Can a Poverty-Reducing and Progressive Tax and Transfer System Hurt the Poor?

Sean Higgins and Nora Lustig

Abstract

Whether the poor are helped or hurt by taxes and transfers is generally determined by comparing income distributions before and after fiscal policy using stochastic dominance tests and measures of progressivity and horizontal inequity. We formally show that these tools can fail to capture an important aspect: that a substantial proportion of the poor are made poorer (or non-poor made poor) by the tax and transfer system. We call this fiscal impoverishment, and axiomatically derive a measure of its extent. An analogous measure of fiscal gains of the poor is also derived, and we show that changes in the poverty gap can be decomposed into our axiomatic measures of fiscal impoverishment and gains. We also establish dominance criteria for unambiguous comparisons of fiscal impoverishment and gains under the current system to that under a proposed reform, for a range of possible poverty lines. We illustrate using Brazilian data.

JEL Codes: H2, I32

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Working Paper 405
May 2015

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Sean Higgins
Tulane University

Nora Lustig
Tulane University

For detailed comments on earlier versions of the paper, we are grateful to Alan Barreca, Jean-Yves Duclos, John Edwards, Peter Lambert, Darryl McLeod, Mauricio Reis, Kathleen Short, Rafael Salas, Jay Shimshack, and Harry Tsang. We also thank Adam Ratzlaff and Mel Reitcheck for research assistance. Work on this project was partially completed when S. Higgins was visiting Haas School of Business at UC Berkeley and the Center for Economic Studies at El Colegio de México with funding from the Fulbright–García Robles Public Policy Initiative; the hospitality of these institutions is gratefully acknowledged.

Sean Higgins and Nora Lustig. 2015. "Can a Poverty-Reducing and Progressive Tax and Transfer System Hurt the Poor?." CGD Working Paper 405. Washington, DC: Center for Global Development.
http://www.cgdev.org/publication/can-poverty-reducing-and-progressive-tax-and-transfer-system-hurt-poor%03-working-paper

Center for Global Development
2055 L Street NW
Washington, DC 20036

202.416.4000
(f) 202.416.4050

www.cgdev.org

1 Introduction

Many argue that high tax burdens on the poor are acceptable if they are accompanied by sufficiently large targeted transfers. Efficient taxes that fall disproportionately on the poor, such as a no-exemption value added tax, are often justified with the argument that "spending instruments are available that are better targeted to the pursuit of equity concerns" (Keen and Lockwood, 2010, p. 141). Similarly, Engel et al. (1999, p. 186) assert that "it is quite obvious that the disadvantages of a proportional tax are moderated by adequate targeting" of transfers, since "what the poor individual pays in taxes is returned to her." These taxes "might conceivably be the best way to finance pro-poor expenditures, with the net effect being to relieve poverty" (Ebrill et al., 2001, p. 105).

How can we be sure that what the poor individual pays in taxes is returned to her? Even if the net effect of taxes and transfers is to relieve poverty, are some poor made worse off? A fiscal system can be unambiguously poverty reducing, yet still make a substantial proportion of the poor worse off. This phenomenon does not only occur with regressive taxes: taxes and transfers can be globally progressive, unambiguously equalizing, and unambiguously poverty reducing and *still* make many poor worse off. In other words, conventional tools used to measure how the poor are affected by the tax and transfer system are inadequate to measure whether some of the poor pay more in taxes than they receive in transfers, a phenomenon we call fiscal impoverishment (FI). In light of the debate surrounding taxing the poor, it is necessary to fill this gap in the measurement arsenal. We axiomatically derive a measure of FI, which is uniquely determined up to a proportional transformation, as well as an analogous measure for fiscal gains of the poor (FGP), which captures the extent to which some poor receive more in transfers than they pay in taxes.[1] The commonly-used poverty gap, which is insensitive to the specific amounts of FI and FGP, can be decomposed into FI and FGP components using our axiomatic measures.

The conventional tools that can overlook FI include stochastic dominance tests, measures of horizontal inequity among the poor, and measures of or tests for the progressivity of taxes and transfers. Stochastic dominance tests are used to determine whether poverty is unambiguously

[1] Our axioms are adapted from the axiomatic poverty and mobility measurement literatures (see Foster, 2006, and Zheng, 1997, for surveys of axiomatic poverty measurement and Fields, 2001, for a survey of axiomatic mobility measurement). Our resulting measure can be viewed as a censored directional version of the mobility measure derived by Fields and Ok (1996).

lower in one income distribution than another for any poverty line and a broad class of poverty measures (Atkinson, 1987; Foster and Shorrocks, 1988); they are used to compare poverty over time (e.g., Ravallion and Huppi, 1991), before and after taxes and transfers (Bishop et al., 1996; Garner and Short, 2002), and under reforms to taxes (Duclos et al., 2008) and social programs (Duclos et al., 2005). Poverty changes can be decomposed into various components including horizontal inequity among the poor (Bibi and Duclos, 2007), a component that—unlike poverty comparisons or stochastic dominance tests—takes into account individuals' positions in the income distribution before taxes and transfers ("pre-fisc"). Tests for progressivity of the tax and transfer system also take pre-fisc positions into account, assessing whether taxes paid minus transfers received, measured as a proportion of pre-fisc income, increase with income (Lambert, 1988; Duclos, 1997).

We make two main contributions to the existing literature. First, we show that stochastic dominance tests, measures of horizontal inequity among the poor, and tests for the progressivity of taxes and transfers do not necessarily capture FI, an important shortcoming given the debate surrounding taxing the poor. For example, in the empirically common scenario where the distribution after taxes and transfers ("post-fisc") dominates the distribution pre-fisc and the system of taxes and transfers is not rank preserving on the domain of poverty lines, we need additional tools to determine whether FI has occurred. In the case of stochastic dominance tests, this occurs because these tests are anonymous with respect to the initial income distribution, so FI and FGP can "cancel out." But even tools that take positions in the pre-fisc distribution into account, such as tests for progressivity and measures of horizontal inequity among the poor, can fail to alert us whether FI has occurred. Combining existing tools, it is sometimes possible to detect FI; however, if the tax and transfer system is not rank preserving among the poor, we can observe unambiguous poverty reduction using stochastic dominance tests and globally progressive, unambiguously equalizing taxes and transfers, while a substantial number of poor are made worse off. Indeed, we observe this empirically in Brazil.

Second, we axiomatically derive a measure that does capture FI, as well as an analogous measure of FGP to view the entire picture of gains and losses among the poor. We propose dominance criteria that can be used to determine whether one fiscal system (such as the one that would occur after a proposed reform) causes unambiguously less FI or more FGP than another (such as the current system). The dominance criteria are closely related to Foster and Rothbaum's (2014) second order downward and upward mobility dominance, and extend the literature on poverty and mobility orderings (e.g., Batana and Duclos, 2010; Fields et al., 2002; Zheng, 1999) to the cases of FI and FGP. We also show how a commonly used scalar measure of poverty that overlooks the extent of FI, the poverty gap, can be decomposed into FI and FGP components using our axiomatic measures.

We illustrate with data from Brazil, which is a pertinent example due to its high burden of taxes on the poor (Baer and Galvão, 2008; Goñi et al., 2011) and its lauded poverty-reducing cash transfer programs: a large-scale anti-poverty program that reaches over one-fourth of all Brazilian households and a non-contributory pension program for the elderly poor that reaches one-third of all elderly (Levy and Schady, 2013, Table 1). Are the Brazilian poor compensated for their high tax burdens by these and other transfers and subsidies? We show that, although the net effect of the tax and transfer system in Brazil is to unambiguously reduce poverty (measured with any poverty line up to \$2.50 per person per day)[2] and taxes and transfers are globally progressive, over 40% of the poor experience FI, paying a total of over \$900 million more in taxes than they receive in transfers.

Section 2 formally defines fiscal impoverishment, proves that FI is not necessarily captured by common tools used to assess the tax and transfer system, and derives conditions under which these tools *can* be used to determine if FI has occurred. Section 3 axiomatically derives a measure that does capture FI; it then proposes a partial FI ordering that can be used to compare the level of FI induced by two fiscal systems for any poverty line. Section 4 derives an analogous measure and partial ordering for FGP and shows that the poverty gap can be decomposed into our axiomatically derived measures of FI and FGP. Section 5 uses data from Brazil to illustrate the shortcomings of conventional tools and the usefulness of FI and FGP measures in practice. Section 6 concludes, and the proofs are collected in the Appendix.

[2]The \$2.50 per person per day poverty line, measured in US dollars adjusted for purchasing power parity, is approximately equal to the median of the poverty lines calculated for the world's low and middle income countries, excluding the fifteen poorest countries for which data are available (Chen and Ravallion, 2010). It is frequently the poverty line used to study the poor across Latin America (e.g., Levy and Schady, 2013) and in other developing countries (e.g., Banerjee and Duflo, 2007).

2 Fiscal Impoverishment and Conventional Measures

Denote income before taxes and transfers by $y_i^0 \in \mathbb{R}_+$ and income after taxes and transfers by $y_i^1 \in \mathbb{R}_+$ for each $i \in S$, where S, with cardinality $|S|$, is the set of individuals in society. Also lying in the income space $\mathbb{R}_+$ is a poverty line z.[3] Each individual's income before or after taxes and transfers is arranged in the vector y^0 or y^1, both ordered in ascending order of pre-fisc income y_i^0. If individual i occupies position n of y^0, that same individual will also occupy position n of y^1; even if reranking occurs, the order of the y^1 vector reflects the pre-fisc income ranking. In this sense, a comparison between particular elements of the vector y^1 with the corresponding elements of y^0 is, in the taxonomy of Bourguignon (2011b), "non-anonymous." The cumulative distribution functions (CDFs) of pre- and post-fisc income are non-decreasing functions $F_0 : \mathbb{R}_+ \to [0,1]$ and $F_1 : \mathbb{R}_+ \to [0,1]$; since F_1 ranks individuals by their *post*-fisc income, a comparison of CDFs is "anonymous."

Definition 1. There is *fiscal impoverishment* if $y_i^1 < y_i^0$ and $y_i^1 < z$ for some individual $i \in S$. In other words, the individual could be poor before taxes and transfers and made poorer by the fiscal system, or non-poor before taxes and transfers but poor after.

Can the common tools that assess whether the fiscal system hurts the poor—specifically, stochastic dominance tests to determine whether post-fisc poverty is unambiguously lower than pre-fisc poverty, tests of the net fiscal system's progressivity, and measures of horizontal inequity among the poor—be used to determine whether FI has occurred? We define these common measures below, then show that under certain conditions they can, but that in many cases of practical interest they cannot. This occurs because FGP can more than compensate for FI, in which case poverty measures and stochastic dominance tests overlook the fact that some poor are made poorer by the tax and transfer system. Progressivity tests and horizontal inequity measures, in turn, are designed to incorporate information about an individual's pre-fisc position, but are not concerned with whether her net tax burden (taxes paid minus transfers received) is positive or negative.

Our propositions hold regardless of whether income "before" taxes and transfers is calculated

[3] In Section 3.4, we generalize to a range of poverty lines since there will likely be disagreement or uncertainty about the poverty line in practice. Adjusting income using equivalence scales (to adjust for economies of scale within households or different caloric needs based on age, for example) is easily incorporated into our framework with a single poverty line by assuming there exists a function φ that maps into "equivalized income" adjusted for these considerations.

by simple arithmetic—as is often done in empirical studies—or behavioral and general equilibrium effects are taken into account to attempt to estimate pre-fisc income as the distribution that would exist in the absence of taxes and transfers. It is also sufficiently flexible to include adjustments to pre-fisc income that take into account various concerns, such as the non-uniform distribution of transfer benefits within a household (e.g., Jacoby, 2002), the differential impacts of cash transfers and equally costly in kind transfers (e.g., Hidrobo et al., 2014), and the utility cost of redistribution (for example, an individual who is taxed \$20 and receives \$20 in transfers may have lower utility than in the absence of redistribution due to the time cost of filing taxes and applying for the transfer program).

2.1 Stochastic Dominance

Definition 2. The post-fisc income distribution F_1 (weakly) *first order stochastically dominates* the pre-fisc income distribution F_0 on the domain of poverty lines if $F_1(y) \leq F_0(y)\ \forall\, y \in [0, z)$.[4]

If the post-fisc income distribution does *not* first order dominate the pre-fisc distribution on the domain of poverty lines, fiscal impoverishment has occurred. Even in this case, it is impossible to tell the extent of FI without explicit measures like the ones we propose in Section 3. A stark example of this comes from Ethiopia: in a study applying our measures and using the "dollar a day" (\$1.25 per household member per day) poverty line, World Bank (2015) finds that the post-fisc distribution does not dominate the pre-fisc distribution; the two CDFs cross just below \$1 per day. As a result, we know from Proposition 1 that FI has occurred. Looking at CDFs and poverty numbers alone, however, greatly masks the extent of FI: the headcount ratio increases from 31.9% to 33.2% of the population, while the poverty gap and squared poverty gap fall as a result of taxes and transfers. Nevertheless, 27.6% of Ethiopians and over 80% of the post-fisc poor experience FI using a poverty line of \$1.25 per day.

Proposition 1. *If F_1 does not first order dominate F_0 on $[0, z)$, then FI has occurred.*

If, on the other hand, the post-fisc distribution *does* dominate, FI may or may not have occurred. In this case, stochastic dominance tests can only be used to identify FI if the tax and transfer

[4] The interval is open at $y = z$ since we define poverty as $y_i < z$; our results are unaltered if instead we define poverty as $y_i \leq z$, FI as $y_i^1 < y_i^0$ and $y_i^1 \leq z$, and stochastic dominance on the closed interval $[0, z]$.

system is rank preserving among the poor (i.e., if $y_i^0 \geq y_j^0 \Leftrightarrow y_i^1 \geq y_j^1$ for all (i, j) such that, for each $k \in \{i, j\}$, $y_k^t < z$ for some $t \in \{0, 1\}$).[5] When the tax and transfer system is rank preserving among the poor, dominance of the post-fisc over the pre-fisc distribution on the domain of poverty lines is a necessary and sufficient condition for the absence of FI. In practice, however, "rank reversals are typically induced in the transition from the pre-tax to the post-tax income parade" (Lambert, 1994, p. 17). In this case, first order stochastic dominance of the post-fisc over the pre-fisc distribution on the domain of poverty lines is not a sufficient condition to conclude that FI has not occurred; the existence and extent of FI can be assessed using the measures derived in Section 3.

Proposition 2. *If the tax and transfer system <u>is</u> rank preserving among the poor, F_1 dominates F_0 on $[0, z)$ if and only if FI has not occurred.*

Proposition 3. *If the tax and transfer system is <u>not</u> rank preserving among the poor, dominance of F_1 over F_0 on $[0, z)$ is not a sufficient condition for the absence of FI.*

Since, in practice, we often observe first order stochastic dominance of F_1 over F_0 on the domain of poverty lines and a tax and transfer system that is not rank preserving among the poor (in Brazil, for example), Proposition 3 has important implications for distinguishing between FI and changes in measured poverty. First order stochastic dominance on the domain of poverty lines implies an unambiguous reduction in poverty according to any poverty measure in a broad class of additively separable measures (Atkinson, 1987; Foster and Shorrocks, 1988); thus, a corollary of Proposition 3 is that poverty comparisons will not necessarily capture FI. In other words, we could observe an unambiguous reduction in poverty according to a variety of poverty measures and poverty lines when comparing incomes before to after taxes and transfers, while at the same time a significant portion of the poor are made worse off by the tax and transfer system.

Stochastic dominance tests and poverty comparisons can fail to capture FI because they are anonymous with respect to initial income—i.e., measures of post-fisc poverty do not take into account an individual's pre-fisc position. As long as the FI of each impoverished individual i is compensated by the gain of a pre-fisc poorer individual j such that $y_j^0 \leq y_i^1 < y_i^0 \leq y_j^1$ if $y_i^0 < z$,

[5] Our definition for *among the poor* is general enough to include the following cases: (i) both individuals are pre-fisc poor and remain poor; (ii) both are pre-fisc poor and one escapes poverty; (iii) both are pre-fisc poor and escape poverty; (iv) one is pre-fisc poor and escapes poverty while the other is pre-fisc non-poor and falls into poverty; (v) both are pre-fisc non-poor and fall into poverty.

or $y_j^0 \leq y_i^1 < z \leq y_j^1$ if $y_i^0 \geq z$, poverty will be lower or unchanged after taxes and transfers. Nevertheless, even the tools used to assess whether fiscal policy hurts the poor that *do* take into account individuals' pre-fisc positions—such as measures of horizontal inequity and tests of the net fiscal system's progressivity—can fail to capture FI.

2.2 Horizontal Inequity

Horizontal inequity can be defined in two ways: the reranking definition and the classical definition. Under either definition, the existence or absence of horizontal inequity among the poor does not tell us whether FI has occurred.

Definition 3. There is *reranking* if $y_i^0 > y_j^0$ and $y_i^1 < y_j^1$ for some (i, j) pair who, from an ethical viewpoint, should not be treated so unequally by the fiscal system that they change ranks. There is reranking *among the poor* if $y_i^0 > y_j^0$ and $y_i^1 < y_j^1$ for some such (i, j) pair where $y_k^t < z$ for some $t \in \{0, 1\}$ for each $k \in \{i, j\}$.

Definition 4. There is *classical horizontal inequity* if equals are treated unequally by the tax and transfer system, i.e., if $y_i^0 = y_j^0$ and $y_i^1 \neq y_j^1$ for some pair (i, j) of ethically equal individuals.[6] There is classical horizontal inequity *among the poor* if $y_i^0 = y_j^0$ and $y_i^1 \neq y_j^1$ for some such (i, j) pair where $y_k^t < z$ for some $t \in \{0, 1\}$ for each $k \in \{i, j\}$.

Proposition 4. *Horizontal inequity among the poor (using either the reranking or classical definition) is neither a necessary nor sufficient condition for FI.*

Even if some pre-fisc poor are impoverished by the tax and transfer system, the ranking among the poor may not change (so there is no horizontal inequity due to reranking among the poor) and pre-fisc equals may be impoverished to the same degree (so there is no classical horizontal inequity among the poor). For example, $z = 10, y^0 = (5, 5, 8, 8, 20), y^1 = (6, 6, 7, 7, 20)$. Nor does the presence of horizontal inequity among the poor necessarily imply FI, because there could be reranking among the poor or unequal treatment among pre-fisc equals when the tax and transfer system lifts incomes of some of the poor without decreasing incomes of any poor. For example,

[6] The requirement that exact equals be treated equally can be replaced with a requirement that approximate equals be treated equally, where approximate equality is determined using bins (Lambert and Ramos, 1997) or a weighting function of the incomes neighboring y_i^0 (Duclos and Lambert, 2000; Auerbach and Hassett, 2002).

$z = 10, y^0 = (5, 5, 6, 20), y^1 = (5, 7, 6, 18)$. In sum, measures of horizontal inequity among the poor do not necessarily capture whether FI has occurred despite that, unlike stochastic dominance tests and poverty comparisons, they take individuals' pre-fisc positions into account.

2.3 Progressivity

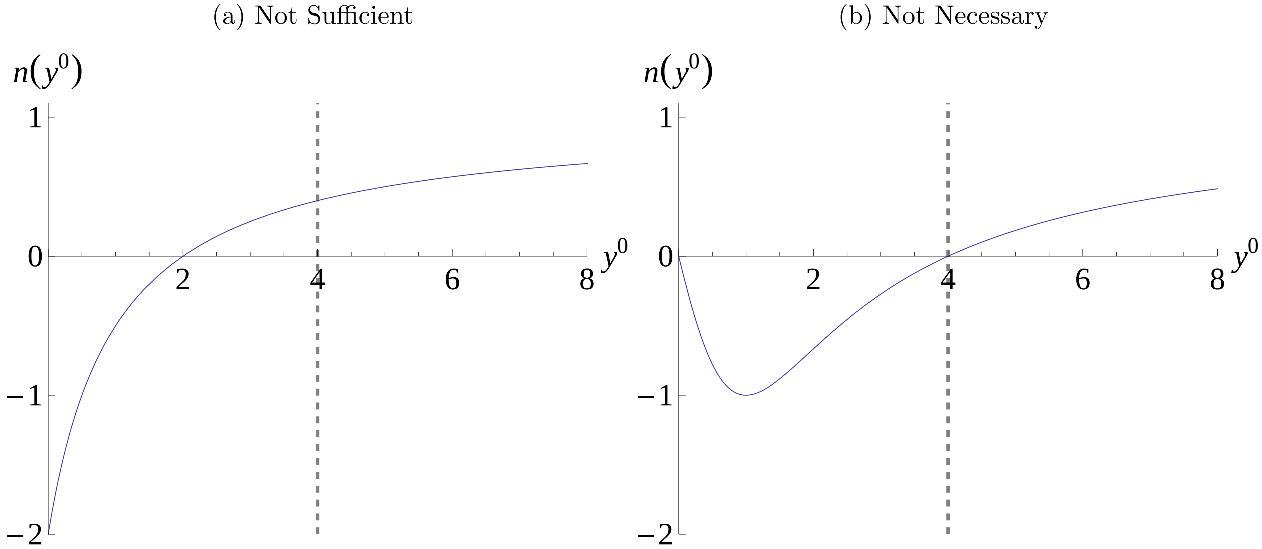
A tax and transfer system is progressive when net taxes (i.e., taxes minus benefits), relative to pre-fisc income, increase with income (Duclos, 1997; Lambert, 1988). Like horizontal inequity among the poor, knowledge about whether the tax and transfer system is progressive does not tell us whether FI has occurred. Let net taxes be determined by a non-stochastic and continuously differentiable function of pre-fisc income denoted $N(y^0)$, where $y^1 = y^0 - N(y^0)$. Note that the function $N(y^0)$ will be negative for individuals who receive more in transfers than they pay in taxes. Denote net taxes relative to pre-fisc income as $n(y^0) \equiv N(y^0)/y^0$ and denote its derivative with respect to y^0 at a given pre-fisc income point $\hat{y}$ as $n'(\hat{y})$. In the definitions below, we follow the taxonomy of Duclos (2008).

Definition 5. The tax and transfer system is *locally progressive* at $\hat{y}$ if $n'(\hat{y}) > 0$. The tax and transfer system is *everywhere progressive* if $n'(y^0) > 0 \; \forall \, y^0 \in \mathbb{R}_+$.

Proposition 5. *An everywhere progressive tax and transfer system is neither a necessary nor sufficient condition for no FI.*

Everywhere progressivity does not imply that FI has not occurred because net proportional taxes $n(y^0)$ may be increasing in pre-fisc income but positive for some poor, as in Figure 1a for poor households with y^0 between \$2 and $z =$ \$4. Conversely, the absence of FI does not imply an everywhere progressive tax and transfer system: even if no poor become poorer, i.e. $n(y^0) \leq 0$ for all $y^0 \in [0, z)$, and no non-poor become poor, i.e. $n(y^0) \leq (y^0 - z)/y^0$ for all $y^0 \geq z$, it could be the case that the government has trouble targeting its benefit programs to the poorest of the poor, who might be marginalized, so $n'(y^0) < 0$ on some interval $(a, b) \subset [0, z]$, as in Figure 1b for $y^0 \in [0, 1)$. Because a tax and transfer system that is everywhere progressive decreases inequality using any inequality measure consistent with the Lorenz criterion (Fellman, 1976; Jakobsson, 1976), an important corollary of Proposition 5 is that a tax and transfer system can be unambiguously equalizing while still causing some of the poor to experience FI.

Figure 1: Graphical Illustration of Proposition 5



Note: Dashed vertical line indicates poverty line.

In practice, tax and transfer systems are not everywhere progressive due to tax schedules that depend on non-income attributes (Lambert, 1993), the combination of various independent taxes and transfers (Ebert and Lambert, 1999), non-compliance and tax evasion (Bishop et al., 2000), and the imperfect targeting and take-up of transfers (Bibi and Duclos, 2007; Duclos, 1995). Thus, practitioners resort to tests that are less stringent than those for everywhere progressivity, such as tests for global progressivity, which compare the Lorenz curve of pre-fisc income with the concentration curve of post-fisc income (to analyze taxes and transfers together), or with the concentration curves of taxes and transfers separately, using the pre-fisc ranking.[7] Because everywhere progressivity is a sufficient condition for global progressivity, another corollary of Proposition 5 is that the tax and transfer system can be globally progressive while still causing substantial FI.

Combining tools, any tax and transfer system that meets the stringent requirement of everywhere progressivity is rank preserving, so if F_1 first order stochastically dominates F_0 on $[0, z)$ and the fiscal system is everywhere progressive, FI has not occurred (Proposition 2). It is straightforward to show, however, that using the less stringent requirement of global progressivity—which allows ranks to change—a tax and transfer system can be globally progressive and unambiguously poverty reducing with F_1 dominating F_0 on $[0, z)$, yet still impoverish a substantial number of poor. In addition to our illustration in Brazil, this phenomenon has been found to occur empirically in

[7] For the formulation of the Lorenz and concentration curves, see Kakwani (1977).

other developing countries by studies applying the measures we derive here, including Armenia using the $1.25 per day poverty line (Younger and Khachatryan, 2014) and El Salvador using the $2.50 poverty line (Beneke et al., 2015).

3 Measures of Fiscal Impoverishment

When FI occurs, it is important to have measures of its extent to inform tax debates. For example, in countries that heavily tax the poor, it is important to know the extent to which some of the poor are not compensated for these taxes by transfers. In this section, we begin by presenting some possible FI measures that have the advantage of simplicity (and could thus be useful in policy debate) but also suffer from limitations, which we discuss. We then axiomatically derive a measure of FI, adapting axioms from the literature on poverty and mobility measurement. Our measure can be viewed as a censored directional version of Fields and Ok's (1996) mobility measure and the partial ordering suggested by the measure is equivalent to the ordering given by Foster and Rothbaum's (2014) second order downward mobility dominance.

3.1 Simple Measures of Fiscal Impoverishment

Two straight-forward measures of FI are the proportion of the total population experiencing FI and the proportion of the post-fisc poor experiencing FI. Using the notation from Section 2, we have $h(y^0, y^1; z) = |A|^{-1} \sum_{i \in S} \mathbb{I}(y_i^1 < y_i^0)\mathbb{I}(y_i^1 < z)$, where $A = S$ gives the proportion of people that are impoverished, while $A = \{i \in S \,|\, y_i^1 < z\}$ gives the proportion of the post-fisc poor that are impoverished. The indicator function $\mathbb{I}$ takes a value of one if its argument is true and zero otherwise.

Given its simplicity and straight-forward interpretation, h can be useful for policy discussions. Nevertheless, like the poverty headcount ratio, h is a crude measures that fails to satisfy some basic desirable properties. In the context of poverty measurement, Sen (1976, p. 219) proposes a monotonicity axiom requiring that, all else equal, "a reduction in income of a person below the poverty line must increase the poverty measure." We propose a similar axiom for FI measures requiring that a larger decrease in post-fisc income for an impoverished person, all else equal, must increase the FI measure. Monotonicity is violated by h: the measure does not increase when an impoverished person becomes more impoverished.[8]

3.2 Axioms

As before, consider pre- and post-fisc incomes $y_i^0, y_i^1 \in \mathbb{R}_+$ for each i in the set of individuals S with cardinality $|S|$; denote the vectors of pre- and post-fisc income for these individuals by y^0 and y^1, both ordered by pre-fisc income y_i^0. Now consider income vectors for the same individuals under different pre- and post-fisc scenarios, denoted by x^0 and x^1, both ordered by pre-fisc income x_i^0. We assume that income is measured in real terms and has been converted to a common currency such as US dollars adjusted for purchasing power parity, thereby simplifying away concerns about inflation or currency conversions if comparing FI over time or across countries. The sets of impoverished individuals in scenarios (y^0, y^1) and (x^0, x^1) are denoted $I_y \equiv \{i \in S \,|\, y_i^1 < y_i^0 \text{ and } y_i^1 < z\}$ and $I_x \equiv \{i \in S \,|\, x_i^1 < x_i^0 \text{ and } x_i^1 < z\}$. A measure of FI is a function $f : \bigcup_{n=1}^{\infty} \mathbb{R}_+^n \times \bigcup_{n=1}^{\infty} \mathbb{R}_+^n \times \mathbb{R}_+ \to \mathbb{R}$, which takes as arguments the pre- and post-fisc income vectors and the poverty line.

Axiom 1 (FI Monotonicity)**.** If $y_i^0 = x_i^0$ for all $i \in S$ and there exists $j \in I_y \cup I_x$ such that $y_j^1 > x_j^1$, while $y_k^1 = x_k^1$ for all $k \in I_y \cup I_x \setminus \{j\}$, then $f(y^0, y^1; z) < f(x^0, x^1; z)$.

FI monotonicity implies that the measure is strictly increasing in the extent to which an impoverished individual is impoverished (*ceteris paribus*), and is analogous to Sen's (1976) monotonicity axiom for poverty measurement. It also implies that the FI measure must be strictly increasing in the number of individuals that are impoverished, holding fixed the amount of FI experienced by others.

Axiom 2 (Focus)**.** If $y_i^0 = x_i^0$ and $y_i^1 = x_i^1$ for all $i \in I_y \cup I_x$, then $f(y^0, y^1; z) = f(x^0, x^1; z)$.

The focus axiom, which is analogous to Sen's (1981) focus axiom for poverty measurement, says that different income changes to the non-impoverished—provided that they remain non-impoverished—leave the FI measure unchanged.

Axiom 3 (Normalization)**.** $I_y = \varnothing \Rightarrow f(y^0, y^1; z) = 0$.

[8] Another simple measure of FI that fails to satisfy monotonicity is the $q \times q$ transition matrix P, whose typical element p_{kl} represents the probability of being in post-fisc income group $l \in \{1, \dots, q\}$ for an individual in pre-fisc income group $k \in \{1, \dots, q\}$, with the cutoffs for at least two income groups below the maximum poverty line. Perhaps the largest drawback of the transition matrix is that it does not capture FI among the poorest pre-fisc group ($k = 1$).

Given the focus axiom, it is natural to normalize our measure so that if the set of impoverished individuals is empty, the FI measure equals zero. Nevertheless, this axiom is not instrumental to our result: if we did not use the axiom, our result would be that our axioms uniquely determine a measure of FI up to a linear (rather than proportional) transformation.

Axiom 4 (Continuity)**.** f is jointly continuous in y_i^0, y_i^1, and z.

The continuity axiom resembles Chakravarty's (1983) continuity axiom for poverty measures; here we require the FI measure to be continuous in pre-fisc income, post-fisc income, and the poverty line (since we may want to assess FI for a range of possible poverty lines). This is stronger than Foster and Shorrocks's (1991) restricted continuity axiom which only requires the measure to be left-continuous in incomes at z; for a recent argument in favor of using the stronger continuity axiom in the context of multidimensional poverty measures, see Permanyer (2014).

Axiom 5 (Permutability)**.** $f(y^0, y^1; z) = f(y_\sigma^0, y_\sigma^1; z)$ for any permutation function $\sigma : S \to S$, where $y_\sigma^0 \equiv (y_{\sigma(1)}^0, \ldots, y_{\sigma(|S|)}^0)$ and $y_\sigma^1 \equiv (y_{\sigma(1)}^1, \ldots, y_{\sigma(|S|)}^1)$.

We use the term "permutability" rather than symmetry or anonymity because symmetry and anonymity have taken on different definitions in different contexts. Both have been used in the same way we use permutability above (e.g., Cowell, 1985; Fields and Fei, 1978; Plotnick, 1982). Symmetry can instead mean, for two income distributions X and Y and a distance measure d, that $d(X, Y) = d(Y, X)$; the two income distributions are treated symmetrically: losses are not distinguishable from gains (Ebert, 1984; Fields and Ok, 1999). Anonymity can instead mean that the measure compares F_0 to F_1 without regard to where a particular individual at position j in F_0 ended in F_1 (e.g., Bourguignon, 2011a,b). In other words, an anonymous measure would compare the pre-fisc income of the jth poorest individual in F_0 to the post-fisc income of the jth poorest individual in F_1, even though "they are not necessarily the same individuals" because of reranking (Bourguignon, 2011a, p. 607)

Next, we must decide whether our measure of FI should be absolute or relative (recalling that we assume income to be in real terms of a constant currency, so arguments about inflation or currency exchange should not affect the decision). Suppose each poor individual's pre-fisc income increases by \$1, taxes and transfers are held fixed, and the price of one essential good in the basic goods basket, normalized to have one unit in the basket, also increases by \$1 per unit.[9] Each poor individual remains the same distance below the poverty line; that distance represents the amount of additional income she needs to afford adequate nutrition and other basic necessities. For those who experience FI, it is the absolute increase in the distance between that individual's income and the poverty line that matters in terms of the quantity of basic goods she can buy. Hence, we assume that if all pre- and post-fisc incomes increase by \$1 and the poverty line also increases by \$1, FI should remain unchanged. We thus impose the following translation invariance axiom, denoting a vector of ones with length $|S|$ by $1_{|S|}$.

Axiom 6 (Translation Invariance)**.** $f(y^0 + \alpha 1_{|S|}, y^1 + \alpha 1_{|S|}; z + \alpha) = f(y^0, y^1; z)$ for all $\alpha \in \mathbb{R}$.

Given our above argument for absolute measures, we also impose linear homogeneity. Instead, specifying homogeneity of degree zero (scale invariance) would be incompatible with translation invariance for the reasons explored in Zheng (1994). Since we assume income is expressed in real terms and a common currency, our measure is nevertheless insensitive to inflation or currency changes.

Axiom 7 (Linear Homogeneity)**.** $f(\lambda y^0, \lambda y^1; \lambda z) = \lambda f(y^0, y^1; z)$ for all $\lambda \in \mathbb{R}_{++}$.

By requiring translation invariance and linear homogeneity, we are deriving a measure of *absolute* FI; from there, the measure can nevertheless be modified to obtain other types of desired measures such as a scale invariant measure. This is similar to the approach taken by Fields and Ok (1996), who axiomatically derive a measure of absolute mobility from which other desired measures such as mobility proportional to income can be obtained. The translation invariance and linear homogeneity axioms have been used together in axiomatic derivations of measures of inequality (Kolm, 1976), poverty (Blackorby and Donaldson, 1980), economic distance (Chakravarty and Dutta, 1987; Ebert, 1984), and mobility (Fields and Ok, 1996; Mitra and Ok, 1998).

Our final axiom is based on a concept introduced to the poverty literature by Foster et al. (1984, p. 761), who argue that "at the very least, one would expect that a decrease in the poverty level of one subgroup *ceteris paribus* should lead to less poverty for the population as a whole." Similarly, it would be desirable for a measure of FI if a decrease in the measured FI for one subgroup of the

[9] To avoid inflation in this thought experiment, assume that there is an offsetting fall in the price of a good *not* in the basic good basket and not consumed by the poor.

population and no change in the measured FI for all other subgroups results in a decrease in the measured FI of the entire population. Hence, we impose a subgroup consistency axiom analogous to the one used for poverty measurement by Foster and Shorrocks (1991); partition S into m subsets $S_1, \ldots, S_m$, and denote the vectors of pre- and post-fisc incomes for individuals belonging to subset S_a, $a \in \{1, \ldots, m\}$, by y_a^0 and y_a^1 or x_a^0 and x_a^1.

Axiom 8 (Subgroup Consistency)**.** If $f(y_a^0, y_a^1; z) < f(x_a^0, x_a^1; z)$ for some $a \in \{1, \ldots, m\}$ and $f(y_b^0, y_b^1; z) = f(x_b^0, x_b^1; z)$ for all $b \in \{1, \ldots, m\} \setminus \{a\}$, then $f(y^0, y^1; z) < f(x^0, x^1; z)$.

Note that y_a^0, x_a^0, y_a^1, and x_a^1 have the same dimension for a given $a \in \{1, \ldots, m\}$ because they are based on the same partition of S; a similar fixed-size subgroup condition is included in Foster and Shorrocks' (1991) axiom. In his survey of axiomatic poverty measurement, Zheng (1997, p. 137) notes that subgroup consistency "has gained wide recognition in the literature."

3.3 An Axiomatic Measure of Fiscal Impoverishment

Proposition 6. *A measure satisfying FI monotonicity, focus, normalization, continuity, permutability, translation invariance, linear homogeneity, and subgroup consistency is uniquely determined up to a proportional transformation, and given by*

$$f(y^0, y^1; z) = \kappa \sum_{i \in S} \big(\min\{y_i^0, z\} - \min\{y_i^0, y_i^1, z\}\big). \tag{1}$$

The summand for individual i behaves as follows. For an individual who was poor before taxes and transfers and is impoverished ($y_i^1 < y_i^0 < z$), it is equal to her fall in income, $y_i^0 - y_i^1$. For an individual who was non-poor before taxes and transfers and is impoverished ($y_i^1 < z \leq y_i^0$), it equals her post-fisc poverty gap, or the amount that would need to be transferred to her to move her back to the poverty line (equivalently, to prevent her from becoming impoverished), $z - y_i^1$. For a non-impoverished pre-fisc non-poor individual ($y_i^0 \geq z$ and $y_i^1 \geq z$) it equals $z - z = 0$. For a non-impoverished pre-fisc poor individual ($y_i^0 < z$ and $y_i^1 \geq y_i^0$) it equals $y_i^0 - y_i^0 = 0$. Hence, f sums the total amount of FI, multiplied by a factor of proportionality. This constant can be chosen based on the preferences of the practitioner: for example, $\kappa = 1$ gives total FI, while $\kappa = |S|^{-1}$ gives per capita FI.[10]

[10] We do not impose a population invariance axiom; this axiom is commonly imposed but is criticized by Hassoun

3.4 Fiscal Impoverishment Dominance Criteria

Having identified the existence of FI in a country, a useful implementation of our FI measure would be to compare the degree of FI in two situations, e.g. by comparing the current fiscal system to a proposed reform. The choice of poverty line might, however, influence our conclusion about which situation entails higher FI. We thus present a partial FI ordering that can be used to determine if FI is unambiguously lower in one situation than another for any poverty line and any measure that satisfies FI monotonicity, focus, normalization, continuity, permutability, translation invariance, linear homogeneity, and subgroup consistency. Since we have already shown that a FI measure satisfies these axioms if and only if it takes the form in (1), a simple way to test for FI dominance for any measure satisfying those axioms and any poverty line up to some maximum poverty line z^+ is to simply compare the curves $f(y^0, y^1; z)$ and $f(x^0, x^1; z)$ across the domain of poverty lines $[0, z^+]$. Interestingly, there is an alternative (equivalent) way to test whether FI is unambiguously lower in one situation than another that uses a dominance test already developed in the mobility literature: Foster and Rothbaum's (2014) second order downward mobility dominance.

Proposition 7. *The following are equivalent.*

a) *FI is unambiguously lower in* (y^0, y^1) *than* (x^0, x^1) *for any poverty line in* $[0, z^+]$ *and any measure satisfying FI monotonicity, focus, normalization, continuity, permutability, translation invariance, linear homogeneity, and subgroup consistency.*

b) $f(y^0, y^1; z) < f(x^0, x^1; z)\ \forall\, z \in [0, z^+]$.

c) (y^0, y^1) *second order downward mobility dominates* (x^0, x^1) *on* $[0, z^+]$.

4 Fiscal Gains of the Poor

Most likely, we will be interested in more than just the extent to which some poor are not compensated for their tax burden with transfers: we will also want to know about the gains of other poor families, and the way in which a comparison of poverty before and after taxes and transfers[11] can

and Subramanian (2012). A subset of measures of form (1) are population invariant: choosing $\kappa = |S|^{-1}$ gives a measure that satisfies population invariance, while $\kappa = 1$ gives a measure that does not.

[11] Comparisons of poverty before and after taxes and transfers are common (e.g., Hoynes et al., 2006, Table 4).

be decomposed into the losses and gains of different poor households. In this section, we formally define fiscal gains of the poor, briefly present the axioms for a measure of FGP analogous to those in Section 3.3 for a measure of FI, and present an axiomatic measure and partial ordering of FGP. We then show that a commonly used measure of poverty, the poverty gap, can be decomposed into our axiomatic measures of FI and FGP.

4.1 Definitions, Axioms, and Measures of Fiscal Gains of the Poor

Definition 6. There are *fiscal gains of the poor* if $y_i^0 < y_i^1$ and $y_i^0 < z$ for some individual $i \in S$. The individual may or may not receive enough in net transfers to be post-fisc non-poor (i.e., it is possible that $z \leq y_i^1$ or $y_i^1 < z$).

A simple measure of FGP would be the proportion of the population or proportion of the pre-fisc poor that begin poor and gain income due to the tax and transfer system: $\ell(y^0, y^1; z) = |A|^{-1} \sum_{i \in S} \mathbb{I}(y_i^0 < y_i^1)\mathbb{I}(y_i^0 < z)$, with $A = S$ for the proportion of the total population or $A = \{i \in S \,|\, y_i^0 < z\}$ for the proportion of the pre-fisc poor. This simple measure, however, violates the following monotonicity axiom for FGP. Let the sets of pre-fisc poor individuals experiencing fiscal gains under two scenarios be denoted $G_y \equiv \{i \in S \,|\, y_i^0 < y_i^1 \text{ and } y_i^0 < z\}$ and $G_x \equiv \{i \in S \,|\, x_i^0 < x_i^1 \text{ and } x_i^0 < z\}$. A measure of FGP is a function $g : \bigcup_{n=1}^{\infty} \mathbb{R}_+^n \times \bigcup_{n=1}^{\infty} \mathbb{R}_+^n \times \mathbb{R}_+ \to \mathbb{R}$, which takes as arguments the pre- and post-fisc income vectors and the poverty line.

Axiom 1′ (FGP Monotonicity)**.** If $y_i^0 = x_i^0$ for all $i \in S$ and there exists $j \in G_y \cup G_x$ such that $y_j^1 < x_j^1$, while $y_k^1 = x_k^1$ for all $k \in G_y \cup G_x \setminus \{j\}$, then $g(y^0, y^1; z) \leq g(x^0, x^1; z)$, with strict inequality if $y_j^1 < z$.

Consider a pre-fisc poor individual who receives more in transfers than she pays in taxes. If she is given even more transfer income, while the pre- and post-fisc incomes of all others experiencing FGP do not change, FGP must not decrease; if she would have remained in poverty post-fisc without the additional transfer income, FGP must increase. The remaining axioms from Section 3.3 are desirable for a measure of FGP as well, and carry over directly to FGP after replacing f with g, I_y with G_y, and I_x with G_x. These axioms lead to a measure of FGP that is unique up to a proportional transformation.

Proposition 8. *A measure satisfying FGP monotonicity, focus, normalization, continuity, permutability, translation invariance, linear homogeneity, and subgroup consistency is uniquely determined up to a proportional transformation, and given by*

$$g(y^0, y^1; z) = \kappa \sum_{i \in S} \big(\min\{y_i^1, z\} - \min\{y_i^0, y_i^1, z\}\big). \tag{2}$$

An individual who is pre-fisc poor and gains income from the tax and transfer system, but remains post-fisc poor ($y_i^0 < y_i^1 < z$), contributes the amount of her income gain, $y_i^1 - y_i^0$, to the measure of FGP. A pre-fisc poor individual that gains income and as a result has post-fisc income above the poverty line ($y_i^0 < z \leq y_i^1$) contributes the amount of net transfers that pulled her pre-fisc income to the poverty line, $z - y_i^0$. Someone who is pre-fisc poor and does not gain income ($y_i^1 \leq y_i^0 < z$) contributes $y_i^1 - y_i^1 = 0$. Someone who is pre-fisc non-poor ($z < y_i^0$) also contributes 0 (for her, the summand equals $z - z$ if she remains non-poor or $y_i^1 - y_i^1$ if she loses income and becomes poor).

As with fiscal impoverishment orderings, a fiscal gain partial ordering can be used to make unambiguous FGP comparisons for any poverty line and any measure satisfying our axioms. The ordering compares $g(y^0, y^1; z)$ to $g(x^0, x^1; z)$ for all $z \in [0, z^+]$, and coincides with Foster and Rothbaum's (2014) second order upward mobility dominance (the proof proceeds similarly to the proof of Proposition 7 for FI).

4.2 Decomposition of the Difference between Pre-Fisc and Post-Fisc Poverty

The most common measures of poverty used in both policy circles and scholarly papers (e.g., Chen and Ravallion, 2010; Ravallion, 2012) are the poverty headcount ratio, which enumerates the proportion of the population that is poor, and the poverty gap, which takes into account how far the poor fall below the poverty line. The latter might be expressed in absolute terms, summing the gap between each poor person's income and the poverty line, in which case it can be thought of as the total amount that would need to be given to the poor to eliminate poverty (if targeting were perfect). Or it can be normalized, dividing the absolute poverty gap by the poverty line and population size, for example, to create a scale- and population-invariant measure. We use a general definition of the poverty gap that encompasses its absolute and normalized forms.

Definition 7. The *poverty gap* is given by

$$p(y; z) = \nu(S, z) \sum_{i \in S} (z - y_i) \mathbb{I}(y_i < z), \tag{3}$$

where $\nu(S, z)$ is a normalization factor. Two special cases are the *absolute poverty gap*, where $\nu(S, z) = 1$, and the *poverty gap ratio*, where $\nu(S, z) = (z|S|)^{-1}$.

For simplicity and because a comparison of pre- and post-fisc poverty usually occurs for a fixed population and given poverty line, we assume that S and z are fixed in what follows.

Proposition 9. *A change in the poverty gap before and after taxes and transfers is equal to the difference between the axiomatic measures of FI and FGP from* (1) *and* (2)*, multiplied by a constant.*

Given the assumption that the population and poverty line are fixed, $\nu(S, z)$ is a constant that we denote $\bar{\nu}$. The poverty gap in (3) can be rewritten as $p(y; z) = \bar{\nu} \sum_{i \in S} (z - y_i) \mathbb{I}(y_i < z) = \bar{\nu} \sum_{i \in S} (z - \min\{y_i, z\})$, so we have

$$\begin{aligned}
p(y^1; z) - p(y^0; z) &= \bar{\nu} \sum_{i \in S} \left(z - \min\{y_i^1, z\}\right) - \bar{\nu} \sum_{i \in S} \left(z - \min\{y_i^0, z\}\right) \\
&= \bar{\nu} \sum_{i \in S} \left(\min\{y_i^0, z\} - \min\{y_i^1, z\}\right) \\
&= \bar{\nu} \left[\sum_{i \in S} \left(\min\{y_i^0, z\} - \min\{y_i^0, y_i^1, z\}\right) - \sum_{i \in S} \left(\min\{y_i^1, z\} - \min\{y_i^0, y_i^1, z\}\right) \right] \\
&= \frac{\bar{\nu}}{\kappa} \left[f(y^1, y^0; z) - g(y^1, y^0; z) \right].
\end{aligned}$$

Comparisons of pre- and post-fisc poverty are often used to assess whether the tax and transfer system helps or hurts the poor. This decomposition can be used to dig deeper into that net effect and observe the extent to which a net reduction in poverty masks the offsetting gains of some poor and impoverishment of others at the hands of the (possibly progressive) tax and transfer system.

5 Illustration with Brazilian Data

Stochastic dominance tests would lead us to conclude that Brazil's tax and transfer system is unambiguously favorable to the poor; in spite of this, the extent of FI is significant. We use the

Pesquisa de Orçamentos Familiares (Family Expenditure Survey) 2008–2009 and, using the income concept definitions from Higgins et al. (forthcoming), compare market income (before taxes and transfers) to post-fiscal income (after direct and indirect taxes, direct cash and food transfers, and indirect subsidies). The precise taxes, transfers, and subsidies included in our analysis are described in detail in Higgins and Pereira (2014).

5.1 Conventional Tools and Fiscal Impoverishment in Brazil

Figure 2a shows the cumulative distribution functions of pre-fisc and post-fisc income. If we set the poverty line z at, say, \$4 per household member per day—a poverty line frequently used by the World Bank when studying middle-income Latin American countries (e.g., Ferreira et al., 2013), with a value slightly higher than the population-weighted average of regional poverty lines calculated by the Brazilian government's Institute of Applied Economic Research—we know from Proposition 1 that FI has occurred because F_1 does not dominate F_0 on $[0, 4)$.

If, however, we set z at \$2.50 per household member per day—a poverty line equal to the median of those calculated for the world's low and middle income countries, excluding the fifteen poorest countries for which data are available (Chen and Ravallion, 2010), frequently the poverty line used to study the poor across Latin America (e.g., Levy and Schady, 2013), and sometimes the poverty line used for other developing countries (e.g., Banerjee and Duflo, 2007)—we do observe that the post-fisc distribution first order dominates the pre-fisc distribution. We verify that this first order dominance is statistically significant at the 5% level using the asymptotic sampling distribution derived by Davidson and Duclos (2000) with a null hypothesis of non-dominance; the result is also robust to the type of data contamination considered in Cowell and Victoria-Feser (2002).[12]

In addition, the tax and transfer system is globally progressive and unambiguously equalizing.[13] Figure 2b shows the Lorenz curve of pre-fisc income, which plots the proportion of total pre-fisc income accruing to the poorest x% of the population, as well as the concentration curve of post-fisc income, which plots the proportion of total post-fisc income accruing to the pre-fisc poorest

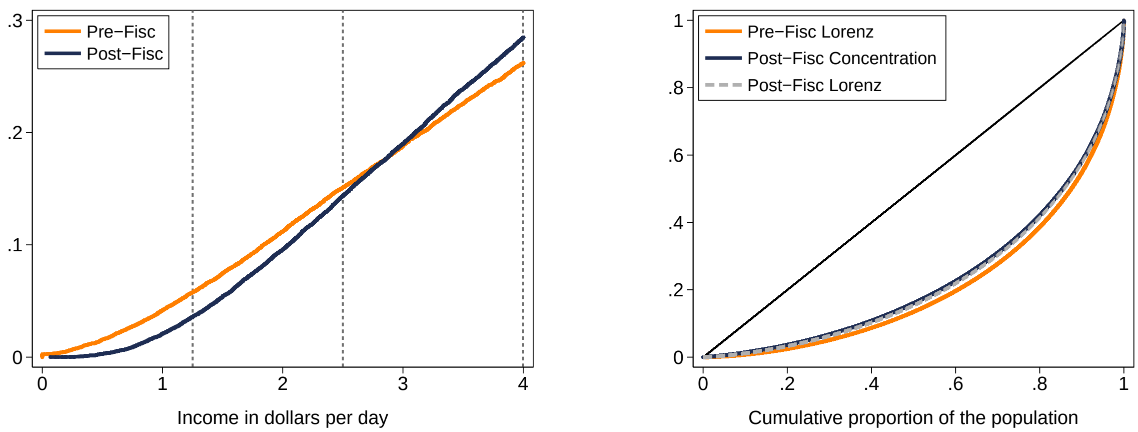
[12] Davidson and Duclos (2013) argue that a null hypothesis of non-dominance is preferable to one of dominance (e.g., Barrett and Donald, 2003) because failure to reject the null of dominance does not necessarily imply dominance. Since we only use first order dominance here, by Cowell and Victoria-Feser (2002, theorem 3) our dominance result is robust to data contamination.

[13] It is not everywhere progressive, a more stringent condition that is essentially never met in practice and requires the tax and transfer system to be rank preserving.

Figure 2: Conventional Tools to Assess the Tax and Transfer System in Brazil

(a) First Order Stochastic Dominance (Cumulative Distribution Functions)

(b) Global Progressivity (Lorenz and Concentration Curves)

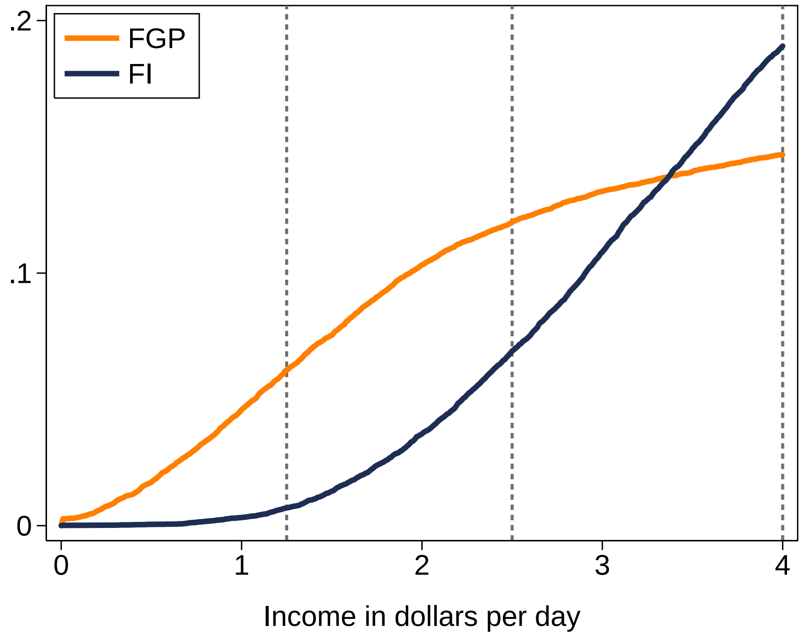


Note: Dashed vertical lines included at common "international" poverty lines of \$1.25, \$2.50, and \$4 per person per day.

$x\%$. The post-fisc concentration curve lies everywhere above the pre-fisc Lorenz curve, indicating that the system is globally progressive (in the income redistribution sense; see Duclos, 2008). This dominance is statistically significant at the 5% level using the asymptotic sampling distribution from Davidson and Duclos (1997). The post-fisc Lorenz curve, which plots the proportion of total post-fisc income accruing to the post-fisc poorest $x\%$, also lies everywhere above the pre-fisc Lorenz, indicating that the tax and transfer system is unambiguously equalizing (Atkinson, 1970).

These conventional measures mask the FI induced by Brazil's tax and transfer system. Using the \$2.50 per person per day poverty line, for which we observe first order dominance of the post-fisc over the pre-fisc distribution, the proportion of the population experiencing FI—i.e., $h(y^0, y^1; z = 2.5)$ with $A = S$—is 6.9%. Considering that the post-fisc poverty headcount ratio is 16.7%, this is substantial: over 40% of the post-fisc poor experience FI. More precisely, $h(y^0, y^1; z = 2.5)$ with $A = \{i \in S \,|\, y_i^1 < z\}$ is 41.4%. Figure 3 shows how the proportion of the population that is fiscally impoverished changes across poverty lines: for low poverty lines, FI is essentially non-existent. This is not surprising in light of the unconditional component of the government cash transfer program *Bolsa Família*, available to households with income below 70 *reais* per person per month (\$1.22 per day), regardless of whether the household has children or elderly members, and without conditions. Between the \$1.25 and \$2.50 per day poverty lines, FI begins to increase rapidly, and for poverty lines above \$3.37 per day, more poor Brazilians experience fiscal impoverishment than fiscal gains.

Figure 3: Proportion of Population Experiencing FI and FGP



Note: Dashed vertical lines included at common "international" poverty lines of \$1.25, \$2.50, and \$4 per person per day.

At the upper bound poverty line of \$4 per day, entirely 19.0% of the population or 59.7% of the post-fisc poor are fiscally impoverished.

Turning to our axiomatically derived measure of fiscal impoverishment, total FI using the \$2.50 per day poverty line (i.e., $f(y^0, y^1; z = 2.5)$ with $\kappa = 1$) is over \$900 million, or about 14% of the 2009 budget of *Bolsa Família*, Brazil's flagship anti-poverty program that reaches over one-fourth of the country's population. Nevertheless, FI is again low for very low poverty lines, at \$55 million—or 0.8% of the 2009 budget of *Bolsa Família*—for the \$1.25 per day poverty line. The per capita amount of FI (setting $\kappa = |S|^{-1}$) is \$0.01 per person per day; this amount divides by the *total* population, not just the impoverished. The average FI of the impoverished, $|I_y|^{-1} f(y^0, y^1; z = 2.5)$, $\kappa = 1$, is \$0.19 per person per day, a loss that averages 8.9% of the pre-fisc incomes of the impoverished.

This FI is mostly caused by consumption taxes, which make up 88.0% of the total taxes paid by impoverished individuals. Only 14.6% of the impoverished report paying income taxes, likely because many of them work in the informal sector, while those who work in the formal sector often have income below the minimum exemption; a similarly low number of impoverished (13.0%) report paying property taxes. Meanwhile, the multiple consumption taxes levied at the state and federal levels are high and often cascading, and consumption tax exemptions for basic goods are almost non-existent (Corbacho et al., 2013). The poor in Brazil thus end up paying a large portion of their incomes in consumption taxes (Baer and Galvão, 2008; Goñi et al., 2011; Higgins and Pereira, 2014). Interestingly, many of those experiencing FI are not excluded from the safety net; they *do* receive government transfers or subsidies: 64.9% receive cash transfers from *Bolsa Família*, for example. Benefits paid to recipient families are low, however, compared to conditional cash transfer programs in other Latin American countries with similar living standards (Levy and Schady, 2013), and thus in some cases do not compensate poor households for the consumption taxes they pay.

In sum, for the $4 per day poverty line, we could not assess the extent of FI in Brazil without the measures from Section 3 (although we would be alerted to its existence by Proposition 1). More importantly, for the $2.50 per day poverty line, we would *not* be alerted to the existence of FI using conventional tools, and might conclude that the tax and transfer system unambiguously benefits the poor since it causes an unambiguous reduction in poverty and is globally progressive. Doing so, we would be overlooking an important aspect: that many poor are made poorer by the tax and transfer system.

5.2 Decomposing the Reduction in the Poverty Gap

Table 1 further illustrates how comparing poverty before and after taxes and transfers can overlook FI due to the offsetting effects of the impoverishment of some households and gains of other poor households. We fix the poverty line at $2.50 per person per day; because the poverty line and population are fixed, we can make the decomposition simpler by setting $\kappa = \nu(S, z)$.

The amount of FI—which, as discussed above, is over $900 million in total or about 9% of the pre-fisc incomes of those who experience FI—is dwarfed by FGP from Brazil's transfer programs, which totals over $3.4 billion. The absolute poverty gap, or the minimum amount that would need to be transferred to the poor to eliminate poverty if transfers were perfectly targeted, falls from $12.6 billion before taxes and transfers to $10.1 billion after. The change in the absolute poverty gap, $2.5 billion, looks impressive, but masks differential trends in two groups of the poor: those who gain (a total of $3.4 billion) and those who lose (a total of $934 million).

The change in the absolute poverty gap, a measure of poverty reduction equivalent to the

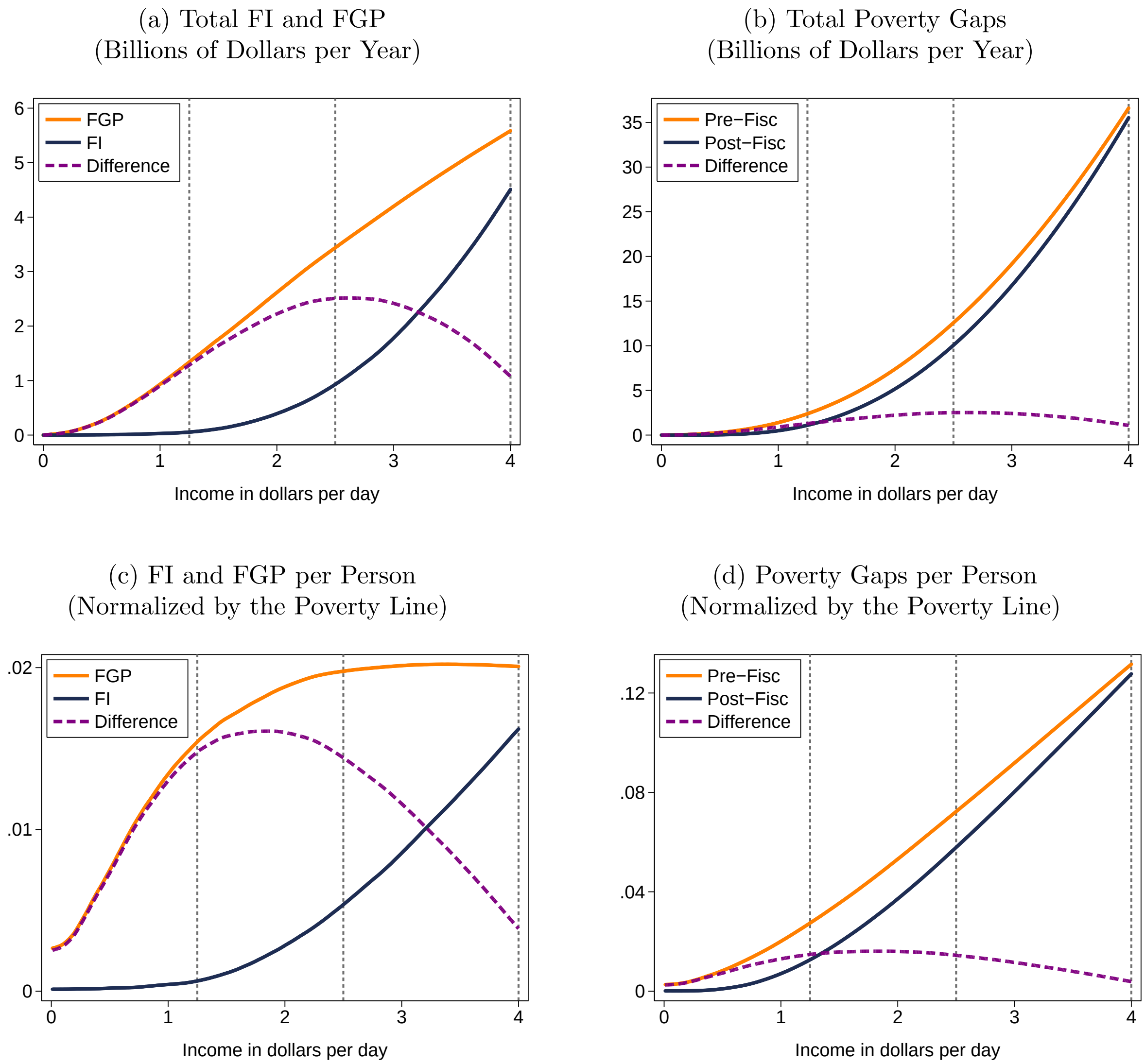
Table 1: Pre-Fisc and Post-Fisc Poverty Gaps, FI, and FGP in Brazil (z = \$2.50)

	Absolute totals $\kappa = \nu(S,z) = 1$ (US dollars/year)	Normalized $\kappa = \nu(S,z) = \frac{1}{z\lvert S\rvert}$ (Unit free)
$p(y^1; z)$	10,063,263,731	0.0579
$p(y^0; z)$	12,567,596,206	0.0723
$p(y^1; z) - p(y^0; z)$	−2,504,332,475	−0.0144
$f(y^0, y^1; z)$	934,039,521	0.0054
$g(y^0, y^1; z)$	3,438,371,997	0.0198
$f(y^0, y^1; z) - g(y^0, y^1; z)$	−2,504,332,475	−0.0144

difference between FGP and FI by Proposition 9, is actually near its peak at the \$2.50 per day poverty line, as shown in Figures 4a and 4b. Maximum poverty reduction is achieved at a poverty line of \$2.61 per person, where the difference between the pre-fisc and post-fisc poverty gaps is \$2.52 billion. After this point, the rate of increase of total FI surpasses the rate of increase of total FGP (Figure 4a), causing the government's reduction in the total poverty gap to fall (dashed line in Figure 4b). That the difference between FGP and FI reaches its maximum around the \$2.50 poverty line is not surprising: the eligibility cut-off for the conditional component of *Bolsa Família*, available to families with children who comply with certain education and health requirements, is \$2.45 per person per day. Just above this line, a number of families still receive benefits due to program leakages, variable and mismeasured income, or components of income we are measuring that are not taken into account in the estimation of eligible income; not far above the line, however, families become ineligible and we see a simultaneous deceleration of fiscal gains and acceleration of impoverishment.

Expressed as a proportion of the poverty line and in per capita terms (where per capita refers to dividing by the entire population, not just the poor), the poverty gap ratio falls from 7.23% of the poverty line before taxes and transfers to 5.79% after. This change of 1.44 percentage points can be decomposed into FGP of 1.98% of the poverty line and FI of 0.54%. Figures 4c and 4d show this decomposition for a range of poverty lines: at the lowest poverty lines, although there are some pre-fisc poor individuals, most experience FGP and escape poverty, while FI is essentially zero; as a result, there is little to no post-fisc poverty for very low poverty lines. As the poverty line approaches its upper limit of \$4 per person per day, FGP levels off and FI increases sharply:

Figure 4: FI, FGP, and Poverty Gaps in Brazil for Various Poverty Lines



Note: Dashed vertical lines included at common "international" poverty lines of \$1.25, \$2.50, and \$4 per person per day.

the gap between the normalized pre- and post-fisc poverty gaps plummets toward zero.

These decompositions and FI measures add an additional layer to the debate on taxing the poor: in the case of Brazil, they reveal that the net effect of the tax and transfer system is indeed to reduce poverty and inequality, but while the fiscal gains of some poor are larger in magnitude than the impoverishment of other poor, a significant proportion of the poor pay more in taxes than they receive in transfers, and lose an average of 9% of their incomes to the fiscal system.

6 Conclusions

Stochastic dominance tests used to make unambiguous conclusions about poverty changes, measures of horizontal inequity among the poor, and tests of the net fiscal system's progressivity can fail to capture that some of the poor are made poorer (or some of the non-poor made poor) by the tax and transfer system. Specifically, in the frequent scenario that the fiscal system is not rank preserving and the post-fisc distribution first order dominates the pre-fisc distribution, stochastic dominance tests cannot be used to alert us to whether fiscal impoverishment has occurred. Likewise, tests for horizontal inequity among the poor or the progressivity of the tax and transfer system do not tell us about the existence of FI.

Hence, conventional measures of a tax and transfer system might lead us to conclude that it unambiguously benefits the poor, when in fact a substantial number of poor are not compensated with transfers for their tax burdens. Indeed, in our illustration with Brazilian data, the fiscal system unambiguously reduces poverty for any poverty line up to \$2.50 per person per day and is globally progressive. Over 40% of the post-fisc poor, however, experience FI, paying a total of over \$900 million more in taxes than they receive in transfers—a phenomenon that is completely overlooked by stochastic dominance tests.

Given this shortcoming of conventional criteria and the debate about high taxes on the poor and transfers compensating them for those taxes, we propose a set of axioms that should be met by a measure of FI, and show that these uniquely determine the measure up to a proportional transformation. We also propose a partial ordering to determine when one fiscal system, such as that under a proposed reform, induces unambiguously less FI than another, such as the current system, over a range of possible poverty lines. To obtain a complete picture of the fiscal system's effect on the poor, we propose an analogous measure of fiscal gains of the poor, and show that the difference between the pre-fisc and post-fisc poverty gaps can be decomposed into our axiomatic measures of FI and FGP.

Our results can be extended to comparisons between two points in time or before and after a policy reform, rather than pre- and post-fisc. In comparison to the tools used to assess whether the tax and transfer system hurts the poor, tools from the literatures on pro-poor growth and policy reforms (tax reforms, trade liberalization, etc.) suffer from similar limitations. For pro-poor growth, stochastic dominance tests are often used to assess whether poverty is unambiguously reduced over time; it directly follows from Proposition 3 that these will not necessarily capture that some of the poor become poorer over time.[14] Hence, growth can appear unambiguously pro-poor even if a significant proportion of the poor are immiserized.[15] Growth incidence curves (Ravallion and Chen, 2003) and related pro-poor partial orderings (Duclos, 2009) can fail to capture impoverishment for the same reason that stochastic dominance tests do: they are anonymous with respect to $t = 0$ income. Although their non-anonymous counterparts (Bourguignon, 2011a; Grimm, 2007; Van Kerm, 2009) resolve this issue in theory, in practice—to become graphically tractable—they average within percentiles, and hence impoverishment can still be overlooked if within some percentiles, some poor are "hurting behind the averages" (Ravallion, 2001, p. 1811).

For policy reform, Besley and Kanbur (1988) derive poverty-reducing conditions for reallocating food subsidies; these results are extended to commodity taxes and a broader class of poverty measures by Makdissi and Wodon (2002) and Duclos et al. (2008). Again, by Proposition 3, unambiguous poverty reduction does not guarantee that a substantial portion of the poor are not hurt by the reform. In the literature on trade liberalization, Harrison et al. (2003, p. 97) note that "even the most attractive reforms will typically result in some households losing," and recent efforts to measure welfare impact at the household level have been made following Porto (2006). Nevertheless, because results are presented at some aggregate level (e.g., by state or percentile), impoverishment due to trade reform could still be overlooked. For example, Nicita's (2009, p. 26) finding that "on average all income groups benefited from [Mexico's] trade liberalization, but to a varying extent" does not tell us whether some households within each income percentile were made worse off by the reform.

In each of these cases, our axiomatically derived FI measure could be used to quantify the impoverishment of those becoming poorer over time or the extent to which losers are hurt by policy reforms. Our decomposition could be used to examine the extent to which a decrease in poverty

[14] That is, unless ranks of the poor are preserved, in which case Proposition 2 can be used to determine if some poor are impoverished.

[15] Here, we are using the poverty-reducing or weak absolute definition of pro-poor (in the respective taxonomies of Kakwani and Son (2008) and Klasen (2008)), by which "growth is pro-poor if the poverty measure of interest falls" (Kraay, 2006, p. 198). We could instead adopt a relative definition of pro-poor growth (Kakwani and Pernia, 2000), perhaps out of the concern that "throughout the developing world, poverty reduction has been below what distributionally neutral growth would have achieved" (Groll and Lambert, 2013, p. 783). Growth-adjusted stochastic dominance tests can be used to determine when growth is unambiguously relatively pro-poor (Duclos, 2009), and it can be shown that this type of dominance can also occur despite a significant portion of the poor becoming poorer.

over time or due to a reform balances out the gains and losses of different households. Doing so, we will cease to overlook cases where growth, policy reform, or the tax and transfer system is poverty-reducing and progressive, yet hurts a substantial proportion of the poor.

Appendix

Proof of Proposition 1 If F_1 does not first order stochastically dominate F_0 on $[0, z)$, then there exists $\hat{y} \in [0, z)$ such that $F_1(\hat{y}) > F_0(\hat{y})$. By the definition of CDFs, this implies that the proportion of individuals with $y_i^1 \leq \hat{y}$ is higher than the proportion of individuals with $y_i^0 \leq \hat{y}$. Since the total number of individuals is identical in the pre-fisc and post-fisc distributions, there exists $j \in S$ such that $y_j^0 > \hat{y}$ and $y_j^1 \leq \hat{y}$, which implies that FI has occurred. □

Proof of Proposition 2 For sufficiency, suppose not. Then FI has occurred, which implies that $y_i^1 < y_i^0$ and $y_i^1 < z$ for some i. If there does not exist $j \in S$ such that $y_j^0 \leq y_i^1 < y_j^1$ then $F_1(y_i^1) > F_0(y_i^1)$ which implies $F_1 \not\succeq_0^z F_0$. If there does exist such an individual j, then the tax and transfer system is not rank preserving among the poor. Both possibilities are contradictions. Necessity follows immediately from the contrapositive of Proposition 1. □

Proof of Proposition 3 By counterexample: $y^0 = (5, 8, 20), y^1 = (9, 6, 18), z = 10$. F_1 first order stochastically dominates F_0 on $[0, z)$, yet there is FI. □

Proof of Proposition 4 Counterexamples given in text. □

Proof of Proposition 5 Not sufficient: consider the example $n(y^0) = 3y^0/(y^0 + 1) - 2$, $z = 4$, shown in Figure 1a. The system is everywhere progressive since $n'(y^0) > 0$ for all $y^0 \in \mathbb{R}_+$, while $n(y^0) > 0$ for $y^0 \in (2, 4)$ implying that FI has occurred. Not necessary: consider the example $n(y^0) = (y^0 - 1)^2/(.5(y^0)^2 + 1) - 1$, $z = 4$, shown in Figure 1b. Since $n(y^0) \leq 0$ for all $y^0 \in [0, z)$ and $n(y^0) \leq (y^0 - z)/y^0$ for all $y^0 \geq z$, there is no FI; nevertheless, $n'(y^0) < 0$ on $[0, 1)$ so the system is not everywhere progressive. □

Proof of Proposition 6 We begin with a lemma analogous to one of the propositions in Foster and Shorrocks (1991). To simplify notation, $y_a \equiv (y_a^0, y_a^1)$ for a subset S_a of a partition of S into

m subgroups $a = 1, \ldots, m$; similarly, $x_a \equiv (x_a^0, x_a^1)$. We also define vectors $y_{-a}^t \equiv (y_b^t)_{b \neq a \in \{1,\ldots,m\}}$, $t \in \{0, 1\}$ as the vector of pre- or post-fisc incomes of all $i \notin S_a$ (similarly for x_{-a}^t) and $y_{-a} \equiv (y_{-a}^0, y_{-a}^1)$, $x_{-a} \equiv (x_{-a}^0, x_{-a}^1)$.

Lemma. $f(y_a, y_{-a}; z) \geq f(x_a, y_{-a}; z) \Rightarrow f(y_a, x_{-a}; z) \geq f(x_a, x_{-a}; z)$.

Proof. By subgroup consistency, $f(y_a, y_{-a}; z) \geq f(x_a, y_{-a}; z) \Rightarrow f(y_a; z) \geq f(x_a; z)$. (Suppose not. Then $f(y_a; z) < f(x_a; z)$, which by subgroup consistency implies $f(y_a, y_{-a}; z) < f(x_a, y_{-a}; z)$, a contradiction.) $f(y_a; z) \geq f(x_a; z)$ implies either $f(y_a; z) > f(x_a; z)$ or $f(y_a; z) = f(x_a; z)$. In the former case, it immediately follows by subgroup consistency that $f(y_a, x_{-a}; z) \geq f(x_a, x_{-a}; z)$. In the latter case, the implication is shown by contradiction. Suppose that $f(y_a, x_{-a}; z) < f(x_a, x_{-a}; z)$. Then by subgroup consistency we have (since $f(y_a; z) = f(x_a; z)$) $f(y_a, x_{-a}, x_a; z) < f(x_a, x_{-a}, y_a; z)$, which contradicts permutability. $\square$

This lemma shows that a subgroup-consistent and permutable measure of FI is separable by group, using a definition of separability analogous to that used for preferences in the utility literature. Because the lemma can be reiterated within any particular subgroup to further separate individuals in that subgroup, we have that each set of individuals is separable (which is analogous to the "each set of sectors is separable" requirement in Gorman (1968, p. 368)). Hence, from Debreu (1960, theorem 3), there exists a continuous FI function determined up to an increasing linear transformation of the form

$$f(y^0, y^1; z) = \alpha + \beta \sum_{i \in S} \phi_i(y_i^0, y_i^1, z)$$

where ϕ_i is a real-valued function for each $i \in S$. The additional requirement for Debreu's (1960) proof that more than two of the $|S|$ elements of S are essential is satisfied as long as $|S| \geq 3$ and f is non-constant on $[0, z]$, which in turn is implied by monotonicity as long as at least one individual is impoverished.[16]

Permutability implies that $\phi_i = \phi_j$ for all $i, j \in S$, so we have $f(y^0, y^1; z) = \alpha + \beta \sum \phi(y_i^0, y_i^1, z)$

[16] The assumptions of at least three individuals in society and at least one impoverished individual are innocuous for any real-world application.

where ϕ is a real-valued function. By the focus and normalization axioms:

$$\phi(y_i^0, y_i^1, z) = \begin{cases} \tilde{\phi}(y_i^0, y_i^1, z) & \text{if } y_i^1 < y_i^0 \text{ and } y_i^1 < z \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

By the continuity of f, ϕ and $\tilde{\phi}$ must also be continuous. Consider an individual with $y_i^0 > z$ and $y_i^1 = z$. Since y_i^1 is not less than z, i is not impoverished, so by (4), $\phi(y_i^0, y_i^1, z) = 0$. Now consider an alternative situation where $\tilde{y}_i^1 = z - \epsilon$ for a sufficiently small $\epsilon > 0$. In this scenario, $\tilde{\phi}$ cannot be a direct function of y_i^0 or ϕ would be discontinuous at z; instead, $\tilde{\phi}$ must be a direct function of just y_i^1 and z so that an infinitesimal decrease in y_i^1 below z results in an infinitesimal increase in ϕ. By a similar argument, for an individual with $y_i^0 < z$, $y_i^1 = y_i^0$, and $\tilde{y}_i^1 = y_i^0 - \epsilon$, $\tilde{\phi}$ cannot be a direct function of z and instead must directly depend only on y_i^1 and y_i^0 so that an infinitesimal decrease in y_i^1 below $y_i^0 < z$ results in an infinitesimal increase in ϕ.

Given this, we can rewrite $\tilde{\phi}(y_i^0, y_i^1, z) = \tilde{\phi}(\min\{y_i^0, z\}, y_i^1)$. Since $\tilde{\phi}$ is only defined for those who are impoverished (i.e., those for whom $\min\{y_i^0, y_i^1, z\} = y_i^1$), we have

$$\begin{aligned} \tilde{\phi}(y_i^0, y_i^1, z) &= \tilde{\phi}(\min\{y_i^0, z\}, \min\{y_i^0, y_i^1, z\}) && (5) \\ &= \tilde{\phi}(\min\{y_i^0, z\} - \min\{y_i^0, y_i^1, z\}, 0) && (6) \\ &= \big(\min\{y_i^0, z\} - \min\{y_i^0, y_i^1, z\}\big)\tilde{\phi}(1, 0) && (7) \end{aligned}$$

where (6) follows from translation invariance and (7) from linear homogeneity. Noting that $\tilde{\phi}(1,0)$ is a constant (that is positive by monotonicity) and denoting it γ, we have

$$\phi(y_i^0, y_i^1, z) = \begin{cases} \big(\min\{y_i^0, z\} - \min\{y_i^0, y_i^1, z\}\big)\gamma & \text{if } i \in I_y \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

For $i \notin I_y$ we can also write $\phi(y_i^0, y_i^1, z) = (\min\{y_i^0, z\} - \min\{y_i^0, y_i^1, z\})\gamma$ since the non-impoverished are either non-poor before taxes and transfers and non-poor after ($\Rightarrow \min\{y_i^0, z\} = \min\{y_i^0, y_i^1, z\} = z$) or poor before taxes and transfers but do not lose income to the fiscal system ($\Rightarrow \min\{y_i^0, z\} = \min\{y_i^0, y_i^1, z\} = y_i^0$). Therefore $f(y^0, y^1; z) = \alpha + \beta\gamma \sum_{i \in S} \big(\min\{y_i^0, z\} - \min\{y_i^0, y_i^1, z\}\big)$. By normalization, $\alpha = 0$, which completes the proof. $\square$

Proof of Proposition 7 (a)$\Leftrightarrow$(b) follows immediately from Proposition 6. For (b)$\Leftrightarrow$(c), we begin by defining Foster and Rothbaum's (2014) second order downward mobility dominance.

Definition 8. (y^0, y^1) *second order downward mobility dominates* (x^0, x^1) on $[0, z^+]$ if

$$\int_0^z m(y^0, y^1; c)dc < \int_0^z m(x^0, x^1; c)dc \; \forall \, z \in [0, z^+],$$

where $m(y^0, y^1; z) = |S|^{-1} \sum_{i \in S} \mathbb{I}(y_i^1 < z < y_i^0)$ is Foster and Rothbaum's (2014) *downward mobility curve*, measuring the proportion of the population that begins with income above each poverty line and ends with income below the line.

A sufficient condition for (b)$\Leftrightarrow$(c) is $f(y^0, y^1; z) \propto \int_0^z m(y^0, y^1; c)dc$. For a given poverty line $z = \hat{z}$, partition the set S into four subsets: $S_1 = \{i \in S \,|\, y_i^1 < y_i^0 < \hat{z}\}$, $S_2 = \{i \in S \,|\, y_i^1 < \hat{z} \leq y_i^0\}$, $S_3 = \{i \in S \,|\, y_i^0 \geq \hat{z}, y_i^1 \geq \hat{z}\}$, $S_4 = \{i \in S \,|\, y_i^0 < \hat{z}, y_i^0 \leq y_i^1\}$. For any subset $S_a \subset S$, denote $f_a(\cdot; z) \equiv \kappa \sum_{i \in S_a} \big(\min\{y_i^0, z\} - \min\{y_i^0, y_i^1, z\}\big)$ and $m_a(\cdot; z) \equiv |S|^{-1} \sum_{i \in S_a} \mathbb{I}(y_i^1 < z < y_i^0)$.

Each $i \in S_1$ experiences downward mobility on the interval $[0, \hat{z}]$ for all $z \in (y_i^1, y_i^0) \Rightarrow$ individual $i \in S_1$ increases $m_1(\cdot; z)$ by $|S|^{-1}$ for $z \in (y_i^1, y_i^0)$ and by zero for $z \leq y_i^1$ and $z \geq y_i^0 \Rightarrow$ individual $i \in S_1$ increases $\int_0^{\hat{z}} m_1(\cdot; c)dc$ by $|S|^{-1}(y_i^0 - y_i^1)$. Summing over all $i \in S_1$, $\int_0^{\hat{z}} m_1(\cdot; c)dc = \sum_{i \in S_1} |S|^{-1}(y_i^0 - y_i^1)$.

$y_i^1 < y_i^0 < \hat{z} \; \forall \, i \in S_1 \Rightarrow f_1(\cdot; \hat{z}) = \kappa \sum_{i \in S_1}(y_i^0 - y_i^1) \Rightarrow$

$$f_1(\cdot; \hat{z}) = \kappa |S| \int_0^{\hat{z}} m_1(\cdot; c)dc. \tag{9}$$

Each $i \in S_2$ experiences downward mobility on the interval $[0, \hat{z}]$ for all $z \in (y_i^1, \hat{z}]$, which increases $m_2(\cdot; z)$ by $|S|^{-1}$ for $z \in (y_i^1, \hat{z}]$ and by zero for all other $z \Rightarrow$ individual $i \in S_2$ increases $\int_0^{\hat{z}} m_2(\cdot; c)dc$ by $|S|^{-1}(\hat{z} - y_i^1)$. Summing over all $i \in S_2$, $\int_0^{\hat{z}} m_2(\cdot; c)dc = \sum_{i \in S_2} |S|^{-1}(\hat{z} - y_i^1)$.

$y_i^1 < \hat{z} \leq y_i^0 \; \forall \, i \in S_2 \Rightarrow f_2(\hat{z}, \cdot) = \kappa \sum_{i \in S_2}(\hat{z} - y_i^1) \Rightarrow$

$$f_2(\cdot; \hat{z}) = \kappa |S| \int_0^{\hat{z}} m_2(\cdot; c)dc. \tag{10}$$

Each $i \in S_3$ does not experience downward mobility on the interval $[0, \hat{z}]$; summing over all

$i \in S_3$ and integrating over our domain, we have $\int_0^{\hat{z}} m_3(\cdot; c)dc = 0$. $y_i^0 \geq \hat{z}$ and $y_i^1 \geq \hat{z}\ \forall\, i \in S_3 \Rightarrow$

$$f_3(\cdot; \hat{z}) = \kappa \sum_{i \in S_3} (\hat{z} - \hat{z}) = 0 = \kappa |S| \int_0^{\hat{z}} m_3(\cdot; c)dc. \tag{11}$$

Similarly $\int_0^{\hat{z}} m_4(\cdot; c)dc = 0$ because each $i \in S_4$ does not experience downward mobility on $[0, \hat{z}]$. $y_i^0 < \hat{z}$ and $y_i^0 \leq y_i^1\ \forall\, i \in S_4 \Rightarrow$

$$f_4(\cdot; \hat{z}) = \kappa \sum_{i \in S_4} (y_i^0 - y_i^0) = 0 = \kappa |S| \int_0^{\hat{z}} m_4(\cdot; c)dc. \tag{12}$$

Given the definitions of $f_a(\cdot; z)$ and $m_a(\cdot; z)$ and that $S = S_1 \cup S_2 \cup S_3 \cup S_4$ and $S_1 \cap S_2 \cap S_3 \cap S_4 = \varnothing$, we have $f(y^0, y^1; z) = \sum_{a=1}^{4} f_a(\cdot; z)$ and $m(y^0, y^1; z) = \sum_{a=1}^{4} m_a(\cdot; z)$. Hence, by (9)–(12), $f(y^0, y^1; \hat{z}) = \kappa |S| \int_0^{\hat{z}} m(\cdot; c)dc$. This holds for all $\hat{z} \in [0, z^+]$ since the choice of $\hat{z}$ was arbitrary, which completes the proof. □

Proof of Proposition 8 Analogous to the proof of Proposition 6 for FI. □

Proof of Proposition 9 Given in text. □

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