect both outputs equally. 1/

The maximization problem for the representative firm becomes:

$$(10) \quad \max_{\{Y_t^s, (Y^*)_t^s\}} \; at E[\frac{f_{\pi_t}}{PI_{t+1}}] =$$

$$\max_{\{Y_t^s, (Y^*)_t^s\}} \; at E \{\frac{P_t}{PI_{t+1}} Y_t^s + \frac{P_t^* S_t}{PI_{t+1}} (Y^*)_t^s - \frac{W_t}{PI_{t+1}} L_t^d - \frac{i_t}{PI_{t+1}} C_t^d\}$$

where: Y_t^s = domestic supply of the nontradable good;

$(Y^*)_t^s$ = domestic supply of the tradable good;

P_t = price level of the nontradable good;

P_t^* = price level of the tradable good; and

S_t = exchange rate: the domestic currency price of the foreign exchange.

The firm maximizes expected real profits. Once more, the expected price index of period t+1 is the relevant deflator because the firm's profits realized during period t are distributed to its owners (households) at the beginning of period t+1. The total cost faced by the firm is the total wage bill plus the interest payments made on bank loans needed to pay labor in advance.

Equation (10) is maximized subject to the product transformation curve (equation (9)) and to the firms' financial constraint:

$$(11) \quad C_t^d = W_t L_t^d$$

1/ The productivity term in the denominator means that a positive random increase in productivity will result in a lower amount of labor required to produce a given amount of output. The sequence $\{\phi_t\}$ is assumed to be a strictly positive stationary stochastic process and its distribution will be specified in Section IV.

Given that the only reason for a firm to demand loans is to finance labor, this constraint has to be satisfied as an equality every period. 1/

The information set that the firm uses at the beginning of the period to form its expectations (Ω_{at}) consists of the information on variables up to period t-1, plus the current wage rate, the interest rate, and the productivity term. 2/ Thus:

$$(12) \quad \Omega_{at} = \{\Omega_{t-1}, W_t, i_t, \phi_t, \mu_t\}$$

The maximization procedure leads to the following supply function for the nontradable good:

$$(13) \quad Y_t^s = \frac{{}_{at}E(P_t)}{W_t(1+i_t)} \phi_t$$

and to the following supply function for the tradable commodity:

$$(14) \quad (Y^*)_t^s = \frac{{}_{at}E(P_t^* S_t)}{W_t(1+i_t)} \phi_t$$

From these decisions rules, it is clear that the supply of each commodity depends positively on its own price and on a "common" productivity shock, and negatively on the "effective" cost of labor: $W_t(1+i_t)$.

Substituting equations (13) and (14) into equation (9) yields the demand for labor:

$$(15) \quad L_t^d = \frac{1}{2}\phi_t \left[\frac{({}_{at}E(P_t))^2 + ({}_{at}E(P_t^* S_t))^2}{(W_t(1+i_t))^2}\right]$$

1/ Notice that the assumption of joint production allows us to treat the demand for credit as a single variable, without having to decompose it according to its uses (the production of tradable or nontradable goods).

2/ The information set at the beginning of the period also includes the random term μ_t which affects the household's utility function, and will be considered below.

The demand for labor depends positively on the expected money prices of the final outputs and on the productivity shock, and negatively on the effective cost of labor.

Finally, equation (11) can be used to derive the firms' demand for credit:

$$(16) \qquad C_t^d = \frac{W_t \, \phi_t[({}_{at}E(P_t))^2 + ({}_{at}E(P_t^* S_t))^2]}{2[W_t(1+i_t)]^2}$$

3. The representative household

Each household lives two periods and maximizes expected utility. Consumption commodities and leisure are assumed to yield positive utility. Thus, utility depends inversely on time devoted to work. For simplicity, it is assumed that the young agent maximizes the expected value of the following utility function:

$$(17) \quad U = \mu_1 \ln [Y_1^{\tau}(Y^*)_1^{(1-\tau)}] + \ln(\bar{L}-L_1) + \beta Y_2^{\tau} (Y^*)_2^{(1-\tau)}$$

where the subscript 1 is used to indicate the agent's first period of life and 2 to indicate his second and last period. Y_1 and Y_1^* refer to his demand for nontradables and tradables, respectively, for his first period and Y_2 and Y_2^* refer to his consumption demands for the second period. μ_1 refers to a random shock affecting the utility derived from current consumption, and $(\bar{L}-L_1)$ is the amount of leisure available to the agent after he has supplied labor. Also, $0 < \beta < 1$. The utility function (17) is additive separable in consumption and leisure and shows constant marginal utility of future consumption. 1/

In the first period of his life, the individual works, consumes, and saves money. In his second period, he is old and at the beginning of the period receives net transfers (Tr_2) from the government and profits from the firm (${}^f\pi_1$) and banks (i_1C_1).

1/ In addition, the use of Cobb-Douglas functions implies that the derived elasticities of demand for current consumption of both tradable and nontradable commodities with respect to total expenditure on current consumption is equal to one. By the same token, the elasticities of demand for future consumption of both tradable and nontradable commodities with respect to total expenditure on future consumption is equal to one.

Thus, the lifetime budget constraint of the agent is:

$$(18)\quad P_2Y_2^d + (P_2^*S_2)(Y^*)_2^d = W_1L_1^s + Tr_2 + {}^h\pi_2 - P_1Y_1^d - (P_1^*S_1)(Y^*)_1^d$$

where: ${}^h\pi_2 = {}^f\pi_1 + i_1C_1$.

The problem for the young generation is to maximize the expected value of equation (17) subject to the budget constraint (18) and to the condition that the young agent's demand for money be nonnegative:

$$(19)\quad W_1L_1^s - P_1Y_1^d - P_1^*S_1(Y^*)_1^d > 0.$$

The inequality constraint (19) is necessary to satisfy the assumption that all transactions are carried out with money, given that the young generation starts life with no money.

The representative young agent faces a two-stage decision problem: at the beginning of the period, he chooses his labor supply, taking into account the available and relevant information at his disposal at that moment. At the end of the period, the agent decides on his demand for commodities and money taking into account the information available at that time.

The information set available to the agent at the end of period 1 is:

$$(20)\quad \Omega_{b1} = \{\Omega_{a1},\ P_1,\ S_1,\ P_1^*\}$$

where the information set at the beginning of period 1 (Ω_{a1}) is identical to the set faced by firms and is given by equation (12).

Appendix I shows the derivation of the agent's decision rules. It is enough to state here that the maximization problem leads to the following decision rules for consumption in the first period:

$$(21)\quad Y_1^d = \frac{j\ \tau}{\beta}\ \mu_1\ \frac{b1E(PI_2)}{P_1}$$

$$(22)\quad (Y^*)_1^d = \frac{j'(1-\tau)}{\beta}\ \mu_1\ \frac{b1E(PI_2)}{P_1^*S_1}$$

where: $PI_2 \equiv$ price index in period 2

$$\equiv P_2^{\tau} (P_2^* S_2)^{1-\tau}.$$

That is, current consumption of tradable and nontradable commodities depends positively on expected inflation, where the relevant future price level is the "general" price level or "price index," while the relevant current price level is the own commodity price. The decision rules (21) and (22) also depend positively on a random term (μ_t) which affects the marginal utility of (total) current consumption.

The labor supply decision rule derived in this model is:

$$(23) \quad L_1^s = \bar{L} - \frac{j'}{\beta} \frac{a1E(PI_2)}{W_1}.$$

That is, labor supply depends positively on the expected future real wage where the relevant deflator is the expected future price index.

Equations (21) and (22) represent the consumption demands of the young generation. To obtain the <u>total</u> market demands for consumption goods, we have to add to these the old generation demands. Such demands have already been obtained in Appendix I (equations (1.3) and (1.4)), where the solution to the agent's second period problem was presented. Hence, for every period t, the <u>total</u> current demand for nontradables (Z_t^d) and tradables ($(Z^*)_t^d$), respectively are:

$$(24) \quad Z_t^d = \frac{j'\tau}{\beta} \mu_t \frac{bt E(PI_{t+1})}{P_t} + \tau \frac{aH_t}{P_t}$$

$$(25) \quad (Z^*)_t^d = \frac{j'(1-\tau)}{\beta} \mu_t \frac{bt E(PI_{t+1})}{P_t^* S_t} + (1-\tau)\frac{aH_t}{P_t^* S_t}$$

Finally, the young agent's demand for money ($^yD_t^d$) can be obtained by subtracting the value of his consumption bundle from his wage earnings.

$$(26) \quad {}^yD_t^d = W_t L_t - P_t Y_t^d - P_t^* S_t (Y^*)_t^d$$

and from equations (21) and (22):

$$(26') \quad y_{D_t^d} = \frac{W_t}{PI_t} L_t - \frac{j'}{\beta} \mu_t \quad \frac{{}_{bt}E(PI_{t+1})}{PI_t}$$

That is, the household's demand for the real stock of money depends positively on real wage income and negatively on the expected inflation rate.

IV. The Aggregate Supply and Demand Functions

1. The aggregate supply functions

In Section III, domestic supply equations for the tradable and non-tradable commodities were derived as functions of the wage rate and the interest rate. However, the equilibrium conditions in the labor and credit markets determine both the nominal wage rate and the interest rate. This section will obtain the solutions for the wage rate and the interest rate, and thus the final form of the aggregate supply functions. In order to achieve this, the model will be cast in log-linear terms. Lower case letters will be used to represent the log of a variable (with the exception of i_t, which stands for the observed value of the interest rate).

Based on equation (15) and assuming that the "weights" in the firm's price index equal those in the consumer price index, a log linear version of the demand for labor is:

$$(27) \quad l_t^d = \alpha_o + \alpha_1[{}_{at}E(pi_t) - w_t - i_t] + u_t$$

where: $u_t = \log \phi_t \sim N(0,\sigma_u^2)$, and u_t is independent of the rest of disturbances in the model.

and $pi_t \equiv \tau\, p_t + (1-\tau)(p_t^* + s_t)$

Similarly, based on equation (11) a log-linear approximation for the demand for credit is:

$$(28) \quad c_t^d = \beta_o + \alpha_1[{}_{at}E(pi_t) - i_t] + (1-\alpha_1)\, w_t + u_t$$

Log-linear versions for the supply of labor and credit are based on equations (23) and (8), respectively. Hence:

$$(29) \quad l_t^s = \gamma_o + \gamma_1(w_t - {}_{at}E(pi_{t+1}))$$

and:

$$(30) \quad c_t^s = k + {}_a h_t$$

Equations (27) through (30) and the market equilibrium conditions: $l_t^d = l_t^s$ and $c_t^d = c_t^s$ give solutions 1/ for the levels of the nominal wage rate and the nominal interest rate as functions of the monetary base, the expectations about prices and the random term to productivity. 2/

These solutions in turn can now be substituted into the following log-linear approximations of the supply functions for nontradable and tradable goods obtained in Section III (equations (13) and (14)).

$$(31) \quad y_t^s = - w_t - i_t + {}_{at}E(p_t) + u_t$$

1/ It should be recalled that the wage rate and the interest rate solutions are not final solutions because the price level (and its expectations) are taken as exogenous for the time being.

2/ The solution for the interest rate is:

$$i_t = \gamma_0' - \gamma_1'({}_a h_t - {}_{at}E(pi_t)) + \gamma_2' ({}_{at}E(pi_{t+1} - pi_t)) + \gamma_3' u_t$$

where:

$$\gamma_0' = \frac{\gamma_1\beta_o+\alpha_1\beta_o-\gamma_o+\gamma_o\alpha_1+\alpha_0-\alpha_o\alpha_1}{\alpha_1(1 + \gamma_1)} \gamma_1' k$$

$$\gamma_1' = \frac{\gamma_1 + \alpha_1}{\alpha_1(1 + \gamma_1)} \quad ; \gamma_2' = \frac{(1-\alpha_1)\gamma_1}{\alpha_1(1 + \gamma_1)} \quad ; \gamma_3' = \frac{1}{\alpha_1}$$

and the solution for the wage rate is:

$$w_t = {}_{at}E(pi_t) + \beta_o' + \beta_1'({}_{at}E(pi_{t+1} - pi_t)) + \beta_2'({}_a h_t - {}_{at}E(pi_t))$$

where: $\beta_o' = \dfrac{\alpha_o - \gamma_o - \beta_o + k}{(1 + \gamma_1)} \quad ; \beta_1' = \dfrac{\gamma_1}{(1 + \gamma_1)} \quad ; \beta_2' = \dfrac{1}{(1 + \gamma_1)}$

$$(32) \quad (y^*)_t^s = - w_t - i_t + {}_{at}E(p_t^*) + {}_{at}E(s_t) + u_t$$

The unitary coefficients (in absolute value terms) accompanying the arguments of y_t^s and $(y^*)_t^s$ reflect the specification of the production side of the model, which implies a unitary own price elasticity and a unitary factor price elasticity for the output supplies.

By substituting the solutions for the wage rate and the interest rate into equations (31) and (32), the aggregate supply functions of outputs are obtained:

$$(33) \quad y_t^s = a_o + a_1({}_ah_t - {}_{at}E(pi_{t+1})) + (1-\tau)({}_{at}E(p_t - p_t^* - s_t)) + a_2u_t$$

$$(34) \quad (y^*)_t^s = a_o + a_1({}_ah_t - {}_{at}E(pi_{t+1})) - \tau({}_{at}E(p_t - p_t^* - s_t)) + a_2u_t$$

where: $a_0 = -(\beta_0 + \gamma_0')$; $a_1 = \dfrac{\gamma_1}{\alpha_1 + \alpha_1\gamma_1}$; $a_2 = 1 - \gamma_3'$

The most important features of equations (33) and (34) are as follows. First, the only difference between the two functions lies in the coefficient of the relative price term. This difference is due to the fact that, although both commodities share the input markets indistinguishably, they face separate output markets. Now, the real exchange rate in this model coincides with the relative price term. Hence, an appreciation of the expected real exchange rate (an increase if the term: ${}_{at}E(p_t - p_t^* - s_t)$) affects negatively the supply of tradables and positively that of nontradables. The third term of both equations will be labelled "the relative price effect".

Second, the supply of both types of commodities depend positively on the expected real monetary base evaluated at the future level of the price index. This result is a direct consequence of the "financial constraint" assumption, and hence the second term of the output equations will be labelled "the financial constraint effect." Third, the random term affecting productivity affects the aggregate supplies with a positive coefficient less than one because in this model u_t has a non-proportional effect on the interest rate.

2. The aggregate demand functions

Equations (24) and (25) can be treated as the aggregate demand functions for nontradables and tradables, respectively. Equation (26') represents the households' aggregate demand for money. To obtain the

total demand for money, the end-of-period demand for money by firms, ${}^{f}D_t^d$, (equal to the value of final outputs) has to be added to equation (26'). Thus:

$$(35)\ D_t^d \equiv {}^{y}D_t^d + {}^{f}D_t^d \equiv W_t L_t - P_t Y_t^d - P_t^* S_t (Y^*)_t^d + P_t Y_t^s + P_t^* S_t (Y^*)_t^s$$

The three markets (the two commodities markets and the money market) clear at the end of every period. The equilibrium conditions for the nontradable good and money markets are:

$$(36)\ Y_t^s = Z_t^d$$

and

$$(37)\ D_t^s = D_t^d$$

In addition, the equilibrium condition for the tradable commodity market is:

$$(38)\ \frac{\Delta F_t}{P_t^* S_t} = (Y^*)_t^s - (Z^*)_t^d$$

Under the assumption of zero capital mobility and fixed exchange rates, the overall balance of payments is identical to the balance of trade, which in this model is equal to the difference between the value of the supply and demand for tradable goods; that is, only net exports can be explained in this model, but not the composition between exports and imports. Such is the nature of equation (38).

V. The Complete Macromodel

The results from the previous sections can now be consolidated into the macro-model to be analyzed in the next sections. By Walras' law we can use only the tradable and nontradable commodity markets to represent the entire macro system. 1/ The complete model consists, then, of the following set of equations (expressed in log terms):

1/ Imposing equilibrium in the markets for tradable and nontradable commodities imply equilibrium in the money market.

$$(39)\quad z_t^d = \delta_0 + \delta_1({}_{bt}E(pi_{t+1})-p_t) + \delta_2({}_ah_t-p_t) + \varepsilon_t$$

$$(40)\quad (z^*)_t^d = b_0 + b_1({}_{bt}E(pi_{t+1})-p_t^*-s_t) + b_2({}_ah_t-p_t^*-s_t) + \varepsilon_t$$

$$(41)\quad y_t^s = a_0 + a_1({}_ah_t - {}_{at}E(pi_{t+1})) + (1-\tau)\ ({}_{at}E(p_t-p_t^*-s_t)) + a_2u_t$$

$$(42)\quad (y^*)_t^s = a_0 + a_1({}_ah_t- {}_{at}E(pi_{t+1})) - \tau\ \ ({}_{at}E(p_t-p_t^*-s_t)) + a_2u_t$$

$$(43)\quad pi_t = \tau\ p_t + (1-\tau)(p_t^*+s_t)$$

$$(44)\quad {}_ah_t = \omega_0 + \omega_1\ gc_t + (1-\omega_1)f_{t-1}$$

$$(45)\quad y_t^s = z_t^d$$

$$(46)\quad f_t = d_1(y^*)_t^s - d_2(z^*)_t^d + (1-d_3)(p_{t+}^*s_t) + d_3f_{t-1}$$

Equations (39) and (40) are log-linear approximations of equations (24) and (25), respectively, with ε_t being a serially independent log normal disturbance, which is also independent of the rest of disturbances of the model, where:

$$\varepsilon_t = \log \mu_t \sim N(0,\sigma_\varepsilon^2)$$

Equations (41) and (42) are identical to equations (33) and (34), respectively, and are repeated here only for convenience. These first four equations characterize the domestic markets for tradable and non-tradable commodities.

Equation (43) is a log approximation of the price index. Equation (44) is a log linear approximation (based on a Taylor expansion) of the identity for the beginning-of-period monetary base (equation (1)). As discussed in Sections II and III, a distinction between the beginning and the end-of-period monetary base arises in this fixed exchange rate model. The government credit (gc_t) is decided at the beginning of the period, but transactions leading to a change in the level of foreign reserves occur at the end of the period. This feature of the model implies that the end of previous period level of the foreign reserves

(f_{t-1}) enters into the structural supply and demand equations in the same way as does the current level of government credit (GC_t). 1/

The equilibrium condition in the nontradable goods market is represented by equation (45), while the equilibrium in the market for tradable goods is represented by equation (46). 2/ 3/

VI. Expectations and the Behavior of Prices, Output, and the Stock of Foreign Reserves Under Complete Current Information

In the previous sections, price expectations were not treated as endogenous variables. To solve the model formed by equations (39) through (46) for the output levels, the price level of the nontradable good and the stock of foreign reserves, it will be assumed that expectations are formed rationally in the sense of Muth (1961). To proceed towards the solution of the model, it is now necessary to specify the processes governing the behavior of the exogenous variables.

The exchange rate is assumed to be an exogenous variable in this model. It is assumed (as in Weber (1981)) that the monetary authority sets a target value for the exchange rate according to some known rule that is independent of the behavior of other variables in the model. For purposes of simplification, the analysis in this paper will assume that the exchange rate follows a random walk; that is:

$$(47) \quad s_t = s_{t-1} + \xi_t$$

where ξ_t is a white noise disturbance independently distributed from the other disturbances in the model.

Since the level of foreign reserves and hence, the money supply, are endogenous variables in this fixed exchange rate case, the only exogenous monetary element in the domestic economy is the government credit. It is assumed that the process followed by the government credit involves a constant trend growth rate, m, in addition to stochastic elements that

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1/ With the obvious allowance made for the constant of linearization: ω_1.

2/ Equation (46) is a log approximation of equation (38).

3/ In the remainder of this paper, f_t will always stand for the end of period level of foreign reserves since that is the relevant endogenous variable. In addition, the predetermined level of reserves at the beginning of every period: ${}_{at}f_t$ equals the end of the previous period level of reserves: f_{t-1}.

make the growth rate fluctuate around m. Stochastic shocks will be of two kinds: permanent (v_t) and temporary (x_t). Hence, gc_t is generated in accordance with: 1/

$$(48) \quad gc_t = gc_{t-1} + m + v_t + x_t - x_{t-1}$$

The two random terms v_t and x_t are generated by white noise processes, that is: $v_t \sim N(0, \sigma_v^2)$; $x_t \sim N(0, \sigma_x^2)$ and both v_t and x_t are serially independent distributed.

The process governing the behavior of the price of the tradable good p_t^* is assumed to be similar to the process followed by gc_t. Hence, p_t^* is generated in accordance with:

$$(49) \quad p_t^* = p_{t-1}^* + j^* + n_t + t_t - t_{t-1}$$

where: j^* is a constant trend growth rate

n_t is a stochastic permanent shock

t_t is a stochastic temporary shock

and n_t and t_t are white noise processes, that is: $n_t \sim N(0, \sigma_n^2)$; $t_t \sim N(0, \sigma_t^2)$ and n_t and t_t are serially independent distributed.

In this section, it is assumed that agents posess full current information; that is, they know (or have enough information to infer) all current values of the relevant variables that affect their decisions.

To facilitate the exposition, the derivation of the final solutions for the stock of foreign reserves and for the price and output levels of the nontradable good is presented in Appendix II. Based on Appendix II, Table 1 distinguishes between the "financial constraint effect" (the effects of exogenous shocks on the expected real monetary base) and the "relative price effect" (the effects of exogenous shocks on the real exchange rate) that impinge on the levels of output of both commodities following a change in the exogenous variables of the model.

The behavioral responses of the price level of the nontradable good, the stock of foreign reserves and the levels of output, following a change in the exogenous variables of the model will now be analyzed.

1/ This process is identical to the one presented in Barro (1978).

Table 1. Decomposition of the Effects Impinging on the Domestic Levels of Output in the Full Current Information Case

Endogenous Variables:	y_t		y_t^*	
Exogenous Variables	Financial constraint effect = $a_1\ ({}_ah_t - {}_{at}E(pi_{t+1}))$	Relative price effect = $(1-\tau)(p_t - p_t^* - s_t)$	Financial constraint effect = $a_1\ ({}_ah_t - {}_{at}E(pi_{t+1}))$	Relative price effect = $-\tau(p_t - p_t^* - s_t)$
Increase in v_t	$\frac{a_1[\omega_1 - \tau(\theta_1 Y_2 + \theta_2)]}{1-\theta_1 Y_3}$	$(1-\tau)\theta_2$	$\frac{a_1[\omega_1 - \tau(\theta_1 Y_2 + \theta_2)]}{1-\theta_1 Y_3}$	$-\tau\theta_2$
Increase in m	$\frac{a_1[\omega_1 - \tau(\theta_1 Y_2 + \theta_2 + \theta_3)]}{1-\theta_1 Y_3}$	$(1-\tau)\theta_3$	$\frac{a_1[\omega_1 - \tau(\theta_1 Y_2 + \theta_2 + \theta_3)]}{1-\theta_1 Y_3}$	$-\tau\theta_3$
Increase in x_t	$\frac{a_1[\omega_1 - \tau\theta_1 Y_2]}{1-\theta_1 Y_3}$	$(1-\tau)\theta_5$	$\frac{a_1[\omega_1 - \tau\theta_1 Y_2]}{1-\theta_1 Y_3}$	$-\tau\theta_5$
Increase in n_t or s_t	$\frac{-a_1[\tau\theta_1(Y_4+Y_5) + \tau\theta_7 + (1-\tau)]}{1-\theta_1 Y_3}$	$(1-\tau)(\theta_7-1)$	$\frac{-a_1[\tau\theta_1(Y_4+Y_5) + \tau\theta_7 + (1-\tau)]}{1-\theta_1 Y_3}$	$-\tau(\theta_7-1)$
Increase in j*	$\frac{-a_1[\tau\theta_1(Y_4+2Y_5) + \tau(\theta_7+\theta_8) + 2(1-\tau)]}{1-\theta_1 Y_3}$	$(1-\tau)(\theta_8-1)$	$\frac{-a_1[\tau\theta_1(Y_4+2Y_5) + \tau(\theta_7+\theta_8) + 2(1-\tau)]}{1-\theta_1 Y_3}$	$-\tau\ (\theta_8-1)$
Increase in t_t	$\frac{-a_1\tau\theta_1 Y_4}{1-\theta_1 Y_3}$	$(1-\tau)(\theta_{10}-1)$	$\frac{-a_1\tau\theta_1 Y_4}{1-\theta_1 Y_3}$	$-\tau(\theta_{10}-1)$
Increase 1/ in u_t	$\frac{-a_1\tau\theta_1 Y_6}{1-\theta_1 Y_3}$	$(1-\tau)\theta_{12}$	$\frac{-a_1\tau\theta_1 Y_6}{1-\theta_1 Y_3}$	$-\tau\theta_{12}$
Increase in ε_t	$\frac{-a_1\tau\theta_1 Y_7}{1-\theta_1 Y_3}$	$(1-\tau)\theta_{13}$	$\frac{-a_1\tau\theta_1 Y_7}{1-\theta_1 Y_3}$	$-\tau\theta_{13}$

Source: Appendix II.

1/ A direct effect on the aggregate supply equal to a_2 should be added in order to obtain the total output effect of a productivity shock.

1. An increase in government credit

Consider first an increase in the current level of government credit caused by a positive permanent monetary change (v_t). The price level of the nontradable good will increase and the level of foreign reserves will decrease, because there will be a net "potential" excess demand for each commodity. These changes in p_t and f_t, however, will be less than proportional to the increase in government credit. 1/

The response of outputs to the increase in gc_t, however, will depend on two effects: (1) a financial constraint effect; and (2) a relative price effect. While the increase in gc_t generates an unambiguous relative price change favoring an increase in the output of nontradables and a decrease in that of tradables, the response of the real monetary base (deflated by the expected future general price level) needs some explanation. Since only the predetermined level of foreign reserves (f_{t-1}) enters into the beginning-of-period monetary base, an increase in gc_t will unambiguously result in an increase in the nominal level of the monetary base.

However, the response of the real monetary base can be analyzed by considering the expected inflation rate. 2/ By using the relevant coefficients in equations (2.3) and (2.4) of Appendix II, it is clear that: 3/

$$(50) \quad \frac{\delta({}_{at}E(p_{t+1})-p_t)}{\delta v_t} = \frac{\theta_2+\theta_1 Y_2}{1-\theta_1 Y_3} - \theta_2 = \frac{\theta_1(Y_2+\theta_2 Y_3)}{1-\theta_1 Y_3}$$

which is negative and less than one is absolute terms. Thus, although the current price level of the nontradable good will increase, the public will also expect a deflation in the subsequent period. The intuition behind this result is straightforward: the decrease in the level of reserves which follows an increase in v_t does not affect the current general price level. This is affected solely by the predetermined value of f_{t-1}. However, the decrease in reserves will decrease the future level of the monetary base and hence will impinge negatively on the expected future general price level. Now, this decrease in the expected inflation rate generates an increase in the equilibrium level of the real monetary base (deflated by the future expected general price

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1/ This is so because (from Appendix II): $\theta_2 < 1$ and (in equation (2.18) of Appendix II): $|\delta f_t/\delta gc_{t-1}| < 1$ since $|Y_2| < 1$.

2/ Since p_t^* is an exogenous variable, the analysis of the expected general inflation rate can be conducted by solely considering the expected inflation rate of the nontradable good.

3/ $\frac{\delta(X)}{\delta(Y)}$ refers to the partial derivative of variable X with respect to variable Y.

level) and hence an increase in the firms' demand for labor. Thus, it generates a positive financial constraint effect tending to cause a short-run increase in the output of both commodities. As a result, the response of nontradables output will always be positive following an increase in v_t, but the output of tradables may either increase or decrease. It will increase if the positive financial constraint effect is greater than the negative relative price effect; hence, a strong enough financial constraint effect will result in an increase in both levels of output.

Next, consider an increase in the rate of growth of government credit. The price level of the nontradable good will increase and the level of foreign reserves will decrease, but the magnitude of these changes will be bigger than those corresponding to an increase in v_t. 1/ While the relative price effect generated by an increase in m is similar (but bigger) than the one generated by an increase in the level of the debt, the financial constraint effect might be quite different and might even affect negatively the supply of both commodities. In fact, it is clear from Appendix II, that when the rate of growth of the government credit increases, the effect on the inflation rate is:

$$(51) \quad \frac{\delta({}_{at}E(p_{t+1})-p_t)}{\delta m} = \frac{\theta_2+\theta_3+\theta_1 Y_2}{1-\theta_1 Y_3} - \theta_3 = \frac{\theta_2+\theta_1(Y_2+\theta_3 Y_3)}{1-\theta_1 Y_3}$$

This effect might be positive or negative. If it is negative, the decrease in the expected inflation rate will generate a positive financial constraint effect by increasing the equilibrium level of the real monetary base. Hence, the effects on the level of outputs will be similar whether the increase in the government debt arises from a rise in either v_t or m. However, if equation (51) is positive, the real monetary base (deflated by either p_t or ${}_{at}E(p_{t+1})$) will decrease, generating a negative financial constraint effect on the aggregate supply of both commodities. In the latter situation, the level of output of tradable goods will unambiguously decrease and the level of output of nontradables might decrease if the negative financial constraint effect outweighs the positive relative price effect. 2/ Notice that in this case, output responses are opposite to those resulting from an increase

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1/ From Appendix II, $\theta_3 > \theta_2$ and (in equation (2.18) of Appendix II)

$$\left|\frac{\delta f_t}{\delta_m}\right| > \left|\frac{\delta f_t}{\delta v_t}\right|.$$

This is so because when m increases, there will be an additional increase in the next period level of government credit, which generates additional pressure on the "potential" current excess demands for both commodities.

2/ That is, if the financial constraint effect outweighs the output effects of an appreciation of the real exchange rate.

in v_t and, hence, aggregate output will decrease following the monetary expansion. This example serves to illustrate once more the importance of the nature of an observed increase in the government credit in order to derive conclusions regarding the monetary effects on output.

Finally, consider a positive temporary monetary shock x_t. Once again the price level of nontradables will increase and the stock of reserves will decrease, but this time the magnitude of changes will be smaller than those corresponding to an increase in v_t. 1/ The obvious reason for this result is that economic agents know that the shock is temporary and hence expect the level of government credit to decrease next period. This expectation will negatively affect their expectations of the future general price level and hence will decrease the current excess demand for both goods.

Once more the relative price effect will be similar to that analyzed above, i.e., it will favor an increase in the supply of nontradable goods. It will, however, be of a smaller magnitude. The financial constraint effect will be unambiguously positive in this case because there will be an unambiguous reduction in the expected inflation rate. That is, from Appendix II:

$$(52) \qquad \frac{\delta({}_{at}E(p_{t+1})-p_t)}{\delta x_t} = \frac{\theta_1 Y_2}{1-\theta_1 Y_3} - \theta_5$$

which is negative. The intuition behind this result is as follows. Not only will government credit decrease in period t+1, but the reduction in the current level of foreign reserves will further reduce the level of the monetary base in period t+1. Hence, the expected value of the future general price level will be lower. Thus, as in the case of a permanent increase in the government debt, a strong enough financial constraint effect will generate an increase in the level of aggregate output.

2. A devaluation or an increase in the foreign price of the tradable good

A devaluation or an increase in the foreign price of the tradable good affects the expected general inflation rate through their effects on the domestic price levels of both produced goods. In addition, while

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1/ From equation (2.10) of Appendix II: $\theta_5 < \theta_2$; and (in equation (2.18) of Appendix II):

$$\left|\frac{\delta f_t}{\delta x_t}\right| < \left|\frac{\delta f_t}{\delta gc_{t-1}}\right|.$$

such changes do not affect the current level of the beginning of period monetary base, they do affect the future level of the monetary base through their impact on the level of reserves.

Since s_t enters into the system of structural equations in the same way as p_t^* does, the economic effects of an anticipated permanent devaluation, i.e., a rise in ξ_t, are identical to the effects generated by a permanent rise in the price level of the tradable good (n_t). Thus, consider an increase in either n_t or ξ_t. Both the price level of the nontradable good and the level of foreign reserves will rise 1/ because this kind of increase generates a "potential" excess demand for nontradables and an excess supply of tradables. The effect on the output levels of both commodities can, once more, be decomposed in a "relative price" effect and a "financial constraint" effect. From Table 1 and Appendix II, it can be seen that the relative price effect implies a depreciation of the real exchange rate that favors an increase in the supply of tradables and a decrease in the supply of nontradables (since $\theta_7 < 1$). Since the current nominal monetary base at the beginning of the period remains unchanged when there is a devaluation or a rise in the price of tradables, the financial constraint effect can be evaluated by analyzing the effect on the expected future general price level. 2/ That is, from Appendix II:

$$(53) \qquad \frac{\delta({}_{at}E(pi_{t+1}))}{\delta\xi_t} = \frac{\tau\theta_1(Y_4+Y_5)+\tau\theta_7}{1-\theta_1 Y_3} + (1-\tau)$$

which is unambiguously positive because $|Y_4| > |Y_5|$ and $\theta_7 > 0$. This result is straightforward. A permanent devaluation or a permanent rise in the current price level of the tradable good increases the current level of foreign reserves. This in turn increases the next period monetary base and hence generates an increase in the expected value of the future general price level. Since the change in reserves does not affect the current beginning-of-period monetary base, the increase in the expected value of the future general price level will be larger than the increase in the current general price level; that is, the expected inflation rate will increase. As a result, the current real monetary base (deflated by ${}_{at}E(pi_{t+1})$ will decrease, generating a negative financial constraint

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1/ From Appendix II: $\theta_7 > 0$ and (in equation (2.18) of Appendix II):

$$\frac{\delta f_t}{\delta\xi_t} = \frac{Y_4+Y_5+Y_3\theta_7}{1-\theta_1 Y_3} > 0$$

because $|Y_4| > |Y_5+Y_3|$ and $\theta_7 < 1$.

2/ Since p_{t+1}^* is a component of pi_{t+1}, it is no longer possible to solely concentrate on the effects of a rise in p_t^* on the inflation rate of the nontradable good.

effect. Thus, the supply of the nontradable good will unambiguously decrease, while the response of the supply of the tradable good depends on the importance of the relative price effect compared to the financial constraint effect. If the latter outweighs the former, the supply of tradables also decreases. In this case, and contrary to the standard result, a devaluation or an increase in the price level of the tradable good give rise to a decrease in aggregate output.

Next, consider an increase in the rate of growth of the price level of the tradable good. Although the price level of the nontradable good will increase (on a larger magnitude 1/ than the rise generated by an increase in n_t) this is the only case in which we have not been able to sign the effect on the level of reserves 2/ and thus, the responses of the inflation rate and the real monetary base are ambiguous. These indeterminacies arise because when j^* increases, there is a positive first round effect on the expected rate of inflation which is not present when n_t increases. The "impact" increase in ${}_{at}E(pi_{t+1})$ partially or totally offsets the "potential" excess supply of tradable goods generated by the increase in p_t^*. If the negative effect of a rise of ${}_{at}E(pi_{t+1})$ on f_t is large enough, the level of foreign reserves will decrease. Similarly, if the initial excess supply of tradables is large enough, the level of reserves will increase. In the latter case, the future beginning-of-period monetary base will also increase, as will the expected inflation rate generating a negative financial constraint effect on the output levels of both commodities. However, it is also possible that the level of foreign reserves will decrease following a rise in j^*. In such a case, its effect on the expected inflation rate and hence on the financial constraint effect is indeterminate. The only unambiguous result inpinging on the supply of outputs is a relative price effect favoring the output level of the tradable good. 3/

Finally, consider a temporary rise in the price level of tradables. The price level of the tradable good and the level of foreign reserves will increase, but this time the increase in the level of foreign

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1/ From equation (2.12) of Appendix II: $\theta_8 > \theta_7$.

2/ From equation (2.18) of Appendix II:

$$\frac{\delta f_t}{\delta j^*} = \frac{Y_4 + 2Y_5 + Y_3(\theta_7+\theta_8)}{1-\theta_1 Y_3}$$

which will be negative if $|Y_4| < |{}_2Y_5 + Y_3\ (\theta_7 + \theta_8)|$.

3/ From Appendix II: $\theta_8 < 1$.

reserves will be larger than that corresponding to a permanent increase. 1/

The rise in the level of reserves increases the future beginning-of-period level of the monetary base, and this will result in an increase in the expected future general price level. 2/ As a consequence, the equilibrium level of the current real monetary base (deflated by ${}_{at}E(pi_{t+1})$) will fall generating a negative financial constraint effect on the output levels of both commodities. Thus, the effects of a temporary change in the price level of the tradable good are qualitatively similar (but of a different magnitude) to those generated by a permanent change. The supply of the nontradable good will unambiguously decrease, while the response of the output of the tradable good depends on the importance of the relative price effect as compared to the financial constraint effect. If the financial constraint effect is strong enough, aggregate output will decrease.

3. An increase in productivity or an exogenous change in preferences

An increase in productivity or an exogenous change in preferences that assigns a lower weight to current consumption leads to a decrease in the price level of the nontradable good and to an increase in the stock of foreign reserves 3/ because both shocks generate a potential excess supply of both commodities. In addition, either an increase in u_t or a decrease in ε_t generates: (1) a depreciation of the real exchange rate that results in a relative price effect favoring the supply of tradables; and (2) a negative financial constraint effect on the supply of both commodities. Result (1) above is obvious, and result (2) emerges because of the positive impact of the rise in the current level of foreign reserves on the future expected general price level, which in turn lowers the equilibrium real monetary base.

1/ Notice from equation (2.18) of Appendix II that:

$$\left|\frac{\delta f_t}{\delta t_t}\right| > \left|\frac{\delta f_t}{\delta v_t}\right|.$$

2/ From Appendix II: $\dfrac{\delta({}_{at}E(pi_{t+1}))}{\delta t_t} = \dfrac{\theta_1 Y_4}{1-\theta_1 Y_3}$ which is positive.

3/ From Appendix II: $\theta_{12} < 0$; $\theta_{13} > 0$, and in equation (2.18) of Appendix II:

$$\frac{\delta f_t}{\delta u_t} = \frac{Y_6}{1-\theta_1 Y_3} > 0; \text{ and } \frac{\delta f_t}{\delta \varepsilon_t} = \frac{Y_7}{1-\theta_1 Y_3}.$$

Notice one interesting result which emerges from the above analysis: In the presence of a negative financial constraint effect, it is possible that the output response of the nontradable good to a productivity increase is negative. This will happen if the combined negative "financial constraint" and "relative price" effects outweigh the positive direct effect of a productivity increase on production. 1/ This result highlights once more the importance of the behavior of the expected future general price level in an economy facing a financial constraint. Any change in an exogenous variable leading to a rise in foreign reserves exerts an upward pressure on the expected future general price level, because of its impact on the future beginning-of-period monetary base.

VII. The Incomplete Current Information Case

We will now relax the assumption that agents possess full current information by assuming instead that: (1) agents do not know the decomposition of a monetary shock (a shock to the government credit) between its temporary and permanent components; and (2) at the beginning of the period, although agents know the rule (47), they do not know the value of ξ_t. Moreover, they do not observe the price level of the tradable good that will prevail at the end of the period. The beginning of period observation of the interest rate allows agents to infer the current value of the government credit but they cannot distinguish (during the current period) between permanent and temporary monetary shocks. In addition, knowledge of gc_t does not convey any relevant information about the current exchange rate or the current price level of the tradable good. That is, now the relevant information set is the one specified in equation (12).

The beginning and end-of-period price level expectations are not equal under fixed rates because it is assumed that n_t, ξ_t, and t_t are only known at the end of each period. 2/ This feature of our model makes its solution particularly cumbersome. Hence, rather than presenting specific solutions of the model, the remainder of this section will discuss the effects on the model's endogenous variables of removing the assumption of full current information.

1/ That is, if: $|a_2| < \left|\frac{-a_1\tau\theta_1 Y_6}{1-\theta_1 Y_3} + \theta_{12}(1-\tau)\right|$ (see Table 1 and Appendix I).

2/ The confusion between v_t and x_t does not generate a discrepancy between beginning and end-of-period expectations because the distinction between the shocks is only known with one period lag.

1. The confusion between permanent and temporary monetary shocks

Under incomplete current information, the direction of the responses of the price level of the nontradable good and the stock of foreign reserves to an increase in the government debt will be the same as in the full current information case. This is so because, in the full current information case, the effects of both a temporary or a permanent monetary shock were qualitatively similar. They were of different magnitudes, however. Under full current information, the (positive) response of the price level of the nontradable good and the (negative) response of the level of foreign reserves were larger if they followed a permanent rather than a temporary change in the government debt. It follows from this that if a temporary shock is mistakenly interpreted as permanent, the increase in p_t caused by that shock will be larger than in the full current information case.

In addition, under full current information, both shocks generated an increase in the real monetary base (a positive financial constraint effect) and a relative price effect favoring the supply of the nontradable good. However, while the relative price effect was larger after a permanent monetary shock, the financial constraint effect was larger after a temporary shock. 1/ Thus, while both shocks generated a definite increase in the supply of the nontradable good, it could not be determined which shock generated the greater response. Moreover, although the response of the supply of the tradable good is ambiguous, a temporary monetary shock might lead to an output expansion of the tradable good, even if a permanent shock leads to an output contraction of this good. This result obviously follows because of the larger financial constraint effect and the smaller relative price effect generated by a temporary shock.

From the above considerations, it follows that, under incomplete current information, the confusion between permanent and temporary monetary shocks cannot change the direction of the response of the supply of the nontradable good relative to the full current information case; only the magnitude of the output increase will differ between the two alternative information sets. However, the confusion between monetary shocks might change the direction of response of the supply of tradables relative to the full current information case. In particular, if under full current information, a temporary shock leads to an output expansion while a permanent shock leads to an output contraction of the tradable

1/ Comparing equations (50) and (52), it can be concluded that the decline in the expected inflation rate following a monetary shock was larger if the shock was temporary than if it was permanent. Hence, the increase in the real monetary base was larger under a temporary monetary shock.

good, a permanent shock mistakenly viewed as temporary (when lack of complete current information is assumed) might increase the output level of the tradable commodity.

To sum up, confusion between permanent and temporary monetary shocks can only affect the magnitude of the responses of the price and output levels of the nontradable good, as well as the stock of foreign reserves relative to the corresponding responses under full current information. However, such a confusion can imply an increase in the output level of the tradable good following a permanent shock, even in situations where a fully anticipated permanent change would lead to an output contraction.

2. The lack of beginning-of-period observation of the exchange rate and the foreign price of the tradable good

In our model, output decisions are made at the beginning of every period based on the information set available at the time. If the current period exchange rate and the price level of the tradable good are not known at the beginning-of-the-period, agents must form expectations about them. 1/ In this fixed exchange rate case, agents cannot infer the value of either the exchange rate or the price level of the nontradable good by observing the beginning-of-period current level of the monetary base. Thus, the short-run supplies of both commodities no longer depend on their actual price levels; instead, they depend on their expected price levels. 2/ Hence, the current output levels of both commodities will remain unchanged following either an unanticipated permanent devaluation or an unanticipated permanent or temporary shock to the price level of the tradable good.

Now, since the outputs remain unchanged following an unanticipated shock to p_t^* or a change in ξ_t, the price level of the nontradable good and the stock of foreign reserves have to bear all the adjustment. In particular, an unanticipated positive permanent shock (either a rise in ξ_t or a rise in n_t) will raise the demand for nontradables and will decrease the demand for tradables 3/ implying that the increase in both p_t and f_t will be larger than in the complete current information case and hence, the depreciation of the real exchange rate will be smaller in the incomplete current information case. If, instead, the unanticipated shock is a temporary increase in the price level of tradables, only the *demand for tradable goods will decrease as a first round effect*, because

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1/ Which in the present case are equal to:

$$_{at}E(p_t^*) = p_{t-1}^* + j^* - t_{t-1} \text{ and } _{at}E(s_t) = s_{t-1}.$$

2/ It is important to recall that, in this model, price misperceptions of the Lucas type are not an argument in the supply functions.

3/ The demand for y_t will increase because of a rise in $_{bt}E(p_{t+1}^*)$. The demand for y_t^* will decrease because of a rise in p_t^*.

a temporary shock does not affect ${}_{bt}E(p^*_{t+1})$, which is the foreign price variable impinging on the demand for nontradable goods. However, in subsequent rounds, the increased level of reserves will increase the expected future general price level, leading to a rise in the demand for nontradables. Thus, the price level of nontradables will increase following an unanticipated temporary increase in the price of tradables, but by less than in the case of an unanticipated permanent increase in the price of tradables.

To sum up: relative to the full information case, incomplete beginning-of-period knowledge about the current period exchange rate and the price level of the tradable good results in a larger response of both p_t and f_t to an unanticipated permanent change in either p^*_t or s_t. Also, the increase in the price level of the nontradable good will be larger if the unanticipated increase in the price of the tradable good is permanent rather than temporary. In contrast with the full current information case, no output response will follow an unanticipated devaluation or an unanticipated foreign price shock. This result contrasts with that obtained from models which stress the effects of price misperceptions on aggregate supply. In those models an anticipated devaluation has no effect on the output level, while an unanticipated devaluation does. The opposite result arises here. This is an interesting conclusion because, to the extent that real world economies are properly described by the features of this model, we may conclude that an unanticipated current devaluation will increase the level of foreign reserves without hurting the current levels of output and employment of those economies. However, notice that in period t+1, ξ_t will be part of the information set and hence the unanticipated devaluation of period t might then have the contractionary effects which an anticipated devaluation would have had in period t. That is, the output effects of an unanticipated devaluation will not be eliminated, but only postponed.

VIII. Conclusions

This paper has set up a model for a small open economy under a fixed exchange rate regime, and has attempted to provide an explanation for some of the observed short-run output responses to devaluations and monetary policies in LDCs. Specifically, the paper has considered the effects of changes in the exogenous component of the money supply (which in this model equals the level of government credit), the price of the tradable goods, and the exchange rate (a devaluation) on the outputs of the domestically produced commodities, the price of the nontradable good, and the level of foreign reserves. It has been shown that the domestic firms' need to finance in advance their working capital from a domestic banking sector (the financial constraint) has important implications for the economy's response to such exogenous shocks.

In particular, it has been shown that if the financial constraint effect is strong enough, an anticipated rise in the level of government credit will be expansionary, while an anticipated rise in the rate of growth of government credit might be contractionary. Moreover, an anticipated rise in the foreign price of the tradable good or an anticipated devaluation will be contractionary in the short run.

In addition, in the model developed in this paper, a permanent monetary shock might generate increases in the outputs of both commodities if it is mistakenly viewed as temporary, even in situations where a fully anticipated permanent monetary change would lead to a reduction in the output of the tradable good.

Finally, an unanticipated devaluation, or an unanticipated rise in the foreign price of the tradable good, will result in a depreciation of the real exchange rate, but will have no current output effects. However, it might lead to contractions of the outputs of both goods in the subsequent periods. This result differs from that arising in models where price misperceptions enter output supply functions; in those models, an unanticipated devaluation leads to an immediate, albeit short-run, output expansion.

In summary, this paper has shown that for small open economies facing severe financial constraints under a fixed exchange rate regime, the direction of the output response to exogenous shocks crucially depends on: (1) the nature of the exogenous shock impinging on the economy; and (2) the importance of the financial constraint effect as compared to the relative price effect.

Derivation of the Household's Decision Rules

To illustrate the agent's maximization problem, let us first consider the end of the period when he already knows his labor supply.

At the end of period 1, the agent will choose his current and future consumption levels of tradable and nontradable goods. He knows that in period 2, he will have a nominal wealth: "A_2" and will have to choose Y_2 and Y_2^* in order to:

$$(1.1) \quad \text{Max } Y_2^{\tau} \, Y_2^{*(1-\tau)}$$

subject to:

$$(1.2) \quad A_2 = P_2 \, Y_2^d + (P_2^* S_2)(Y^*)_2^d$$

From equations (1.1) and (1.2), the demand functions for the agent's second period of life are obtained:

$$(1.3) \quad Y_2^d = \tau A_2 / P_2$$

$$(1.4) \quad (Y^*)_2^d = (1-\tau) A_2 / P_2^* S_2$$

As stated above, these demand functions show a unitary elasticity of demand with respect to expenditure on future consumption (A).

Given (1.3) and (1.4), the maximized value of $Y_2^{\tau} \, (Y^*)_2{}^{(1-\tau)}$ is:

$$(1.5) \quad (Y_2^{\tau} \, (Y^*)_2{}^{(1-\tau)})_{max} = A_2 / (\tau^{-\tau} \, (1-\tau)^{-(1-\tau)} P_2^{\tau} (P_2^* S_2)^{1-\tau})$$

$$\equiv A_2 / (j' \, PI_2)$$

where: $j' \equiv \tau^{-\tau} (1-\tau)^{-(1-\tau)}$

$PI_2 \equiv$ price index in period 2 $\equiv P_2^{\tau} (P_2^* S_2)^{1-\tau}$

The price index PI is homogenous of degree one, due to the Cobb-Douglas specification of the utility function. Substituting (1.5) into equation (17) of the main text and noticing that the budget constraint (equation (18) in the main text) requires:

$$(1.6) \quad A_2 = W_1 L_1 + Tr_2 + {}^h\pi_2 - P_1 Y_1^d - (P_1^* S_1)(Y^*)_1^d \equiv {}_aH_2$$

The first period problem is obtained. This requires the agent to choose Y_1^d, $(Y^*)_1^d$ to:

$$(1.7) \qquad \max_{\{Y_1^d,(Y^*)_1^d,\lambda_2\}} = \mu_1 \ln\,[Y_1^{\tau}(Y^*)_1^{(1-\tau)}] + \ln(\bar{L}-L_1)$$

$$+ \frac{\beta}{j'}\, b1^E\, [\frac{W_1L_1+Tr_2+{}^h\pi_2-P_1Y_1^d-(P_1^*S_1)(Y^*)_1^d}{PI_2}]$$

$$+ \lambda[W_1L_1^s - P_1Y_1^d - P_1^*S_1(Y^*)_1^d]$$

where λ is the Kuhn-Tucker multiplier associated with the constraint (19) in the main text.

Imposing the restrictions that consumption of both commodities be positive every period and that the young individual's demand for money be positive, the maximization procedure leads to the following decision rules:

$$(1.8) \qquad Y_1^d = \frac{j'\tau}{\beta}\ \mu_1\ \frac{b1^E(PI_2)}{P_1}$$

$$(1.9) \qquad (Y^*)_1^d = \frac{j'(1-\tau)}{\beta}\ \mu_1\ \frac{b1^E(PI_2)}{P_1^*S_1}.$$

Now, we can turn to the young agent's maximization problem at the beginning of the period. At that time, the individual's demands for commodities are stochastic variables. The agent's problem is:

$$(1.10) \qquad \max_{\{L_1^s\}}:\ \mu_1\ a1^E\ [\ln((\tilde{Y})_1^{\tau}(\tilde{Y}^*)_1^{(1-\tau)})] + \ln(\bar{L}-L_1)$$

$$+ \frac{\beta}{j'}\ a1^E(\frac{W_1L_1+Tr_2+{}^h\pi_2-P_1\tilde{Y}_1-P_1^*S_1\tilde{Y}_1^*}{PI_2})$$

where the "~" on top of (Y_1) and (Y_1^*) is used to indicate the stochastic nature of these variables.

Notice that the inequality constraint (19) in the main text does not apply at the beginning of the period, because the demands for consumption goods and money are made effective at the end of the period. The maximization procedure leads to the following labor supply decision rule:

$$(1.11) \quad L_1^s = \bar{L} - \frac{j'}{\beta}\ \frac{a1^E(PI_2)}{W_1}.$$

Derivation of the Final Solution for the Price and Output Levels of the Nontradable Good and the Level of Foreign Reserves

The macro-model of Section V can be solved to yield the following set of semi-reduced forms for the price level of the nontradable good and the end of period level of foreign exchange reserves.

$$(2.1) \quad p_t = X_0 + X_1 f_{t-1} + X_2 gc_t + X_3 {}_{at}E(p_{t+1}) + X_4 (p_t^* + s_t)$$

$$+ X_5 ({}_{at}E(p_{t+1}^* + s_{t+1})) + X_6 u_t + X_7 \varepsilon_t$$

$$(2.2) \quad f_t = Y_0 + Y_1 f_{t-1} + Y_2 gc_t + Y_3 {}_{at}E(p_{t+1}) + Y_4 (p_t^* + s_t)$$

$$+ Y_5 ({}_{at}E(p_{t+1}^* + s_{t+1})) + Y_6 u_t + Y_7 \varepsilon_t$$

where: $X_0 = [\delta_0 - a_0 - a_1 \omega_0 + \delta_2 \omega_0] / \det$

$X_1 = [\delta_2(1-\omega_1) - a_1(1-\omega_1)] / \det$

$X_2 = [\delta_2 \omega_1 - a_1 \omega_1] / \det$

$X_3 = [\delta_1 \tau + a_1 \tau] / \det$

$X_4 = [1 - \tau] / \det$

$X_5 = [\delta_1(1-\tau) + a_1(1-\tau)] / \det$

$X_6 = - a_2 / \det$

$X_7 = 1 / \det$

and:

$Y_0 = [d_1 \tau(a_0 + a_1 \omega_0 - \delta_0 - \delta_2 \omega_0)$

$+ (1-\tau+\delta_1+\delta_2)(d_1 a_0 - d_2 b_0 + d_1 a_1 \omega_0 - d_2 b_2 \omega_0)] / \det$

$Y_1 = [d_1 \tau(1-\omega_1)(a_1 - \delta_2)$

$+ (1-\tau+\delta_1+\delta_2)\ (d_3 + (1-\omega_1)(d_1 a_1 - d_2 b_2))] / \det$

$Y_2 = [d_1 \tau \omega_1 (a_1 - \delta_2) + (1-\tau+\delta_1+\delta_2)(\omega_1)(d_1 a_1 - d_2 b_2)] / \det$

$Y_3 = - \tau[d_1 \tau(a_1 + \delta_1) + (1-\tau+\delta_1+\delta_2)(d_1 a_1 + d_2 b_1)] / \det$

$$Y_4 = [d_1\tau(\delta_1+\delta_2) + (1-\tau+\delta_1+\delta_2)(1-d_3+d_2b_1+d_2b_2)] \ / \ \det$$

$$Y_5 = -\ (1-\tau)[d_1\tau(a_1+\delta_1) + (1-\tau+\delta_1+\delta_2)(d_1a_1+d_2b_1)] \ / \ \det$$

$$Y_6 = [d_1\tau a_2 + (1-\tau+\delta_1+\delta_2)(d_1a_2)] \ / \ \det$$

$$Y_7 = -\ [d_1\tau + (1-\tau+\delta_1+\delta_2)d_2] \ / \ \det$$

and: $\det = 1/(1-\tau + \delta_1 + \delta_2)$

For stability purposes, it will be assumed that $\delta_2 > a_1$ and $b_2 > a_1$. Also, from equation (2.2) it is clear that (leaving aside the constant d_3) the effects of gc_t and f_{t-1} on the level of foreign reserves are similar. This is, of course, due to the fact that both variables are components of the beginning-of-period monetary base. Hence, the usual requirement for the stability of equation (2.2):

$$\left| \frac{\delta f_t}{\delta f_{t-1}} \right| < 1 \text{ is related to the value of } \left| \frac{\delta f_t}{\delta gc_t} \right|$$

In general, 1/ in this model $|\delta f_t/\delta gc_t| < 1$ and, $1 > (\delta f_t/\delta f_{t-1}) > 0$. Hence the model will be stable. 2/

In order to proceed towards a final solution of the model, that is, taking into account the endogenous property of price expectations, it is important to realize that equations (2.1) and (2.2) are not independent from each other. The method of undetermined coefficients is used

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1/ The restrictions imposed by the microfoundations in Section III of the main text imply the following: First, δ_2 and b_2 are respectively the elasticities of the demand for nontradables and tradables, with respect to the real monetary base. From equations (24) and (25) it is clear that both elasticities are less than one, and that $\delta_2 = b_2$. Second, δ_1 and b_1 are respectively the elasticities of the demand for nontradables and tradables with respect to the expected inflation rate, which again from equations (24) and (25) are less than one and $\delta_1 = b_1$. In addition, $(\delta_1+\delta_2) = (b_1+b_2) = 1$. *Thus, from equation (2.2)* $\delta f_t/\delta gc_t$ *is less than* one in absolute terms for most plausible values of the model's parameters.

In order to avoid a bias in the model arising from the constant of linearization, d_3, the remainder of this paper will assume $d_3 = 1$. Under that assumption, $|\delta f_t/\delta f_{t-1}| < 1$ and the model will be stable.

2/ Notice that if $d_3 < 1$ the model will also be stable; but then $(\delta f_t/\delta f_{t-1})$ might be less than zero, which would imply that the convergence pattern would exhibit dampened cycles.

here to obtain the final solution of the model (see Lucas (1972), Barro (1976, 1978)). Following McCallum (1983), notice that the solution for the price level of nontradables can be written as a linear function of the predetermined state variables f_{t-1}, gc_{t-1}, m, v_t, x_t, x_{t-1}, p^*_{t-1}, j^*, n_t, t_t, t_{t-1}, s_{t-1}, ξ_t, u_t, ε_t, and the constant 1. Hence:

$$(2.3)\quad p_t = \theta_o + \theta_1 f_{t-1}\ \theta_2\ gc_{t-1} + \theta_3 m + \theta_4 v_t + \theta_5 x_t + \theta_6 x_{t-1} + \theta_7 (p^*_{t-1} + s_{t-1}) + \theta_8 j^* + \theta_9 (n_t + \xi_t) + \theta_{10} t_t + \theta_{11} t_{t-1} + \theta_{12} u_t + \theta_{13} \varepsilon_t$$

When the θs are the unknown coefficients.

Leading equation (2.3) once, taking beginning-of-period t expectations, and using equation (2.2), we obtain:

$$(2.4)\quad {}_{at}E(p_{t+1}) = \frac{1}{1-\theta_1 Y_3} \{(\theta_0+\theta_1 Y_0) + (\theta_1 Y_1) f_{t-1} + (\theta_1 Y_2+\theta_2)(gc_{t-1}+v_t-x_{t-1}) + (\theta_1 Y_2+\theta_2+\theta_3) m + (\theta_1 Y_2+\theta_2+\theta_6) x_t + (\theta_1 Y_4+\theta_1 Y_5+\theta_7)(p^*_{t-1}+s_{t-1}+\xi_t+n_t-t_{t-1}) + (\theta_1 Y_4+2\theta_1 Y_5+\theta_7+\theta_8) j^* + (\theta_1 Y_4+\theta_7^N+\theta_{11}) t_t + (\theta_1 Y_6) u_t + (\theta_1 Y_7)\varepsilon_t\}$$

Substituting equation (2.4) into equation (2.1), the final solution for the price level of nontradables is obtained:

$$(2.5)\quad p_t = \frac{1}{1-\theta_1 Y_3} \{[X_0 - X_0\theta_1 Y_3 + X_3\theta_0 + X_3\theta_1 Y_0] + [X_1 - X_1\theta_1 Y_3 + X_3\theta_1 Y_1]\ f_{t-1} + [X_2 - X_2\theta_1 Y_3 + X_3\theta_1 Y_2 + X_3\theta_2]\ (gc_{t-1}+v_t-x_{t-1})$$

$$+ [X_2 - X_2\theta_1Y_3 + X_3\theta_1Y_2 + X_3\theta_2 + X_3\theta_3]\ m$$

$$+ [X_2-X_2\theta_1Y_3+X_3\theta_1Y_2+X_3\theta_2+X_3\theta_6]\ x_t$$

$$+ [(X_4+X_5)(1-\theta_1Y_3)+X_3\theta_1Y_4+X_3\theta_1Y_5+X_3\theta_7]\ (p^*_{t-1}+s_{t-1}+\xi_t+n_t-t_{t-1})$$

$$+ [(X_4+2X_5)(1-\theta_1Y_3)+X_3\theta_1Y_4+2X_3\theta_1Y_5+X_3(\theta_7+\theta_8)]\ j^*$$

$$+ [X_4-X_4\theta_1Y_3+X_3\theta_1Y_4+X_3(\theta_7+\theta_{11})]\ t_t$$

$$+ [X_6-X_6\theta_1Y_3+X_3\theta_1Y_6]\ u_t$$

$$+ [X_7-X_7\theta_1Y_3+X_3\theta_1Y_7]\ \varepsilon_t\}$$

Equating coefficients among equations (2.3) and (2.5), the solution for the θs are obtained. In particular:

$$(2.6) \qquad \theta_1 = \frac{-(X_3Y_1-X_1Y_3-1) \pm \sqrt{(X_3Y_1-X_1Y_3-1)^2 - 4Y_3X_1}}{2Y_3}$$

There are two possible solutions 1/ for θ_1. To choose between them, we will follow McCallum (1983) by imposing the requirement that the solution for θ_1 must be valid for all admissible values of the structural parameters. In particular, f_{t-1} appears in the solution for p_t because it forms part of the system (equation (2.1)). In the special case in which $X_1 = 0$, f_{t-1} would not be an argument for p_t and hence, would not be included in the "minimal set of state variables." Thus, θ_1 would be equal to zero. But from equation (2.6) it is clear that $\theta_1 = 0$ would be obtained (under the assumption $X_1 = 0$) only if the negative root is used.

The parameters of the model implies that θ_1 has a positive value 2/ and hence allows us to sign the rest of θs since they are functions of θ_1. Thus:

1/ The roots from equation (2.6) are real since the term inside the square root is always positive. This is so because $Y_3 < 0$ and $X_1 > 0$. The fact that the roots are real is an indication that the model posses an economically sensible solution.

2/ $(X_3Y_1-X_1Y_3-1)$ will be negative under the simplifying assumptions: $d_1 = d_2$ and $d_3 = 1$. Since a requirement from the microfoundations of the model is: $\delta_2 = b_2$; then:

$$(X_3Y_1-X_1Y_3-1) = \frac{\tau(\delta_1+a_1)}{1-\tau+\delta_1+\delta_2} - 1$$ which is negative and less than one in absolute terms.

$$(2.7)\quad \theta_2 = \frac{\theta_1(X_3Y_2-X_2Y_3) + X_2}{1-\theta_1Y_3-X_3} > 0 \text{ and less than one. } \underline{1}/$$

$$(2.8)\quad \theta_3 = \frac{\theta_2(1-\theta_1Y_3)}{1-\theta_1Y_3-X_3} > 0 \text{ and } \theta_3 > \theta_2$$

$$(2.9)\quad \theta_4 = -\theta_6 = \theta_2$$

$$(2.10)\quad \theta_5 = \frac{\theta_2(1-\theta_1Y_3-X_3)}{1-\theta_1Y_3} > 0 \text{ and } \theta_5 < \theta_2$$

$$(2.11)\quad \theta_7 = \frac{(1-\theta_1Y_3)(X_4+X_5) + X_3\theta_1(Y_4+Y_5)}{1-\theta_1Y_3-X_3} > 0 \text{ since } |Y_4|>|Y_5|; \text{ and } \theta_7 < 1$$

$$(2.12)\quad \theta_8 = \theta_7 + \frac{X_5}{1-\theta_1\ Y_3-X_3} > 0; \text{ and } \theta_8 > \theta_7$$

2/ (Cont'd from p. 38) Next, consider the term under the square root:

$$(X_3Y_1-X_1Y_3-1)^2-4Y_3X_1 = 1 + \frac{\tau^2(\delta_1+a_1)^2}{(1-\tau+\delta_1+\delta_2)^2} - \frac{2\tau(\delta_1+\alpha_1)}{1-\tau+\delta_1+\delta_2} + \frac{4(1-\omega_1)(\delta_2-a_1)(d_1\tau)(a_1+\delta_1)(1+\delta_1+\delta_2)}{(1-\tau+\delta_1+\delta_2)^2}$$

which is positive and greater than one. Since the relevant root in equation (2.6) is the negative root and since $Y_3 < 0$, θ_1 will be positive.

1/ Under the restrictions imposed by the microfoundations, $X_3Y_2=X_2Y_3$. Thus, $\theta_2 = \frac{X_2}{1-\theta_1Y_3-X_3}$, which is positive but less than one.

(2.13) $\theta_9 = -\theta_{11} = \theta_7$

(2.14) $$\theta_{10} = \frac{X_4\ (1-\theta_1 Y_3) + X_3\theta_1 Y_4}{1-\theta_1 Y_3} > 0 \text{ since } Y_4 > 0; \text{ and } \theta_{10} < 1$$

(2.15) $$\theta_{12} = \frac{X_6\ (1-\theta_1 Y_3) + X_3\theta_1 Y_6}{1-\theta_1 Y_3} < 0 \text{ since } X_6 < 0$$

(2.16) $$\theta_{13} = \frac{X_7(1-\theta_1 Y_3) + X_3\theta_1 Y_7}{1-\theta_1 Y_3} > 0 \text{ since } X_7 > 0$$

(2.17) $$\theta_0 = \frac{X_0\ (1-\theta_1 Y_3) + X_3\theta_1 Y_0}{1-\theta_1 Y_3 - X_3} > 0$$

The final solution for the levels of output of both commodities are found by substituting equations (2.3) and (2.4) into equations (41) and (42) of the main text. Those reduced form solutions are presented in Table 1.

Finally, the solution for the level of foreign reserves can be *easily found by substituting equation* (2.4) *into equation* (2.2).

(2.18) $$\begin{aligned} f_t = \frac{1}{1-\theta_1 Y_3} \{ & [Y_0+Y_3\theta_0] + [Y_1]\ f_{t-1} \\ & + [Y_2+Y_3\theta_2]\ (gc_{t-1} + v_t - x_{t-1}) \\ & + [Y_2+Y_3(\theta_2 + \theta_3)]\ m \\ & + [Y_2]\ x_t \\ & + [Y_4+Y_5+Y_3\theta_7]\ (p^*_{t-1}+n_t+s_{t-1} + \xi_t - t_{t-1}) \\ & + [Y_4 + 2Y_5 + Y_3\ (\theta_7+\theta_8)]\ j^* \\ & + [Y_4]\ t_t \\ & + [Y_6]\ u_t + [Y_7]\varepsilon_t \}. \end{aligned}$$

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