CENTER FOR GLOBAL DEVELOPMENT

Revealing the Subsidy: Competitive Allocation of Concessional Finance in Fragile States

Álvaro S. González

Abstract

A firm (the sponsor) planning to enter or expand in a fragile and conflict-affected economy can seek concessional assistance from a development finance institution (DFI). The two negotiate bilaterally over the required support. By June 2023, the International Development Association's Private Sector Window had approved nearly $4 billion through such negotiations across three replenishments.

Development finance institutions must assess the support sponsors require, although some relevant information is private. This paper proposes a Least Present Value of Support auction, analogous to the Least Present Value of Revenue mechanism used in road concessions. Prequalified sponsors bid on the present-value cap on contingent support under a standard contract. Payments occur only against independently verified revenue shortfalls. Contract duration may vary in response to negative shocks, subject to a predetermined limit. The mechanism applies to projects with independently verifiable revenues and irreversible investments, where reference-case returns justify participation but downside risk deters investment.

If participation is feasible under the core contract, acceptable support caps form an interval with an attained minimum. If an award at that minimum leaves the sponsor no better off than its outside option, second-price procurement has a truthful equilibrium, ranking sponsors by their minimum required support. With a contractual accumulation rate below the sponsor's discount rate, no retained terminal value, and a nonnegative outside option, a compensation target at or below appraised capital cost excludes all sponsors. Operating costs, discount rates, and outside options influence minimum acceptable caps. Selecting the lowest cap does not necessarily minimize expected expenditure. Numerical designs show lower expenditure than sequential posted offers at the maximum cap, but fewer concessions reach the compensation target and more end in sponsor exit. Lower expenditure does not establish an efficiency gain.

KEYWORDS
auctions, blended finance, concessional finance, fragile states, IDA Private Sector Window, state-contingent contracts

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Supplementary Materials: The replication package provides Python code, tests, inputs, outputs, and replication instructions for the numerical results and sensitivity analyses, including the termination-penalty variant. Supporting files cover the concessionality exhibit and classification sensitivities, with source records and the addresses and checksums of the archived disclosure pages. The package (version 1.0.0) is archived at https://doi.org/10.5281/zenodo.24013716 and maintained at https://github.com/Econoguayo/revealing-the-subsidy-replication.

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1. Introduction

The Private Sector Window (PSW) of the International Development Association (IDA) sets concessional support project by project. Except in tenders for infrastructure projects similar to public concessions and in open-access programs, the amount of support is set through bilateral negotiation between a deal team of the development finance institution (DFI) and a single sponsor.

In negotiating the subsidy required for a project, the sponsor has information about its own costs, market position, and willingness to bear risk which the DFI's deal team does not possess. The DFI might know a lot about the country and the sector but little about how much subsidy this particular sponsor (firm) needs to put this specific project in place.

When one side of a negotiation knows the answer and the other side must guess, the side with the answer has little incentive to volunteer the information. The sponsor has an incentive to claim more subsidy than needed, and the DFI has no way to verify the minimum requirement. In markets with lots of competitors, a buyer quoted a high price seeks a second quote, and the seller who quoted high risks losing the sale. Competition therefore limits the seller's ability to benefit from the information gap. Without a competing quote, the DFI's alternative is project rejection, a constraint the sponsor anticipates in negotiation.

Pricing a PSW project in a fragile or conflict-affected situation (FCS) requires producing an actual dollar amount (a cardinal figure) from estimates of country risk and the sponsor's private parameters. Allocation requires only ordinal information. The pricing problem asks what the right dollar amount is, and an answer requires information about the sponsor's costs, discount rate and outside option as well as the frequency and severity of shortfalls. A ranking of sponsors by the support each would accept for the same project serves the allocation rule. In the model, the shock distribution is common knowledge. The auction uses bids incorporating sponsors' private parameters rather than requiring the DFI to estimate those parameters. The same ranking can also serve to set and enforce a fixed ceiling on support.[1]

The proposed Least Present Value of Support (LPVS) auction implements the allocation rule explained above. The LPVS is adapted from the Least Present Value of Revenue (LPVR) mechanism which was developed for highway concessions (Engel et al., 2001). Under LPVS, each sponsor bids the support cap at which it is willing to undertake the project, and the lowest bidder wins a cap equal to the second-lowest qualifying bid, or a preset maximum when no other bid qualifies. In FCS economies, where shocks are potentially very costly to sponsors, the LPVS contract would adjust its length, within preset limits, in the event of delays or temporary revenue shortfalls, and it makes no provision for renegotiation.

The question is under what conditions competition for a contingent support cap ranks sponsors by their minimum support requirements, and how the contract's payment and duration rules affect participation, fiscal expenditure and operating outcomes. Indeed, the sponsor's payoff depends on the cap via the path of revenue shortfalls, the stopping rule and the exit decision, and is generally nonlinear in the awarded cap. Under the core contract, if the contractual accumulation rate is below each sponsor's discount rate and terminal values leave a later termination unrewarded, the sponsor's payoff is weakly increasing and right-continuous in the cap. When participation is feasible, the caps

[1]The paper uses "DFI" (development finance institution) for institutions allocating concessional resources under uncertainty. References to IDA, the International Finance Corporation (IFC), and the Multilateral Investment Guarantee Agency (MIGA) concern World Bank Group arrangements. FCS designations follow the World Bank's harmonized classification. Although designed for the PSW, the mechanism addresses an allocation problem shared across DFIs.

a sponsor accepts therefore form an interval whose minimum is attained (Section 5). This same contract imposes on its terms a necessary condition. If no terminal value is retained by the sponsor and its outside option is nonnegative, a compensation target at or below appraised capital cost excludes every sponsor. Once a minimum acceptable cap exists, and provided that an award at that minimum leaves the sponsor no better off than its outside option, second-price procurement, the implementation considered here, ranks sponsors by that minimum via a weak-dominance argument. The paper states the proviso as an assumption and gives a sufficient condition for it, equality of the exit value and the terminal values at handover and at expiry, which the baseline numerical environment meets. The paper leaves open the equilibrium of first-price procurement. The minimum acceptable cap depends on the operating cost, the discount rate and the outside option of the sponsor, and selection of the lowest minimum cap need not select the sponsor with the lowest expected expenditure: in a numerical example, a higher-cost sponsor that needs no support wins against a lower-cost sponsor and, under the cap that the auction awards, has an expected disbursement of 0.689 against 0.587 (Section 6). With access to sponsors held equal, the auction spends less per procurement opportunity than sequential offers of the maximum cap, reaches the compensation target less often and produces more sponsor exit, in every design examined under costless exit; relative to an approach to a single sponsor, the higher probability of reaching the target comes from access to more sponsors. Lower disbursement accompanies different operating outcomes, so the reduction in expenditure does not establish an efficiency gain.

The LPVS mechanism works when the allocation problem is narrowly scoped. Project revenue needs to be independently verified because the contract turns on the ability to confirm whether revenue is falling short—that is what triggers an extension of the concession. To justify concessional support, the sponsor must have invested in assets that are irreversible enough to need protection against a prolonged loss of cash flow. When such a loss occurs, as during an extended period of utility arrears, capital is trapped, because the sponsor can neither redeploy the asset nor recover its cost from depressed revenue. When firms must commit substantial investments to irreversible assets, the prospect of such losses is likely to deter many investors from entering FCS markets. For those reasons, the LPVS mechanism insures part of the cost of temporary cash-flow losses up to the support cap, and losses beyond the cap or from the destruction of the asset itself remain outside its coverage. As well, the proposed project must be viable in its reference case, with returns sufficient to support participation before downside risk is taken into account. The mechanism protects against bad luck, but it does not turn a bad project into a good one. No insurance does unless it is a subsidy so excessive that it ends up paying a firm to do something that is unprofitable.

Absent that narrow scope, bilateral negotiation is the right instrument to set subsidy support and select a sponsor. An auction needs a buyer that understands what it is buying and multiple sellers willing to provide it. Most of what DFIs do (supporting one sponsor with an idea no competitor is ready to replicate) satisfies neither condition a good auction requires. When Chile auctioned rural-telephony subsidies in 1995, operators bid zero for 16 of the 46 projects offered, projects that the government had expected to require support (Wellenius, 1997).

2. The Allocation Problem

The World Bank Group's 2016 Forward Look included the creation of the Private Sector Window. The Forward Look aimed to pool IDA, IFC, and MIGA funds and expertise together to maximize

use of development finance in the poorest countries (World Bank Group, 2016). The IDA Deputies approved it during IDA18 (FY2017). The initial allocation was Special Drawing Rights (SDR) 1.8 billion ($2.5 billion), for use in IDA-only countries, with priority given to FCS (International Development Association, 2017).

IDA established four facilities under the PSW to address barriers to private investments including the Blended Finance Facility (BFF) to enable investments in small and medium enterprises (SMEs) and earlier stage investments; the Risk Mitigation Facility (RMF) to support deals without sovereign indemnity; the Local Currency Facility (LCF) to allow for hedging; and Political Risk Insurance from MIGA, facilitated through the MIGA Guarantee Facility (MGF). By IDA20, the window expanded to $5.5 billion. Through June 2023, an estimated $3.9 billion had been approved across 238 transactions in over 60 percent of eligible countries (International Development Association, 2023). Each investment must follow the Enhanced Principles for Blended Concessional Finance that DFIs agreed on (DFI Working Group on Blended Concessional Finance, 2017).[2]

There has been some expansion in scope, with IFC and MIGA expanding into seven new countries, including Yemen, South Sudan, and Somalia (International Development Association, 2023). The Independent Evaluation Group's (IEG) staggered difference-in-differences analysis suggests IFC commitment volumes in these PSW-supported countries are roughly three times greater than expected without the facility (Independent Evaluation Group, 2024).

IFC's share of commitments to PSW-eligible markets has been flat between the three years before the PSW (9.9 percent, 2015–2017) and the six years after (9.8 percent, 2018–2023). Average annual commitments in eligible countries that received PSW support fell 28 percent between 2012–2017 and 2018–2023 (Kenny et al., 2024; Kenny, 2026).

Reported ex ante projections put average concessionality at 6.7 percent of total project cost (International Development Association, 2023; Independent Evaluation Group, 2024). Public project disclosures now report some of these estimates but omit the reference and concessional prices separately and the pricing-model inputs. There is no agreed methodology for establishing minimum concessionality (Mathiasen et al., 2024, p. 11). The Independent Evaluation Group concluded that PSW projects meet the minimum concessionality principle, on the basis of a review of approval and pricing documents, case studies and interviews with staff and clients (Independent Evaluation Group, 2024, pp. xxviii and 45–46). Mathiasen et al. (2024, p. 11) observe that the sources made available to the evaluation team are not public, which precludes outside verification of that finding. Deal teams and the PSW Secretariat use internal models and bilateral negotiation to set support. They have no competitive price against which to assess the negotiated amount.[3]

Of the 204 Board-approved PSW transactions under IDA18 through IDA20 on the project list that IDA publishes (Appendix E), 119 disclose a concessionality estimate. The analysis uses 92 IFC transactions reporting estimates as shares of total project cost. MIGA estimates are excluded because eleven of thirteen are upper bounds and all cover a guarantee period. Table 1 reports predictability from recorded project characteristics.

Disclosed estimates span a factor of nine between the tenth and ninetieth percentiles (Appendix E). Facility alone explains 32 percent of the variation in log concessionality. Adding industry

[2]The transaction count in the IDA20 Mid-Term Review is 238, comprising 166 IFC projects, 17 IFC programs, and 55 MIGA executed guarantee contracts. The Independent Evaluation Group (IEG) review cited below uses a project count of 220, comprising 181 IFC projects and 39 MIGA guarantees.

[3]IFC began publishing private capital mobilization data for PSW projects in April 2024; the first release identified one FY2023 project with external mobilization (Paxton, 2024).

Table 1. Predictability of Disclosed Concessionality from Recorded Project Characteristics

Conditioning set	n	R^2	Adj. R^2	Residual factor	Prediction error		Reduction (percent)
					Transaction	Sponsor	
None (unconditional)	92	0.000	0.000	2.46	2.47	2.47	0.0
Facility	92	0.318	0.303	2.12	2.15	2.15	15.6
Facility, industry	92	0.409	0.375	2.04	2.10	2.10	18.3
Facility, industry, cycle	92	0.410	0.361	2.05	2.13	2.13	16.6
Facility, industry, cycle, fragility	92	0.413	0.356	2.06	2.15	2.15	15.6
Adding log project cost	79	0.555	0.497	1.95	2.08	2.09	22.3

Notes: The dependent variable is the natural logarithm of the concessionality estimate disclosed at approval, expressed as a share of total project cost, for the 92 IFC transactions in the IDA18 through IDA20 Private Sector Window registry whose disclosure states that denominator. The residual factor is the exponentiated residual standard deviation of the fitted regression, an in-sample quantity and not a cross-validated one. The prediction-error columns report the exponentiated root mean squared cross-validated prediction error, $\exp(\sqrt{n^{-1}\sum_i (y_i - \widehat{y}_{-i})^2})$, where $\widehat{y}_{-i}$ omits the held-out observation from estimation. The measure summarizes log prediction error on a multiplicative scale; it is neither a prediction interval nor a statement that individual estimates fall within the stated factor. The transaction column holds out one transaction at a time. The sponsor column holds out all transactions of the same sponsor, which affects the 11 transactions belonging to the four sponsors that appear more than once. The reduction column reports the proportional fall in the root mean squared error relative to an unconditional benchmark fitted and validated on the same observations, measured in log points; for the full conditioning set the corresponding reduction in mean squared error is 28.8 percent. The final row is estimated on the 79 transactions reporting a project cost and is measured against the benchmark for those 79 observations, which is 2.57 rather than the 2.47 of the full sample; its prediction-error factor of 2.08 is not comparable with the rows above.
Industry is the IFC industry group, cycle is the IDA replenishment, and fragility follows the registry's designation. The disclosures do not report separate reference and concessional prices or the inputs to the pricing model, so the exercise measures the predictability of the disclosed estimate from recorded characteristics. Residual variation is not evidence about information held by sponsors and unavailable to IFC.
Source: IFC project disclosures matched to the IDA PSW project registry. Author's compilation. Appendix E describes the data construction.

raises this to 41 percent. Replenishment cycle and fragility add little explanatory power. The full specification reduces the leave-one-out root mean squared error (RMSE) of log concessionality by 15.6 percent relative to an intercept-only model, or 15.4 percent when all transactions of a sponsor are held out together.

Recorded characteristics explain some of the variation, but the published details do not include separate reference prices, concessional prices and pricing inputs. The unexplained residual therefore does not parse out information unavailable to IFC from information used by IFC but not included in the dataset. Project-cost definitions and estimation methodologies also influence the ratios.

The exercise does not test the accuracy or stability over time of these estimates, nor does it identify sponsors' private information or rents that could be recovered. The informational case for allocating competitively depends on project eligibility, bidding and verification procedures.

The DFI Principles for Blended Concessional Finance require that interventions "catalyze market development and mobilize private sector resources" with "minimum concessionality" while "reinforcing markets" (DFI Working Group on Blended Concessional Finance, 2017). U.S. congressional oversight invoked the principle that subsidies should be "competitively based" and "guarded against rent-seeking opportunities" (Waters, 2019). IFC states that, under the PSW, it ensures open and, where possible, competitive approaches to the allocation of funds to infrastructure projects similar

to public concessions. Where a market or project is not suited to a competitive tender, IFC follows an open-access approach, under which all eligible companies may obtain similar concessional terms. In competitive tenders IFC provides specific terms of blended concessional finance, including its amount, to all prequalified bidders, so that the amount of support is set by IFC and is not itself the object of competition (International Finance Corporation, 2026).

Guasch (2004) found renegotiation to be frequent in infrastructure concessions. Of roughly 1,000 concessions awarded in Latin America, 30 percent were renegotiated during the first few years they were in force—55 percent for transportation projects, 74 percent for water—with 85 percent of renegotiations taking place within four years of award. Renegotiated contracts are made bilaterally after competition is over and government leverage has long since evaporated.

The evidence accumulated over the decades on competition and renegotiation distinguishes competition within a sector from competition for a contract. Guasch (2004) reports less renegotiation in electricity and telecommunications than in transportation and in water and sanitation, a difference that he attributes in part to the more competitive nature of the first two sectors. Competition for the contract shows the opposite pattern: excluding telecommunications, 46 percent of competitively awarded concessions were renegotiated, against 8 percent of bilaterally negotiated ones, which Guasch explains by the more favorable terms that bilateral negotiation gives operators.

South Korea's program of minimum revenue guarantees illustrates the fiscal exposure that contingent revenue support can create. Under a minimum revenue guarantee, the government pays a concessionaire the difference between revenue and a guaranteed minimum. For a privately financed road linking Seoul to the new airport at Incheon, the government guaranteed 90 percent of a 20-year revenue forecast; when the road opened in 2000, traffic revenue was less than half of the forecast, and the government has had to pay tens of millions of dollars every year (Irwin, 2007, pp. 1–2). South Korea abolished minimum revenue guarantees for new unsolicited projects in 2006 and for new solicited projects in 2009 (Kim et al., 2011, pp. 20–22). Under LPVS, the awarded cap fixes at award the largest present value of support to which the DFI is exposed.

Contestable markets theory (Baumol et al., 1982) demonstrates that price can be driven toward the competitive level by potential competition even in markets served by only a few firms, as long as entrants can enter at low cost. Incumbent firms that have already entered absorbed the sunk costs and therefore are more likely to exhibit low marginal costs of bid preparation. Thus, a small observed bidder pool may lead to a price near the competitive level if other firms could enter and bid at low cost.

Scaling Solar records participation from responses to the request for qualifications through prequalification, final proposals, award and implementation. Zambia Round 1 received seven final proposals from eleven prequalified firms. Senegal received proposals from eight of thirteen firms, with fourteen bids across two sites. Together, the two tenders yielded final proposals for 15 of 24 prequalified bidder opportunities, or 62.5 percent. These figures do not provide independent probabilities of firm-level participation. Other recovered records cover Zambia Round 2, Madagascar, Ethiopia and three opportunities from Uzbekistan. The number of final bidders was reported for five opportunities, with 5 to 8 bidders per opportunity. Ethiopia subsequently awarded the project and signed a power purchase agreement (PPA) in 2019 (Ethiopia Ministry of Finance, 2021). Table 2 summarizes recorded procurement stages and unresolved outcomes. Participation under LPVS terms is yet to be assessed.[4]

[4]Tender-level conversion rates were 63.6 percent in Zambia Round 1 and 61.5 percent in Senegal. ACWA won Gad and Dicheto, each 125 megawatts, and signed their PPAs in December 2019 (African Energy, 2019; ACWA Power, 2019a;

Table 2. Scaling Solar Participation and Recorded Procurement Stages

Country	Entry	Year	Interest	Prequalified	Final bidders	Recorded outcome
Zambia	Round 1	2016	48	11	7	Award
Senegal	Round 1	2018	28	13	8	Award
Zambia	Round 2	2017	21	12	n.r.	Later outcome unverified
Madagascar	Round 1	2018	n.r.	6	n.r.	Request for proposals in preparation, 2020
Ethiopia	Round 1	2019	28	12	n.r.	Award and PPA signing
Uzbekistan	Navoi	2019	n.r.	11	5	Award
Uzbekistan	Samarkand	2021	n.r.	15	6	Award
Uzbekistan	Jizzakh	2021	n.r.	15	7	Award

Notes: This table audits eight recovered procurement entries, not a complete universe of Scaling Solar tenders. Entries do not have identical units: Zambia Round 1 and Senegal cover two sites each; Uzbekistan's 2021 entries are separate sites sharing a qualification pool. Interest denotes responses to the request for qualifications (RFQ), prequalified denotes the screened pool, and final bidders denotes firms submitting final proposals where the sources establish that count. n.r. means not reported or not verified; it is not zero. Senegal's eight firms submitted fourteen bids across two sites. Zambia Round 1 and Senegal yield fifteen final bidders across twenty-four prequalified bidder opportunities (62.5 percent). These are descriptive opportunities, not independent firms. No pooled confidence interval is reported. Ethiopia's 2019 award and PPA signing contradict classification as a tender that never reached bidding. Award is not evidence of subsequent financial close, construction or operation. Madagascar's preparation status is the October 2020 Implementation Completion Report (ICR) snapshot, not a current status assertion. Suspension causes and the timing of IFC's withdrawal in Ethiopia remain unverified. The record does not identify the binding constraint across the program or a fragility effect.
Sources: IDC Zambia account of the first Scaling Solar auction (Industrial Development Corporation Zambia, 2018) and PV Tech report of the Round 2 prequalification (Kenning, 2017); Senegal award announcements of April 5, 2018 (Private Infrastructure Development Group, 2018) and April 10, 2018 (International Finance Corporation, 2018); Madagascar and Ethiopia prequalification announcements (February 28 and March 23, 2018), reported by PV Tech (Kenning, 2018b,a); IFC Scaling Solar presentation (May 2018); World Bank Zambia ICR (October 2020); ACWA Power Ethiopia award announcement of October 28, 2019 (ACWA Power, 2019b); Ethiopia Ministry of Finance account of the December 2019 PPA signing (Ethiopia Ministry of Finance, 2021); IFC Uzbekistan award announcements of October 4, 2019 and May 20, 2021 (International Finance Corporation, 2019, 2021); Uzbekistan Ministry of Energy announcement of the 15 prequalified bidders for Samarkand and Jizzakh (December 2020) (Ministry of Energy of the Republic of Uzbekistan, 2020).

3. Related Literature

The problem of allocation through negotiation has received ample scrutiny in the infrastructure concession literature. LPVR-type mechanisms were developed in response to demand uncertainty and incomplete contracting problems, two issues that bilateral allocation addresses not so well (Engel et al., 2001; Klein, 1998; Guasch, 2004).

One key result supporting the argument below is from Bulow and Klemperer (1996). A seller

Ethiopia Ministry of Finance, 2021). Missing final-bid counts elsewhere do not establish failure before bidding. The Ministry of Finance account of April 8, 2021 records the PPA signing: https://www.mofed.gov.et/blog/exploring-utilizing-sustainable-energy-sources-meet-economic-development-needs/. ACWA Power's December 22, 2019 release names both sites and records signing: https://acwapower.com/en/media-center/latest-news/acwa-power-to-drive-solar-energy-development-in-ethiopia/. African Energy reports the named-project award on September 27, 2019: https://www.africa-energy.com/news-centre/article/ethiopia-acwa-power-awarded-projects-after-record-bid.

faces N bidders. Each bidder has an independent private value for the good. Each bidder knows their own valuation, and valuations are independently distributed across bidders. The result compares two strategies for raising the seller's expected revenue: They can run the most elaborate selling mechanism that game theory allows—including reserve prices, threats to keep the object unsold, carefully orchestrated bargaining, and so on—and invite N bidders to the auction. Alternatively, the seller does away with much of that sophistication and runs an auction with no reserve price but adds one additional bidder (N+1). Bulow and Klemperer show that the latter strategy yields higher revenue. That extra bidder is worth more than any refinement of selling strategy among the original N.

Bilateral negotiation is a degenerate auction with one bidder. The Bulow and Klemperer results discussed above indicate that, in their setting, even modest competition improves on that benchmark without requiring the DFI to know things it cannot know.

Hong and Shum (2002) estimate a model of auctions with both common- and private-value cost components using data from procurement by state governments. They find that for a substantial subset of auctions in their sample, the median amount paid by the government increases with the number of bidders.

Those results concern common-value elements of cost, which the private-value environment of Section 4 excludes; common values are among the deferred questions (Section 8.2).

Using negotiations to set subsidies is most of what DFIs do and there are many cases in which negotiations beat auctions. Bajari et al. (2009) study private sector building contracts in Northern California and find that buyers have a rational basis for preferring negotiation over an auction when the terms of the contract cannot be fully specified in advance. An auction can compare sellers' offers only for the object described by the buyer. When the object to be purchased is imperfectly described, the low bid tells the buyer nothing; they would do better negotiating directly with the single seller they judge capable of completing the project. Sponsor-led development finance fits this characterization perfectly. Only one firm shows up with a project (which no rival is currently proposing); there is no object for competitors to bid on; there are no competitors to bid. For such projects, negotiation is not a deficiency to be corrected but the appropriate procedure. In what cases auctions are better than negotiations is taken up in Section 8.1.

Klein (1998) explains why renegotiation was so ubiquitous: even well-designed concession contracts usually require adjustment during their lives, and much of the remaining renegotiation could have been avoided with greater care in contract design. Klein argued that renegotiation takes place in a quasi-bilateral monopoly, since the concessionaire can negotiate only with the government; the contract cannot specify how to deal with every eventuality that might arise; and the combination of the two means that ex post, each party has incentives to take advantage of the other's inability to leave the relationship if circumstances change. Modifying what firms bid on was the solution Klein discussed for these contractual problems. By bidding the total revenue they require rather than a price or a term, firms allow the concession's duration to adjust to realized revenue.

That solution is Engel, Fischer, and Galetovic's (1996, 2001) Least Present Value of Revenue auction. Firms bid how much toll revenue they require in present value terms; lowest bidder wins; and the concession lasts until it has paid that revenue amount to the concessionaire. Low traffic lengthens the concession, high traffic truncates it, and in either case the firm ultimately walks away with the amount that it bid. Revenue shortfalls extend the concession's duration, reducing the incentive to renegotiate over lower demand.

The mechanism proposed in this paper applies this idea to the allocation of concessional support.

Under LPVR, the winning firm receives revenue directly from road users, while endogenous duration absorbs demand fluctuations.

Under LPVS, concessional support is provided, and the sponsor bids the present value of support required. The DFI makes payments against verified revenue shortfalls, and its exposure is limited to the cap awarded in the auction. The sponsor receives market revenue plus partial support from the DFI when revenue falls below the specified threshold, subject to the contractual limits. Some of the properties of LPVR carry over, but their applicability depends on features of the highway model absent from concessional finance. Table 3 summarizes the differences between the two mechanisms.

Table 3. Structural Comparison: LPVR and LPVS Mechanisms

Property	LPVR (Engel et al.)	LPVS (This Paper)
Bidding variable	Present value of toll revenues (V_R)	Present value of support accepted (V_S)
Termination rule	Franchise ends when $\int P_t Q_t e^{-rt} dt = V_R$; the bid is the stopping variable	Concession ends at $T = \min\{\tau_W, \tau_A, \bar{T}\}$; the awarded cap limits support but does not terminate the concession
Residual claimant	Concessionaire: low demand → longer franchise, capital tied up	Sponsor: low performance → longer concession, equity tied up; DFI exposure is limited to the awarded cap and does not lengthen with duration
Demand risk bearing	Direct: concessionaire's revenue varies with demand	Indirect: sponsor bears via co-financing exposure and opportunity cost
Price structure	Concessionaire collects at regulated tolls	Sponsor operates under PPAs or regulated tariffs

Notes: "Direct" risk bearing refers to balance-sheet exposure; "indirect" refers to exposure through co-financing and reputational channels.

Economists have long debated whether auctions are appropriate for awarding long-term contracts. Demsetz's (1968) proposal to auction contracts begins with the premise that government does not need to regulate a monopoly if it can auction away the right to be a monopolist: firms bid away the monopoly profit in the franchise, and the winner provides the service at roughly competitive prices. Williamson (1976) applies this idea to long-term contracts and argues that franchise bidding will not work in this setting. The auction disciplines the award price, but the contract lasts for decades. Circumstances change and no contract accounts for every possible contingency, so each renegotiation between the government and the firm awarded the franchise involves bilateral bargaining between mutually dependent parties. The firm has assets specific to the project (sunk costs), and government relies on the service provided by those assets. Neither the firm nor government can credibly walk away from the relationship, so they continually renegotiate the terms of the contract that governs their relationship. In that setting the firm's private information about its costs puts it at an advantage. Auction competition at award turns into monopoly at execution, and monopoly means government must negotiate with an incumbent. Williamson's critique raises a question for development projects involving specific, sunk assets: under which conditions does the critique apply?

Riordan and Sappington (1987) examined the conditions under which franchise bidding is feasible. Riordan and Sappington's result means Williamson's failure is a problem with the conditions of contract, not a property of contract itself. Where the thing being franchised can be described fully enough that competing bids are meaningful comparisons, and where there are enough qualified bidders that the award is competitive, franchise bidding produces the efficiency it originally promised;

where either of those conditions is absent, apply Williamson's critique. Riordan and Sappington replaced Williamson's general presumption against franchise bidding with a narrow case where it applies. Those two conditions define the boundary between the two columns in Section 8.1. And within that boundary Williamson's critique, far from undermining franchise bidding, supports it: his concern was that long-term contracts devolve into the bilateral bargaining he describes. Development finance, though, has always allocated its resources through bilateral bargaining. The costs Williamson foresaw are the costs generated by the current system. Guasch (2004) documents renegotiation after award, including competitively awarded concessions. His findings do not establish whether auctions or negotiations are preferable at award. Auctions represent not a change in the basic technology of procurement so much as its correction: where Riordan and Sappington's conditions are met, they introduce no new problem of bilateral negotiation and offer an alternative to the status quo.

4. Contract and Environment

4.1 Sponsors, revenue and contract terms

The auction has $n \geq 2$ prequalified sponsors bidding for a standardized project. Before bidding, the DFI announces the common contract terms $(W, R^*, \kappa, \bar{s}, r_c, r_f, \bar{T})$ and a maximum admissible cap $\bar{V}$. Sponsor i's private project parameter $\theta_i = (c_i, \rho_i, \omega_i)$ determines its operating cost c_i per period, its discount rate ρ_i and its outside option ω_i. Revenue $R_t = R(z_t)$ in periods $t = 1, \ldots, \bar{T}$ is common to all sponsors, and z_t denotes the country shock process, a Markov chain on a finite set Z. The sponsor incurs the sunk capital cost I at award and receives terminal asset value Λ_W at handover.

The shock process $\{z_t\}$ and its distribution are common knowledge. Sponsors' project parameters θ_i are independently distributed, so values are private and independent conditional on this public environment. Bilateral pricing still requires the individual parameter θ_i, which the DFI does not observe. Auction bids incorporate sponsors' private parameters and their assessment of the common shocks.[5]

The coverage ratio satisfies $\kappa \in (0, 1]$, caps are nonnegative, and the sponsor's upfront investment satisfies $I > 0$. Support is paid against verified shortfalls, up to a per-period limit and the remaining cap. With $g_t = \min\{\bar{s},\ \kappa \max(0, R^* - R_t)\}$, support in period t and cumulative discounted support are

$$s_t = \min\left\{g_t,\ (V - S_{t-1})\,(1 + r_f)^t\right\}, \qquad S_t = S_{t-1} + s_t\,(1 + r_f)^{-t}, \qquad S_0 = 0. \tag{1}$$

The cap applies to gross disbursement and does not revolve. Before settlement, $S_t = \min\{V, G_t\}$, where $G_t = \sum_{k \leq t} g_k (1 + r_f)^{-k}$ is the discounted support that the rule would pay without a cap. The core contract excludes the recovery of arrears: the DFI holds no reimbursement claim on payments that the offtaker makes after support has been paid (recovery is the subject of Section 7).

The contract disentangles two objects which get fused together by a fixed-term subsidy: the value of compensation that the sponsor is entitled to accumulate, and the amount of concessional support to which the DFI is exposed. Support payments count toward the sponsor's accumulated compensation, which determines the concession's duration. Subject to the contractual limits, the

[5] Private signals about an unobserved country fundamental introduce common-value informational asymmetry. Winning conveys information about the winner's signal and induces an upward adjustment of procurement bids (Hong and Shum, 2002). Section 8.2 lists common values among the deferred questions.

concession ends upon reaching the compensation target, while cumulative concessional support is limited to the awarded cap.

4.2 Accumulation, settlement and termination

Pre-settlement accrual and the date at which the compensation target is reached are

$$\widetilde{A}_t = \sum_{k=1}^{t} (R_k - c_i + s_k)\,(1 + r_c)^{-k}\,, \qquad \tau_W = \min\{t : \widetilde{A}_t \geq W\}. \tag{2}$$

Before τ_W the sponsor retains $x_t = R_t - c_i + s_t$, and accumulated compensation is $A_t = \widetilde{A}_t$. At τ_W the sponsor retains only $x_{\tau_W} = (W - A_{\tau_W - 1})(1 + r_c)^{\tau_W}$, so that $A_{\tau_W} = W$. Settlement reduces the support payment first. Under a common contractual rule, the asset owner or successor receives the excess operating cash. Only support actually disbursed counts against the cap.

At the end of each period $t < \bar{T}$ in which the target is not reached, the sponsor may exit. An exit policy σ is a stopping rule adapted to the history of states, and τ_A denotes the exit date it selects. The concession ends at $T = \min\{\tau_W, \tau_A, \bar{T}\}$; reaching the target takes precedence over exit and expiry in the same period. The sponsor then receives a terminal value Θ_T, equal to Λ_W at handover, $\Lambda_{\bar{T}}$ at expiry and L at exit. The residual cash passed to the asset owner at settlement is separate from Λ_W. Along a state path ζ and under exit policy σ, the payoff is $\pi_i(V; \sigma, \zeta) = \sum_{t \leq T} x_t (1 + \rho_i)^{-t} + \Theta_T (1 + \rho_i)^{-T}$. The value of the contract to the sponsor is

$$U_i(V) = -I + \max_{\sigma} \mathbb{E}\left[\pi_i(V; \sigma, \zeta)\right], \tag{3}$$

the participation set is $P_i = \{V \leq \bar{V} : U_i(V) \geq \omega_i\}$, and the minimum acceptable cap is $V_i^* = \min P_i$ where the minimum exists. With Z and $\bar{T}$ finite, the set of exit policies is finite and the maximum exists.

If termination is triggered when the compensation target is reached, so that $T = \tau_W$, then the support payment possible in the final period is capped by the amount of compensation remaining after net operating cash: support falls to the amount needed to bring A_T to W. If net operating cash exceeds the remaining compensation required, no support is paid and retained operating cash is split up ("prorated") so that A_T still equals W. Termination at τ_A or at $\bar{T}$ does not involve any proration, and the accumulated compensation upon termination is then in general below W. If there is no proration at τ_W, a larger support cap might cause the concession to terminate a period earlier and the sponsor to lose some operating cash flow. In that case, sponsor value would not be weakly increasing in the cap, and the above-defined participation threshold would be invalid.

The DFI determines the compensation target W by marking up the appraised capital cost I of the standardized project. Tender documents specify W for the life of the contract on an equal basis for all bidders, regardless of country-shock distributions. Sponsors internalize gaps between realized and appraised capital costs and maintain incentives to minimize construction costs. Bilateral estimation of support needs requires the sponsor's private parameters as well as the shock distribution. Competition limits the support awarded through bids incorporating both. The model's informational advantage concerns the private parameters, since the shock distribution is common knowledge. The DFI still needs to determine verifiable contractual quantities.

When the support cap is exhausted at $\tau_V = \inf\{t : S_t = V\}$, support ends. The concession continues to build up compensation toward W out of project cash flow no longer supported. The

sponsor exits at τ_A if the continuation value becomes less than the exit value. DFI exposure is capped at V. Exhaustion of the cap ends the DFI's payment obligation but provides no evidence of a breach and leads to neither penalties nor transfer. The concession agreement continues on its original terms; breach, insolvency and service interruption are governed by separate clauses. The annex of the companion policy paper (González, 2026) sets out clauses of this kind for a pilot, including termination and transfer to a replacement operator after an uncured breach. Exit by the sponsor is still possible, and after support ends the residual risk of failure remains with the project and the users of its service. Revenues consistently above R^* result in no support payments and allow for earlier completion.

Partial coverage retains the duration response (within the supported range) and the effort incentives. A negative revenue realization lowers R_t and raises s_t. Since $\kappa < 1$ and there is a per-period limit $\bar{s}$, only part of the shortfall is replaced with support. Net accumulation decelerates and τ_W rises. Full coverage would remove this duration response while support remained unconstrained. Duration would only respond to revenue above R^*, a binding per-period limit, or exhaustion of the cap.

The sponsor discounts cash flow at the private rate $\rho_i > r_c$, while compensation accrues toward W at r_c. Delayed completion thus imposes a cost on the sponsor but little cost to the DFI, even after an aggressive winning bid followed by poor performance. Setting r_c substantially below the sponsor's cost of capital would systematically undercompensate the sponsor and raise bids. The fiscal discount rate r_f governs how DFI expenditures are valued and enters the stopping rule only through the limit on cumulative support.

Shortfalls verified as within the control of the operator are excluded from the payment rule. Revenue lost because plant availability falls below the contracted standard, or through deficiencies in billing and collection, is treated as received in the shortfall calculation, although no corresponding cash accrues to the sponsor. Without this exclusion, a guarantee of revenue shortfalls would weaken the incentive of the operator to maintain collection effort.

4.3 Measures

Committed exposure is the awarded cap. Expected expenditure is $E[S_T]$, with S already discounted at r_f, reported per procurement opportunity (zero when there is no award) and conditional on award. Sponsor surplus is U_i at the awarded cap less ω_i. The operating outcomes are the probabilities that the concession ends by reaching the compensation target at τ_W, by the sponsor's exit at τ_A and at the duration ceiling $\bar{T}$, together with expected operating periods under the sponsor. Reaching W triggers contractual handover, reaching $\bar{T}$ ends the concession possibly with compensation below W, and exit records the sponsor's early departure. Neither reaching W nor reaching $\bar{T}$ by itself establishes successful delivery. Operation after any of these events lies outside the core model. Continuity of service would require assumptions about the financing and operating capacity of a successor.

4.4 Why contingent support

The paper restricts attention to support that is paid only when verified revenue falls below a threshold. The restriction follows existing practice in part: according to the IDA20 Mid-Term Review, the window supports private investment through the International Finance Corporation

(IFC) and the Multilateral Investment Guarantee Agency (MIGA) "by backstopping or blending with IFC investments or MIGA guarantees" (International Development Association, 2023, p. 37), and its guarantee instruments pay only when an adverse event occurs. A comparison with upfront subsidies, which may reduce project debt or finance a liquidity reserve, requires a specification of those alternatives and is deferred.

4.5 A worked example

Consider a situation where the government has a goal of connecting a 50-megawatt solar power station to the national grid. Construction and operation by a private sponsor for the term of the concession are planned. Under a power purchase agreement (PPA), the private plant operator sells power to the state utility at the per kilowatt-hour price specified in the contract. The agreement is a take-or-pay contract: the utility owes payment for the contracted capacity whether or not demand requires the output of the plant, so that demand risk rests with the utility. With the contracted tariff, expected output, and assessed costs, the project can meet the minimum required return for the sponsor and clear the financing hurdle.

Given the country's recent history of conflict and severe economic downturns, the sponsor will expect some safeguards against unforeseen disasters to protect their investment. Anything that pushes realized revenues below what is needed to pay debt service on the concession and provide a return on equity will threaten the project company with bankruptcy. Because there is no back end to a fixed-term contract, there is no mechanism to make up shortfalls in revenues. The DFI must decide how much of that type of protection to offer. The DFI would estimate the appropriate level of support and offer that amount in bilateral negotiations. Under LPVS, the DFI turns that decision over to the sponsors.

Say two qualified sponsors decide to bid in the LPVS auction. Sponsor A bids $42 million, within the maximum admissible cap. Sponsor B bids $35 million. Sponsor B wins. The second-lowest bid becomes the maximum contractual payment amount, and $42 million (present value) is the committed DFI exposure for the life of the concession regardless of what happens. Support is not paid up front; disbursements are made as needed by the project and verifiable by a third party from the metering and payment records of the plant.

The threshold for contract viability is $26 million per year, which must cover debt service plus all operating costs. Revenue predicted under the PPA is just under $30 million per year. If the plant realizes revenues above $26 million in any year, the DFI pays zero. Below that amount, the DFI makes up 80 percent of the shortfall. Since the sponsor bears the remaining 20 percent, it has some incentives to run the operation well, and the whole deal is capped on a per-period basis.

For the first six years the DFI disburses nothing because the plant performs as expected. However, in year seven a fiscal crisis following a regional security incident leaves the utility able to pay only $20 million of the roughly $30 million it owes each year; the plant remains available and the invoices are undisputed. The DFI covers $0.8 \times (\$26\text{m} - \$20\text{m}) = \$4.8$ million. The same holds for years eight and nine before the utility resumes full payment. Discount those outlays back to award, at the DFI's cost of 5 percent, and the shock costs the DFI about $9.8 million in present value. That is well below the cap of $42 million.

Complete nonpayment over the same three years would produce a different result: 80 percent of a $26 million shortfall is $20.8 million a year, about $42.3 million in present value before any per-period limit, which would exhaust the $42 million cap.

A three-year collapse in revenues like this would prompt the sponsor to want to renegotiate the contract because its return does not cover its costs. When tied to a conventional fixed-term contract, the DFI may have no choice other than to concede and renegotiate, giving the sponsor better terms. The LPVS contract makes no provision for renegotiation, and the concession continues under its original terms. Absent renegotiation, a bad period lengthens the time the sponsor has in the concession to operate and earn toward the compensation target, subject to the pre-agreed duration limit and exit option. The shock gets absorbed over time rather than in larger payments. That time comes at a cost to the sponsor. The sponsor discounts expected future cash flows at 15 to 20 percent, while its compensation accumulates only at the contractual rate r_c, set below that discount rate; each period of delay therefore lowers the value of the compensation target to the sponsor.

The example leaves the compensation target unspecified and takes the two bids as given. The example therefore determines neither the duration of the concession nor the minimum acceptable caps of the sponsors. The figures for support assume that the concession runs beyond year nine, so that neither settlement nor exit ends support earlier.

5. Participation Threshold and Ranking

5.1 The participation threshold

The participation threshold and the reference bound rest on five assumptions.

A1 *Exogenous, observed revenue.* The state process does not depend on V or on the sponsor's actions, and the sponsor observes its history.

A2 *Rates.* $0 \le r_c < \rho_i$, so that $d_i \equiv (1+r_c)/(1+\rho_i) \in (0,1)$ and a nonnegative terminal value is worth more when received earlier.

A3 *Settlement.* Settlement at τ_W follows the support-first rule above, with no recoveries and no reimbursement claims.

A4 *Terminal values.* $\Lambda_W \ge 0$ and $\Lambda_W \ge \max\{\Lambda_{\bar{T}}, L\}$; all three are independent of V and of the termination date.

A5 *Cap independence.* Operating cost, financing terms and all other obligations of the sponsor are independent of V.

The argument compares two sequences of pre-settlement cash with the same exit date. Write $a_t = x_t(1+r_c)^{-t}$ for the accrual increment, so that $x_t(1+\rho_i)^{-t} = a_t d_i^t$, and use primes for the second sequence.

Lemma 1 (Pathwise dominance)**.** *Fix an exit date $\tau_A \in \{1,\dots,\bar{T}-1\} \cup \{\infty\}$ and two sequences of pre-settlement cash (y_t) and (y'_t), each settled at its own first passage to W. Under A2 and A4, if $y'_t \ge y_t$ for every $t \le \min\{T, T'\}$, then $\pi' \ge \pi$.*

Proposition 1 (Participation threshold)**.** *Under A1–A5, with Z and $\bar{T}$ finite:*

(a) for every state path and every exit policy, the payoff is weakly increasing in the cap, and U_i is weakly increasing in V;

(b) U_i *is right-continuous in* V*;*

(c) the participation set P_i *is either empty or the interval* $[V_i^*, \bar{V}]$*, and the minimum acceptable cap* V_i^* *is attained;*

(d) U_i *is constant for* $V \geq V^{\max} = \sum_{t=1}^{\bar{T}} \bar{g}\,(1+r_f)^{-t}$*, where* $\bar{g} = \max_{z \in Z} \min\{\bar{s}, \kappa \max(0, R^* - R(z))\}$.

The larger cap allows for more support under the shortfall rule and can postpone exhaustion of the cap. Support also speeds accumulation toward W, which the sponsor values if $\rho_i > r_c$. A bid below V_i^* risks receiving an award at a cap at which the payoff to the sponsor is less than its outside option. The minimum acceptable bid and the price of underbidding thus follow from the contract itself, without need to assume an additional penalty.

U_i can jump upward where a higher cap moves the handover date earlier. Thus U_i need not be left-continuous, and attainment requires right-continuity (proofs in Appendix A). The terminal-value step requires that rates be nonnegative (condition A2). With negative rates it is possible that a later terminal value is worth more than an earlier one, and the ordering fails (numerical example in Appendix A). Part (a) only establishes weak monotonicity, and the second-price rule of Section 5.4 uses the threshold in part (c) together with assumption A6 (Section 5.4).

5.2 A necessary condition for participation

The contract caps the sponsor's retained cash at the compensation target. This value is measured at the contractual rate. Discounting above the contractual rate implies a sponsor values this stream at less than W, regardless of the cap.

Proposition 2 (Reference bound)**.** *Suppose a state* z^N *yields the highest revenue* $R_N = \max_z R(z)$ *and* $R_N \geq R^*$*, and let the reference path be* $z_t = z^N$ *for all* t*. Under A1–A5, for every cap* V*,* $U_i(V) \leq U_i^{ref} \equiv -I + \max_{\tau_A} \pi_i^{ref}(\tau_A)$*, where* π_i^{ref} *is the payoff on the reference path, on which no support is paid. A nonempty participation set therefore requires* $U_i^{ref} \geq \omega_i$*.*

Corollary 1. *Suppose* $W \leq I$*,* $\Lambda_W = 0$*,* $\rho_i > r_c$ *and* $\omega_i \geq 0$*, with the other conditions of Proposition 2. Then* P_i *is empty for every cap.*

The wedge between ρ_i and r_c that penalizes delay also reduces the value of the target to the sponsor. For $\omega_i \geq 0$, a nonempty participation set implies either a compensation target above appraised capital cost, a terminal value retained by the sponsor, or both, sufficiently large to compensate for the gap between ρ_i and r_c over the length of the reference path. The compensation target is therefore a first-order design parameter.

5.3 Participation in a numerical environment

A computer implementation calculates the contract value exactly, by enumerating the 2^{20} histories of the two-state revenue process and solving the exit problem by backward induction. All parameters take assumed values; they take the values from the worked example where those are specified. Time is in annual periods, $\bar{T} = 20$; revenue is 30 in the normal state and 20 in the arrears state; $R^* = 26$; $\kappa = 0.8$; $\bar{s} = 10$; $r_f = 0.05$; arrears begin with probability 0.05 or 0.10 per year and persist with probability $2/3$, so that an episode has expected duration three years; $I = 20$; $\Lambda_W = \Lambda_{\bar{T}} = 0$; $\omega = 0$.

Exit is costless in the baseline calibration, so that $L = 0$, and incurs a termination penalty of 2 in one sensitivity run, so that $L = -2$. Sponsor types combine one of three operating costs $c \in \{24, 25, 26\}$ with one of four discount rates $\rho \in \{0.10, 0.12, 0.15, 0.20\}$, broader ranges than the worked example. With $c = 26$ and a concession lasting $\bar{T}$ years, the reference project yields a return of about 19 percent on I. Appendix B checks the ledger, settlement and stopping decisions.

With $r_c = 0.08$, the reference path requires a compensation target between 5 and 54 percent above I across the types for which a target suffices, and for the high-cost type at $\rho = 0.20$ no target suffices within $\bar{T}$ (Table D1). At $W = I$, the participation set is empty for all twelve types under both shock processes, as Corollary 1 requires (Table 4). Reference-case viability is necessary but not sufficient: in 10 of the 60 combinations of type, target and shock process in which the reference path meets the outside option, no cap restores participation. Support covers only part of each shortfall and does not compensate the sponsor for delay. For $W > I$, an interior minimum cap arises in 5 of 36 cells when arrears begin with probability 0.05 and in 13 of 36 cells when they begin with probability 0.10; in the other cells the sponsor either participates without support or rejects every cap. At the minimum acceptable cap, expected disbursement ranges from 21 to 74 percent of the cap across the 18 interior cases (Table D2). A termination penalty of 2 leaves the classification unchanged and moves no minimum cap by more than 0.4 (Table D3).

Table 4. Minimum acceptable cap V^* by sponsor type, compensation target and shock process (baseline: costless exit)

		Arrears entry 5 percent a year				Arrears entry 10 percent a year			
c	ρ	$W = 20$	27	30	34	$W = 20$	27	30	34
24	0.10	–	0	0	0	–	0	0	0
24	0.12	–	0	0	0	–	0	0	0
24	0.15	–	0	0	0	–	8.4	1.1	0
24	0.20	–	–	13.4	0	–	–	–	11.4
25	0.10	–	0	0	0	–	1.9	0	0
25	0.12	–	0	0	0	–	8.4	3.0	0
25	0.15	–	11.4	0	0	–	–	14.7	7.2
25	0.20	–	–	–	–	–	–	–	–
26	0.10	–	0	0	0	–	14.8	9.7	6.5
26	0.12	–	8.5	1.8	0	–	–	20.9	13.7
26	0.15	–	–	–	10.8	–	–	–	–
26	0.20	–	–	–	–	–	–	–	–

Notes: Entries: 0, the sponsor participates without support; a number, the interior minimum acceptable cap V^*, in the same units as $I = 20$; –, the participation set is empty for every cap. $r_c = 0.08$, $R = (30, 20)$, $R^* = 26$, $\kappa = 0.8$, $\bar{s} = 10$, $r_f = 0.05$, $\bar{T} = 20$, arrears persistence $2/3$, $\Lambda_W = \Lambda_{\bar{T}} = 0$, exit value $L = 0$, $\omega = 0$. The column $W = 20$ is the case $W = I$. Every interior case passed the grid diagnostic before the threshold search. Source: `run_tractability.py`, exercise B.

5.4 Ranking under second-price procurement

A bid qualifies if it is no greater than $\bar{V}$. If there are two or more qualifying bids, the lowest bidder wins and is awarded a cap equal to the second-lowest qualifying bid; if there is one qualifying bid,

that bidder is awarded $\bar{V}$; if there are none, there is no award. Ties are broken randomly. A sponsor whose participation set is empty abstains. The ranking argument uses one assumption in addition to A1–A5.

A6 *Threshold payoff.* If $V_i^* > 0$, then $U_i(V_i^*) = \omega_i$.

With private values, an attained minimum acceptable cap, participation at every cap above it and A6, reporting the threshold is weakly dominant; thus, truthful reporting is an equilibrium in which the sponsor whose V_i^* is lowest wins. Assumption A6 matters only when the lowest rival bid equals V_i^*. If $U_i(V_i^*) > \omega_i$ at a positive threshold, a sponsor facing a rival bid of V_i^* obtains more from a slightly lower bid, which wins outright at the same cap, than from the truthful bid, which wins only the tie-break; truthful reporting is then neither weakly dominant nor an equilibrium at that profile. Because caps are nonnegative, no lower bid exists when $V_i^* = 0$, and A6 imposes nothing in that case. Proposition 1 does not imply A6, since U_i can jump upward at a cap that moves the handover date earlier. The following remark, proved in Appendix A, gives a sufficient condition for A6.

Remark 1 (Payoff at the threshold)**.** *Under A1–A5, with Z and $\bar{T}$ finite, if $L = \Lambda_W = \Lambda_{\bar{T}}$, then U_i is continuous in V, and A6 holds.*

The baseline numerical environment of Section 5.3 sets $L = \Lambda_W = \Lambda_{\bar{T}} = 0$, which meets the condition of Remark 1; the sensitivity run with a termination penalty, $L = -2$, does not meet it. Weak dominance permits other equilibria. The argument from weak dominance does not extend automatically if bids influence project selection under a budget constraint or the sponsor's other deals.

An analytical variant of the award rule incorporates the two safeguards proposed for a pilot in the companion policy paper (González, 2026). The DFI fixes and documents a maximum cap before inviting bids, independently of any submission, does not disclose it and does not adjust it during the tender, and a competitive award requires two qualified bids at or below that cap. If at least two bids qualify, the lowest bidder wins and is awarded a cap equal to the second-lowest qualifying bid; otherwise there is no competitive award. Ties are broken randomly. Fix the bids of the other sponsors and the maximum cap, and let m denote the lowest rival bid. If m exceeds the maximum cap, no submission by the sponsor produces two qualifying bids, and there is no award. If m does not exceed the maximum cap, a sponsor that wins is awarded the cap m whatever its own bid; truthful reporting secures the award when m exceeds V_i^* and avoids it when m falls below V_i^*, and when m equals V_i^* the truthful bid ties and the tie is broken randomly, as under the rule above. The argument for reporting the threshold therefore carries over to this variant under the assumptions of this section. Because the maximum cap is fixed before bids are invited and does not depend on the bids submitted, the argument holds for every value of the maximum cap. The exercises of Section 6 apply the rule stated above, under which a single qualifying bidder is awarded $\bar{V}$, and do not cover this variant. With recovery of arrears, the participation set need not be an upper interval, and the argument no longer establishes truthful reporting (Section 7).

5.5 First-price procurement

An equilibrium analysis of first-price procurement, which the paper defers, would assume that types differ in a single dimension, giving a monotone mapping from θ_i to V_i^*. This assumption permits analysis using the one-dimensional type $v_i \equiv V_i^*$, with distribution F on $[\underline{v}, \bar{v}]$ and density f.

Under first-price procurement, the lowest bidder wins and receives a support cap equal to its bid. With a symmetric increasing strategy β, a sponsor of type v submitting bid b earns

$$\Pi(b, v) = [1 - F\left(\beta^{-1}(b)\right)]^{n-1} [U(b; v) - \omega(v)] \tag{4}$$

The winning bid b limits a contingent stream of payments. Since b is not itself a payment, the winner's surplus is $U(b; v) - \omega(v)$, rather than bid minus cost. The first-order condition evaluated at $b = \beta(v)$ gives

$$\beta'(v) = \frac{(n-1)f(v)}{1 - F(v)} \cdot \frac{U\left(\beta(v); v\right) - \omega(v)}{U_V\left(\beta(v); v\right)} \tag{5}$$

subject to $\beta(\bar{v}) = V^*(\bar{v})$. The highest type bids its participation cap as its probability of winning approaches zero. For surplus linear in the bid, $U(b; v) - \omega(v) = b - v$ and $U_V \equiv 1$, the condition reduces to standard procurement and integrates to the familiar closed-form solution (Milgrom, 2004). The contingent payments considered here do not satisfy this linearity requirement. The differential equation holds only where $U(b; v)$ is differentiable in the bid and $U_V(\beta(v); v) > 0$ along the equilibrium path. Proposition 1 establishes weak monotonicity only, and only under A1–A5, which must hold across the caps and policies being compared; U is flat in V wherever a higher cap alters neither support payments nor continuation decisions, as occurs above the largest discounted support that the per-period limit and the duration cap permit. Within a flat range, a higher bid lowers the probability of winning without raising the payoff conditional on winning, so a bid in the interior of the range is weakly dominated by the bid at its lower end. Bids may then accumulate at that lower end, ties may occur with positive probability, and a strictly increasing equilibrium is not established. Sufficient conditions for $U_V > 0$ have not been established, so the differential equation applies only over ranges of bids on which $U_V > 0$, not throughout the admissible range, and the analysis does not characterize equilibrium on flat ranges.

5.6 Posted offers

Two posted offers serve as comparators, under the same contract and the same maximum admissible cap. Under the single offer, the DFI draws one sponsor from the same distribution of types as the auction bidders and offers it $\bar{V}$; the sponsor accepts if its minimum acceptable cap does not exceed $\bar{V}$ and then receives $\bar{V}$, and otherwise there is no award. The single offer coincides with the award rule of Section 5.4 applied to one bidder. Under sequential offers, the DFI offers $\bar{V}$ to up to n independently drawn sponsors, one after another, and awards the contract to the first sponsor that accepts. The single offer and sequential offers differ only in the number of sponsors approached. Sequential offers and the auction have access to the same number of sponsors, but use different rules to select the recipient and set the cap. Both posted offers are benchmarks at the ceiling on the cap, rather than optimized offers or negotiated support.

Remark 2. *Let a denote the probability that a sponsor drawn from the type distribution abstains and hold the contract and $\bar{V}$ fixed.*

(a) The auction with n bidders and sequential offers to n sponsors both result in an award with probability $1 - a^n$, which rises with n whenever $0 < a < 1$; the single offer is the case $n = 1$.

(b) Under sequential offers, for a type j with population probability p_j and $V_j^* \leq \bar{V}$, the probability that an award occurs and the first sponsor to accept is of type j equals $\sum_{k=1}^{n} a^{k-1} p_j = p_j(1 -$

$a^n)/(1-a)$, *so that conditional on award it equals* $p_j/(1-a)$, *the probability for a single sponsor conditional on acceptance. Sequential offers therefore have the outcomes of the single offer conditional on award, and their measures per opportunity equal* $1-a^n$ *times those outcomes. They have the award probability of the auction but not its outcomes conditional on award.*

(c) Conditional on award, the awarded cap does not exceed $\bar{V}$*, so committed exposure under the auction never exceeds that under a posted offer.*

(d) For every auction winner, the payoff at $\bar{V}$ *is at least the payoff at the awarded cap, by Proposition 1(a).*

Because sequential offers and the auction have the same award probability, each difference between them per opportunity equals $1-a^n$ times the corresponding difference conditional on award, and the two comparisons have the same sign. With access to sponsors matched, both describe the difference between the procedures. Remark 2 orders neither expected expenditure nor the operating outcomes; Section 6 compares those measures numerically for the designs examined.

6. Expenditure and Operating Outcomes

6.1 Design of the exercises

The exercises use the numerical environment of Section 5.3, with $r_c = 0.08$ and a maximum admissible cap $\bar{V} = 25$. The compensation target is a common tender term, with three scenarios, $W = 27$, 30 and 34, and the six designs combine the three targets with arrears entry of 0.05 or 0.10 a year. Bidder types are drawn independently, with equal probability for each of the twelve types. The pool retains all types: sponsors that participate without support, sponsors with a positive minimum cap, and sponsors that abstain because no admissible cap meets their outside option. Auctions have $n = 2$ or $n = 3$ bidders under the award rule of Section 5.4. Each sponsor bids its minimum acceptable cap. Under A6 these bids are weakly dominant (Section 5.4). The baseline, where $L = \Lambda_W = \Lambda_{\bar{T}} = 0$, meets the sufficient condition of Remark 1. The termination-penalty run considered in Section 6.4 does not, and the outcomes reported for that run describe allocations under threshold bids. All expectations over bidder draws are computed exactly over the 12^n profiles.

6.2 Minimum cap and expected expenditure

In the design with arrears entry of 0.10 and $W = 30$, under costless exit, the sponsor with operating cost 25 and discount rate 0.10 needs no support and bids zero, whereas the sponsor with operating cost 24 and discount rate 0.15 requires a cap of 1.125 (Table 5). In an auction between the two, the higher-cost sponsor wins and receives a cap of 1.125, which both sponsors accept. Under that cap, the winner's expected disbursement is 0.689, compared to 0.587 for the lower-cost sponsor. The winner's concession reaches the compensation target with probability 0.60 and ends by exit with probability 0.15, compared to 0.74 and 0.10 for the lower-cost sponsor. The sponsor with the lowest minimum cap need not therefore have the lowest expected expenditure. This comparison uses the cap awarded by the auction and relies neither on the common cap $\bar{V}$ nor on the exit-ordering condition of Proposition 3.

Fixing the discount rate and outside option, lower operating cost weakly increases the payoff at every cap. On a given revenue path, operating under the same cap and the same exit policy, the two

Table 5. Two-sponsor auction: both sponsors evaluated at the awarded cap (arrears entry 0.10, $W = 30$, costless exit)

				At the awarded cap				
c	ρ	Bid V^*	Outcome	$E[S]$	Target	Exit	Expiry	$U - \omega$
25	0.10	0.000	wins	0.689	0.60	0.15	0.25	1.23
24	0.15	1.125	accepts, loses	0.587	0.74	0.10	0.16	0.00

Notes: Second-price award between the two sponsors: the sponsor with the lower bid wins, and the awarded cap equals the other bid, 1.125. Both sponsors are evaluated at that cap. $E[S]$ is expected discounted disbursement; Target, Exit and Expiry are the probabilities that the concession ends by reaching W, by the sponsor's exit and at $\bar{T}$; $U - \omega$ is sponsor surplus, with $\omega = 0$. Source: `c3_selection.py`, saved type stage `results_c3/c3_type_stage.json`.

sponsors receive exactly the same vector of pre-settlement support payments through the earlier of their two termination dates; thus the lower-cost sponsor has at least as much pre-settlement cash in each of those periods; Lemma 1 then orders their payoffs on every path, and expectation and the maximum over exit policies preserve the order. Lower operating cost therefore weakly decreases the minimum acceptable cap, and the minimum caps associated with the twelve types are ordered consistently with operating cost in every design, under both exit specifications. The ordering is weak: any sponsor that needs no support bids zero, and any sponsor that abstains submits no bid; selection by minimum cap may thus pool sponsors with different operating costs, and strict sorting by operating cost need not obtain. When sponsors differ in discount rate as well, ranking by minimum cap need not respect the ranking by operating cost, as in the example above.

Proposition 3 in Appendix A.1 states that a sufficient condition for expected expenditure at a common cap to increase in operating cost is that the higher-cost sponsor does not exit before the lower-cost sponsor's concession ends. When exit is costless, the condition is violated for every pair of participating types with distinct operating costs, at both $\bar{V}$ and at every cap that the auction can award. Expected disbursement is still ranked according to operating cost for every such pair at every awardable cap in these designs.

6.3 Exposure, expenditure and operating outcomes under the award rule

Sponsors that need no support form the largest group in four of the six designs; between two and six of the twelve types abstain (Table D7). Conditional on receiving an award, a single sponsor qualifies in between 7 and 67 percent of cases and receives $\bar{V}$ when it does (Table D5). The expected value of the awarded cap is between 2.6 and 19.5, versus 0.1 to 4.6 for the winner's expected minimum cap. Expected disbursement under the awarded cap is between 0.63 and 6.28 versus 0.05 to 2.21 under the winner's own minimum cap. In ten of the twelve combinations of design and number of bidders, awards where a single qualifier participates account for between 54 and 81 percent of this difference. In the other two combinations, $W = 30$ and $W = 34$ with arrears entry equal to 0.10 and three bidders, they account for 39 and 34 percent, respectively. Conditional on receiving an award, a sponsor that needs no support but is awarded $\bar{V}$ draws support in years of arrears until the concession ends or its cap is exhausted. Expected disbursement in this case is between 3.1 and 9.7 depending on the operating cost and the design.

Contested awards always have at least two qualifying sponsors. For 11 to 91 percent of these

awards, the cap takes a value of zero and there is neither disbursement nor ranking by expenditure. For 7 to 61 percent of contested awards, the cap is positive and the sponsors who accept it do not all have the same operating cost. Among these awards, the winner's expected disbursement using the awarded cap is strictly the lowest in 55 to 100 percent of cases; there are no ties. The average excess of the winner's expected disbursement over the lowest such expected disbursement ranges from zero to 0.46 (see Appendix C).

Relative to the winner's own minimum cap, the awarded cap increases the probability of reaching the target by 0.02 to 0.12 and decreases the probability of exit by 0.01 to 0.06 in each design-number of bidders combination (Table D6). Under the awarded cap, a concession reaches the compensation target with probability between 0.65 and 0.90, terminates by exit with probability between 0.03 and 0.13, and expires prior to reaching W with probability between 0.07 and 0.22; with the winner's own minimum cap, these ranges are 0.57 to 0.85, 0.04 to 0.17, and 0.11 to 0.26. Expected expenditure increases or stays constant across awardable caps for every participating type in every design, as does payoff U; operating outcomes are not monotone in the cap for every type, however. With costless exit, the probability of reaching the target falls between two adjacent awardable caps for 13 of the 50 type-design combinations, by at most 0.028, while the probability of exit increases for 20 of the 50, by at most 0.053 (Table D18). With the termination penalty, no type flips participation category, minimum caps move by at most 0.39 and expected disbursement under the awarded cap changes by at most 0.04. The probability of exit under the awarded cap falls to between 0.008 and 0.022, and the probability of expiry before W rises to between 0.08 and 0.31.

6.4 The auction against posted offers

With access to sponsors matched, the auction has lower expected expenditure per opportunity than sequential offers in every combination of design and number of sponsors under costless exit, by 0.96 to 5.35, a lower probability of reaching the compensation target, by 0.01 to 0.09, and a higher probability of exit, by 0.01 to 0.07 (Table 6). Committed exposure per opportunity is lower by 4.1 to 22.3, sponsor surplus per opportunity by 0.20 to 1.56, and the probability of expiry before W is higher in eleven of the twelve combinations. For $W = 30$, arrears entry of 0.10 and two sponsors, both procedures result in an award with probability 0.889; per opportunity, expected expenditure is 7.44 under sequential offers and 5.58 under the auction, the probability of reaching the target 0.708 and 0.687, and the probability of exit 0.050 and 0.070.

Conditional on award, committed exposure equals 25 under posted offers, against an expected awarded cap of 8.6 to 19.5 with two bidders and of 2.6 to 15.1 with three. Expected expenditure ranges from 3.96 to 9.15 under posted offers and is lower under the auction in every combination, by 1.28 to 5.43, and sponsor surplus is lower by 0.27 to 1.57 (Table D11). The access term, sequential offers to n sponsors minus the single offer, raises expected expenditure per opportunity by 0.69 to 2.72, the probability of reaching the target by 0.12 to 0.33 and the probability of exit by 0.002 to 0.017; the rules term, the auction minus sequential offers, has the sign opposite to the access term for expenditure and the target and the same sign for exit (Table D12). With the two terms summed, the auction reaches the target per opportunity more often than the single offer in every combination, by 0.04 to 0.30, so that, in this split, the higher probability of reaching W relative to an approach to one sponsor comes from access to more sponsors. Expected expenditure per opportunity under the auction exceeds that under the single offer in two of the twelve combinations, both in the design with arrears entry of 0.10 and $W = 27$, in which half of the types abstain.

Table 6. Single offer, sequential offers and auction with two sponsors: outcomes per procurement opportunity (costless exit)

				Exposure	Expenditure	Operating outcomes		
Arrears entry	W	Procedure	Award	Cap	$E[S]$	Target	Exit	Expiry
0.05	27	Single offer	0.667	16.7	2.64	0.624	0.009	0.033
0.05	27	Sequential	0.889	22.2	3.52	0.832	0.013	0.044
0.05	27	Auction	0.889	13.1	2.26	0.803	0.024	0.062
0.05	30	Single offer	0.750	18.8	3.20	0.687	0.011	0.052
0.05	30	Sequential	0.938	23.4	4.00	0.858	0.014	0.066
0.05	30	Auction	0.938	11.1	2.07	0.812	0.031	0.095
0.05	34	Single offer	0.833	20.8	4.15	0.700	0.013	0.120
0.05	34	Sequential	0.972	24.3	4.84	0.817	0.015	0.140
0.05	34	Auction	0.972	8.4	1.84	0.754	0.052	0.166
0.10	27	Single offer	0.500	12.5	3.63	0.438	0.023	0.040
0.10	27	Sequential	0.750	18.8	5.44	0.657	0.034	0.059
0.10	27	Auction	0.750	14.6	4.49	0.643	0.041	0.066
0.10	30	Single offer	0.667	16.7	5.58	0.531	0.037	0.098
0.10	30	Sequential	0.889	22.2	7.44	0.708	0.050	0.131
0.10	30	Auction	0.889	15.6	5.58	0.687	0.070	0.132
0.10	34	Single offer	0.750	18.8	6.86	0.545	0.042	0.163
0.10	34	Sequential	0.938	23.4	8.58	0.681	0.052	0.204
0.10	34	Auction	0.938	13.3	5.51	0.636	0.099	0.202

Notes: Per procurement opportunity; every measure counts zero when there is no award. Single offer: $\bar{V} = 25$ offered to one sponsor drawn from the twelve types. Sequential: $\bar{V}$ offered to up to two independently drawn sponsors in turn, with the award to the first that accepts. Auction: two sponsors under the second-price award rule with the same $\bar{V}$. Award: probability of an award. Cap: expected awarded cap. $E[S]$: expected discounted disbursement. Target, Exit and Expiry: probabilities that the opportunity ends in a concession that reaches W, ends by the sponsor's exit, or reaches $\bar{T}$ before W. Three bidders and the termination penalty: Tables D10 and D14. Source: `c4_posted_offer.py`, `c4_sequential.py`.

Conditional on award, the difference between a posted offer (P) and the auction (A) in any measure m divides into a selection term and a cap term,

$$E^P\left[m_i\left(\bar{V}\right)\right] - E^A\left[m_w\left(V^A\right)\right] = \underbrace{E^P\left[m_i\left(\bar{V}\right)\right] - E^A\left[m_w\left(\bar{V}\right)\right]}_{\text{selection}} + \underbrace{E^A\left[m_w\left(\bar{V}\right)\right] - E^A\left[m_w\left(V^A\right)\right]}_{\text{cap}},$$

where i denotes the recipient of the posted offer, w the auction winner and V^A the awarded cap. Both terms of the difference in expected expenditure are positive in every combination: the selection term lies between 0.02 and 0.53, and the cap term between 1.07 and 5.07, or 84 to 99 percent of the difference in this accounting decomposition (Table D13). Evaluated at $\bar{V}$, the auction winner has a lower expected disbursement than the recipient of a posted offer, a higher probability of reaching the target, by 0.004 to 0.063, and a lower probability of exit, by 0.0002 to 0.011. For both operating outcomes, the selection term has the sign opposite to the total difference, and the cap term exceeds the total difference in absolute value: for the same winners, the probability of reaching the target is lower by 0.03 to 0.14, and the probability of exit higher by 0.01 to 0.08, at the awarded cap than at

$\bar{V}$. Lower disbursement therefore accompanies different operating outcomes, and an efficiency gain cannot be established just by the reduction in expenditure. For sponsor surplus, the selection term is negative and the cap term positive in every combination, the latter consistent with Remark 2(d).

With the termination penalty, the auction has lower expected expenditure per opportunity than sequential offers in every combination, by 0.95 to 5.31, and a lower probability of reaching the target, by 0.01 to 0.08; the difference in the probability of exit lies between -0.004 and 0.017, with the auction higher in eight of the twelve combinations (Tables D14 to D17).

7. Arrears Recovery

In the worked example in Section 4.5, the unpaid invoices are still owed. Under a contract with recovery, if the utility pays back some or all of the arrears, the generator uses the recovered amounts first to reimburse the DFI for the $4.8 million that it paid out for each year. This lowers the net cost to the DFI, but does not replenish the amount under the cap.

Recovery allows for a case where support paid against arrears adds to accumulated compensation, accelerates completion and is later repaid from collections taken after termination. In that case increasing the cap can reduce the sponsor's payoff. See Appendix A for a counterexample.

Recovery can defeat the step on which Proposition 1 is based. Lemma 1 ranks payoffs when cash received under the higher cap is not less in any period up to the earlier termination date and, under the core contract, the sponsor receives nothing after termination except for a terminal value that is unaffected by the cap (assumptions A3 and A4). In the counterexample, support applied to one period of nonpayment advances completion by one period, and settlement of that invoice (received after termination) goes first to the DFI; the sponsor's payoff is higher at the lower cap whenever the terminal value of the asset is less than about 354, against a compensation target of 50.8. The participation set P_i then need not be an upper interval: a sponsor may accept one cap and reject a higher one. Reporting the minimum acceptable cap need not be weakly dominant under second-price procurement, since the award can carry a cap above that minimum at which the payoff of the sponsor falls short of its outside option, and the ranking result of Section 5.4 is not established for a contract with recovery. The analysis leaves open whether a different accrual rule restores the ordering.

8. Scope and Conclusion

8.1 Scope conditions

Three things are required for competitive allocation to work, and they are exacting. The buyer must be able to specify what it is buying so that rival offers can be compared. Several sellers are ready and willing to supply what is specified. And the buyer has to be in a position to rank the offers on one metric, namely the support each would need, without some unintended bias against what they value but have left unspecified. The first two are the franchise-bidding conditions of Section 3.

Consider a sponsor that wants to build a shoe factory in a country where there are none. The DFI is not auctioning off "the right to build a shoe factory here." A pool of rivals must be ready and able to do the same thing for an auction to work well. The proposer cannot be divorced from the proposal because the value is in this particular sponsor's judgment about an opportunity that others

have missed and hence it is not a contestable good. This is the Bajari case (Section 3). The buyer needs to find ahead of time which party is uniquely suited to perform a task that no other can and negotiate with them. In these situations, negotiation dominates bidding; an auction cannot improve on the outcome of that negotiation.

Reference-case viability (Table 7) follows from the payment rule: the DFI pays when revenue falls below R^*.

Table 7. Scope Conditions: When LPVS Applies and When It Does Not

Applies when	*Does not apply when*
Revenue shortfalls are objectively verifiable by third parties	Revenue depends on sponsor-reported figures subject to manipulation
At least two qualified sponsors express credible interest	Markets are so thin that only one sponsor can plausibly bid
Project risks are predominantly flow risks that reduce revenue (offtaker arrears, demand shortfalls)	Stock risks dominate (asset destruction or expropriation more likely than a revenue shortfall)
The DFI can credibly commit to contract terms, including ending support payments when the cap is exhausted	The DFI faces political constraints preventing credible commitment to support caps (strategically visible projects, single-sponsor dependency, substantial host-government co-investment)
Duration flexibility is feasible: contracts can extend without unacceptable political exposure	The project cannot be specified in standard terms (first-of-a-kind projects)
Reference-case project economics support commercial participation, so that what deters the sponsor is the downside rather than the central case	The project carries a viability gap in the reference case and requires subsidy in ordinary states, not only in adverse ones

Notes: Conditions in the left column must all be satisfied; conditions in the right column each independently disqualify the project.

A project which supports commercial participation after the sponsor has priced downside risk needs no concessional support. LPVS addresses downside risk deterring participation in an otherwise viable reference case. An insufficient reference case requires a viability-gap base transfer in ordinary states, outside the LPVS cap. Auctioning such transfers is a separate application not examined here.

The DFI would assess reference-case viability with a standardized financial model using contracted tariff, expected output, appraised cost, financing terms and benchmark return. This assessment requires no shock distribution and does not recreate bilateral pricing's informational problem.

LPVS covers flow risks—those risks that are endemic such as exchange-rate fluctuations or demand shortfalls that decrease project revenue but do not destroy the asset. It does not cover stock risks, the terminal risks such as war or expropriation that terminate a project. Those are covered by MIGA, and any project for which that is the primary exposure is not suitable for LPVS.

Several conditions in Table 7 correspond to assumptions of the model. Assumption A1 requires revenue to depend neither on the cap nor on the sponsor's actions. The payment rule treats revenue lost through failures within the control of the operator as received, so such losses do not raise support (Section 4.2), and verification by third parties makes the shortfall contractible. The model treats the operating costs, discount rates and outside options of sponsors as private values, so that a

rival's bid carries no information about the sponsor's payoff. Assumption A2 requires the contractual accumulation rate to lie below the discount rate of every sponsor. Reference-case viability is the necessary condition of Proposition 2: the payoff of the sponsor on the path on which revenue stays at its highest level, at or above the viability threshold, must reach its outside option. By Corollary 1, a compensation target at or below appraised capital cost, with no terminal value and a nonnegative outside option, fails this condition for every sponsor. Even when these conditions are met, the winning sponsor receives the maximum admissible cap when only one bid qualifies. These situations represent 7 to 67 percent of awards across Section 6 designs. In the 1995 Chilean round of rural telephony subsidies, the Compañía de Telecomunicaciones de Chile (CTC) bid at or near the maximum available subsidy for 23 projects in provinces in which it faced no challengers; in 8 further projects, CTC and Chilesat both bid zero subsidy, and the projects were not awarded because the bidding documents contained no provision for the resolution of ties (Wellenius, 1997, pp. 2–3).

8.2 Deferred questions

Several questions are deferred: formal ambiguity, structural renegotiation and the soft contract of the status quo, a comparison with upfront subsidies, a first-price equilibrium, common values and the winner's curse, collusion, effort and moral hazard, and the effect of auction prices on other bilateral deals. Each of these questions would change the environment of Section 4 and requires its own analysis before it can qualify the results reported here. The measurement plan of the companion policy paper (González, 2026) provides for the comparison of auction bids with subsequent requests for bilateral support, which bears on the effect of auction prices on other bilateral deals. The choice of the coverage ratio κ, the accumulation rate r_c and the duration ceiling $\bar{T}$ is also left open.

8.3 Conclusion

The amount of support a sponsor requires depends on country risk and the sponsor's own parameters. In bilateral negotiation the DFI must assess the minimum support requirement without observing those private parameters. The sponsor has information about its costs, financing terms, operating capability and risk tolerance that is unobserved by the DFI, but under the paper's assumptions has no better information about the country shocks affecting both it and the DFI. A binding bid communicates the offer the sponsor is prepared to honor, which in turn reflects part of this private information. The LPVS auction allocates subsidies using those offers rather than a DFI estimate of each sponsor's minimum support requirement. Country-risk assessment is still needed for project appraisal and contract design.

The analysis shows that, under the core contract and conditional on participation being possible, the caps that a sponsor accepts constitute an interval with an attained minimum and that, when an award at that minimum leaves the sponsor no better off than its outside option, the auction orders sponsors according to their minimum support requirements in the truthful equilibrium of a second-price procurement auction. That ordering does not necessarily equal an ordering by expected expenditure, and, in the designs examined, assuming costless exit and that access to sponsors is held constant, the auction has strictly lower expected expenditure than sequential posted offers at the maximum cap, while the concessions it awards reach the compensation target less frequently and end by exit more frequently. Whether that trade-off improves on posted offers depends on how much the DFI values committed exposure, expenditure, and operating outcomes, which the analysis

leaves open.

A companion policy paper (González, 2026) sets out a proposal for a pilot of the mechanism under the twenty-first replenishment of IDA, together with contract provisions, eligibility tests and a plan for the measurement of participation, support prices, financial close, delivery and payment performance. The proposed pilot terms depart from the core contract analyzed here: the DFI is reimbursed first from arrears collected after support has been paid, the maximum cap is confidential, a competitive award requires two qualified bids at or below that cap, and the award rule remains to be fixed. Section 5.4 extends the ranking argument to the confidential maximum cap and the two-bid requirement. With the recovery rule, the argument of that section no longer establishes truthful reporting, as the counterexample of Section 7 shows, and the exercises of Section 6 do not cover the pilot terms.

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A. Proofs

Proof of Lemma 1.

Proof. Let $Z_t = A'_t - A_t$ and $z_t = Z_t - Z_{t-1}$. For every t before both settlements, $Z_t \geq 0$, because the pre-settlement increments are ordered. If τ_W were smaller than T', then at $t = \tau_W$ the second sequence would satisfy $\widetilde{A}'_t \geq \widetilde{A}_t \geq W$, so that $\tau'_W \leq \tau_W$. Because τ_A and $\bar{T}$ are common to both sequences, it follows that $T' \leq T$.

Case 1: $T' = T$. If the second sequence reaches the target at T, then $A'_T = W \geq A_T$. If it does not, neither does the first, and both remain unsettled at T. In either event $Z_t \geq 0$ for all $t \leq T$. Summation by parts gives

$$\sum_{t \leq T} \left(x'_t - x_t\right)(1+\rho_i)^{-t} = \sum_{t \leq T} z_t d_i^t = Z_T d_i^T + \sum_{t<T} Z_t \left(d_i^t - d_i^{t+1}\right) \geq 0,$$

since $d_i \in (0,1)$. The terminal values coincide when both sequences end by the same event; when only the second reaches the target, $\Theta'_T = \Lambda_W \geq \Theta_T$ by A4.

Case 2: $T' < T$. Then $T' = \tau'_W$, and the first sequence has not reached the target at T'. Let $M = W - A_{T'} > 0$, so that $Z_{T'} = M$ and $Z_t \geq 0$ for $t < T'$. Summation by parts over $t \leq T'$ gives $\sum_{t \leq T'} z_t d_i^t \geq M d_i^{T'}$. After T', only the first sequence continues. Let $B_t = A_t - A_{T'}$ for $T' < t \leq T$. Since $A_t \leq W$ for every $t \leq T$, $B_t \leq M$, and summation by parts gives

$$\sum_{t=T'+1}^{T} a_t d_i^t = B_T d_i^T + \sum_{t=T'+1}^{T-1} B_t \left(d_i^t - d_i^{t+1}\right) \leq M d_i^{T'+1}.$$

The difference in cash payoffs is therefore at least $M d_i^{T'}(1-d_i) > 0$. The difference in terminal values is $\Lambda_W(1+\rho_i)^{-T'} - \Theta_T(1+\rho_i)^{-T} \geq 0$, because $\Lambda_W \geq 0$, $\Lambda_W \geq \Theta_T$, $T' < T$ and $\rho_i \geq 0$. □

Proof of Proposition 1.

Proof. Support ordering. Fix a path and caps $0 \leq V \leq V'$. Before settlement, $s_t(1+r_f)^{-t} = \min\{V, G_t\} - \min\{V, G_{t-1}\}$, which is zero for $V \leq G_{t-1}$, equal to $V - G_{t-1}$ for $G_{t-1} < V < G_t$, and equal to $G_t - G_{t-1}$ for $V \geq G_t$. Each support payment computed without settlement is therefore a continuous, weakly increasing function of V. Hence $s'_t \geq s_t$ in every period up to and including the earlier of the two settlement periods, with payments measured before settlement, and the pre-settlement cash satisfies $y'_t = R_t - c_i + s'_t \geq y_t$. By A5 nothing else in the sponsor's cash depends on the cap.

(a) By A1, an exit policy is a rule on the history of states and can be applied unchanged under either cap. Lemma 1 gives $\pi_i(V'; \sigma, \zeta) \geq \pi_i(V; \sigma, \zeta)$ for every σ and ζ. Taking expectations and then the maximum over policies, $U_i(V') \geq \max_\sigma \mathbb{E}[\pi_i(V; \sigma, \cdot)] - I = U_i(V)$.

(b) Fix a path and a policy. Each s_t computed by the payment rule without settlement is a continuous, weakly increasing function of V, as shown above; so is each $\widetilde{A}_t$. If $\widetilde{A}_t(V_0) \geq W$, then $\widetilde{A}_t(V) \geq W$ for all $V \geq V_0$; if $\widetilde{A}_t(V_0) < W$, then $\widetilde{A}_t(V) < W$ for V close to V_0. Hence τ_W, the termination date and the terminal event are constant on a right neighborhood of V_0, and on that neighborhood the payoff is a continuous function of the support payments and the settlement amount. The path payoff is therefore right-continuous in V. The expected payoff under a policy is a finite sum of such functions, and U_i is the maximum over a finite set of policies of right-continuous, weakly increasing functions; it is therefore right-continuous.

(c) By (a), P_i contains every admissible cap above any of its members. If P_i is nonempty, let $V_i^* = \inf P_i$ and take a sequence in P_i decreasing to V_i^*. Right-continuity gives $U_i(V_i^*) \geq \omega_i$, so the infimum is attained.

(d) No path can draw discounted support above $V^{\max}$, so for $V \geq V^{\max}$ the cap never binds and the payments do not depend on V. □

Remarks on Proposition 1. (i) U_i can jump upward where a higher cap moves the handover date earlier, by one or more periods or from no handover to handover, and the value of handover exceeds the limit of what the sponsor would obtain without the earlier handover; with $\Lambda_W > L$ this can occur even when $\Lambda_W = 0$. U_i need not be left-continuous, and right-continuity is what attainment requires. (ii) The terminal-value step requires $\rho_i \geq 0$. With negative rates a later terminal value can be worth more than an earlier one, and the ordering fails: with $r_c = -0.20$, $\rho_i = -0.10$, $W = 1.5$, $I = 20$, $\Lambda_W = \Lambda_{\bar{T}} = 100$, $L = 0$, $c_i = 19$, revenue of 30 or 20, arrears entry of 0.05 a year with persistence $2/3$ from a normal initial state, $R^* = 26$, $\kappa = 0.8$, $\bar{s} = 10$, $r_f = 0.05$ and $\bar{T} = 2$, raising the cap from 0 to 0.3 brings handover forward and lowers U_i from 93.06 to 92.44. A2 excludes this case. (iii) Part (a) is weak monotonicity only. U_i is flat above $V^{\max}$ and may be flat elsewhere, so the condition $U_V > 0$ on which an equilibrium analysis of first-price procurement would rest is not established; the second-price rule requires the threshold in (c) and assumption A6. (iv) With invoice-level recovery, support paid against arrears can bring handover forward and be repaid after termination; the premise of Lemma 1 then fails, and the counterexample with recovery below shows that monotonicity can fail.

Proof of Remark 1.

Proof. The proof uses three features of the contract of Section 4. First, before termination the ledger along a path depends on the state history and the cap but not on the exit policy: s_t, the unsettled cash $x_t = R_t - c_i + s_t$ and the pre-settlement accrual $\widetilde{A}_t$ are functions of $(z_1, \ldots, z_t)$ and V. By A1 the sponsor observes $(z_1, \ldots, z_t)$ at the end of period t and can therefore compute $\widetilde{A}_t$ under any cap. Second, exit takes place at the end of a period: a sponsor that exits at t receives x_t, including the support s_t paid in that period, and then L, so that its payoff on the path is $\sum_{k \leq t} x_k (1+\rho_i)^{-k} + L(1+\rho_i)^{-t}$. Third, as shown in the proof of Proposition 1, each s_t, and hence each unsettled x_t and each $\widetilde{A}_t$, is a continuous, weakly increasing function of V.

By Proposition 1(a) and (b), U_i is weakly increasing and right-continuous. It therefore suffices to show that $\liminf_{V \uparrow V'} U_i(V) \geq U_i(V')$ for every $V' > 0$. Fix V' and an exit policy σ' that is optimal at V'. The pairs of a state history and a date are finite in number, and each $\widetilde{A}_t$ is continuous in V, so that there is $\eta > 0$ with the following property: for every $V \in (V' - \eta, V')$ and every history and date, $\widetilde{A}_t(V') > W$ implies $\widetilde{A}_t(V) > W$, and $\widetilde{A}_t(V') < W$ implies $\widetilde{A}_t(V) < W$. Fix such a V. Because $\widetilde{A}_t(V) \leq \widetilde{A}_t(V')$, the target is reached under V no earlier than under V'.

Define the exit policy σ_V at the cap V as follows: at the end of each period $t < \bar{T}$ in which the target has not been reached under V, the sponsor exits if σ' prescribes exit after the observed history, or if the target is reached at t under V'. The second condition depends only on $\widetilde{A}_t(V)$ and $\widetilde{A}_t(V')$, which are functions of the observed history. The policy σ_V is therefore a stopping rule adapted to the history of states, and it exits only at dates at which the contract permits exit under V.

Fix a path, and let T' denote the termination date under V' and σ'. Four cases exhaust the possibilities.

(i) Exit at T' under σ'. Then $T' < \bar{T}$ and the target is not reached at or before T' under V', and therefore not under V. The policy σ_V does not exit before T' and exits at T'. Under both caps the payoff consists of the unsettled cash in periods $1, \dots, T'$ and L at T'.

(ii) Expiry at $\bar{T}$ under V' without reaching the target. The target is not reached under V either, neither policy exits, and under both caps the payoff consists of the unsettled cash in periods $1, \dots, \bar{T}$ and $\Lambda_{\bar{T}}$ at $\bar{T}$.

(iii) Handover at T' under both caps. Neither policy exits before T'. Under each cap the payoff consists of the unsettled cash in periods $1, \dots, T'-1$, the settled cash $(W - \widetilde{A}_{T'-1})(1+r_c)^{T'}$ at T', with $\widetilde{A}_{T'-1}$ evaluated at that cap, and Λ_W at T'. This case includes handover at $T' = \bar{T}$.

(iv) Handover at T' under V' but not under V. By the choice of η, $\widetilde{A}_{T'}(V') = W$, so that the settled cash at T' under V' equals the unsettled cash $x_{T'}(V')$. If $T' < \bar{T}$, σ_V exits at T', and the sponsor receives the unsettled cash $x_{T'}(V)$ and then L at T'. If $T' = \bar{T}$, exit is not available, the concession expires under V, and the sponsor receives $x_{\bar{T}}(V)$ and $\Lambda_{\bar{T}}$ at $\bar{T}$. In both subcases the cash in periods $1, \dots, T'$ is the unsettled cash under each cap, and the terminal value, L or $\Lambda_{\bar{T}}$, equals Λ_W by hypothesis.

In every case the termination date and the terminal value are the same under the two caps, and the payoffs differ only through cash amounts that are continuous in the cap. Hence $\pi_i(V; \sigma_V, \zeta) \to \pi_i(V'; \sigma', \zeta)$ as $V \uparrow V'$ on every path. With finitely many paths, $U_i(V) \geq \mathbb{E}[\pi_i(V; \sigma_V, \cdot)] - I \to U_i(V')$, which gives the required inequality, and U_i is continuous. If $V_i^* > 0$, then $U_i(V) < \omega_i$ for every $V < V_i^*$, so that continuity gives $U_i(V_i^*) \leq \omega_i$, and attainment (Proposition 1(c)) gives the reverse inequality. □

Proof of Proposition 2.

Proof. On any path, $s_t = 0$ when $R_t \geq R^*$, and $s_t \leq \kappa(R^* - R_t) \leq R_N - R_t$ when $R_t < R^*$, because $\kappa \leq 1$ and $R^* \leq R_N$. Hence $R_t - c_i + s_t \leq R_N - c_i$ in every period. Fix a path and the exit date that an optimal policy selects on it. Lemma 1, applied with the reference cash as the larger sequence and the same exit date, gives $\pi_i \leq \pi_i^{ref}(\tau_A) \leq \max_\tau \pi_i^{ref}(\tau)$. Taking expectations completes the proof. □

Proof of Corollary 1.

Proof. By A4, $\Lambda_W = 0$ implies $L \leq 0$ and $\Lambda_{\bar{T}} \leq 0$. If $R_N \leq c_i$, the reference cash and every terminal value are nonpositive, and $U_i^{ref} \leq -I < 0$. Otherwise, the reference cash is positive in every period. Whatever the termination date, the accrual increments a_t are nonnegative and sum to at most W, so $\sum_t a_t d_i^t < \sum_t a_t \leq W \leq I$ because $d_i < 1$. Hence $U_i^{ref} < 0 \leq \omega_i$, and Proposition 2 gives the result. □

Counterexample with recovery. The example adds to the core contract the recovery rule of Section 7: a later collection of an invoice for which support was paid goes first to the DFI, up to the support paid, and the generator receives the remainder, including collections after termination. The viability threshold is $R^* = 26$, the utility owes 30 a period, $\kappa = 0.8$, $\bar{s} \geq 20.8$, $c_i = 0$, $r_c = 0$, $\rho_i = 0.05$, $W = 50.8$ and $\bar{T} \geq 3$, and the terminal value of the asset is L at any termination date, so that $\Lambda_W = \Lambda_{\bar{T}} = L$. The path of payments by the utility is deterministic and the same under both caps. The lower cap is $V = 0$, so that no support is paid under it. The fiscal discount rate is $r_f = 0.05$, so that support of 20.8 in nominal terms in period 1 uses $20.8/1.05$ of the cap at the

award date; the higher cap $V' \geq 20.8$ permits the full first-period support payment, and the same holds for any $r_f \geq 0$.

Period	Higher cap V'	Lower cap V
1: utility pays 0	support 20.8; $A = 20.8$	no support; $A = 0$
2: utility pays 30 (current invoice)	$A = 50.8 = W$; completion	$A = 30$
3: utility pays 30 (current invoice)	—	prorated to $A = W$; cash 20.8; completion
4: utility pays 30 (period-1 arrears)	DFI 20.8; generator 9.2	generator 30

Exit is assumed to yield L at the date of exit, with recovery entitlements unchanged. Continuation then dominates exit at every date under both caps whenever L is below 416, the binding comparison being continuation under V at the end of period 2, so the paths shown are optimal for the values of L considered here. With $\delta = 1/(1+\rho_i) = 1/1.05$,

$$U_i(V) - U_i(V') = 30\delta^2 + 20.8\delta^3 + 30\delta^4 + L\delta^3 - \left(20.8\delta + 30\delta^2 + 9.2\delta^4 + L\delta^2\right) = 15.27 - 0.0432\,L,$$

which is positive for any L below about 354. The higher cap brings completion forward with support that is later repaid, and the sponsor loses the operating period it would otherwise have had. Without recovery the same support would be a pure transfer, and the comparison would favor the higher cap.

A.1 Proposition 3 and the exit-ordering condition

Proposition 3. *Under A1 and A3, with revenue that depends on neither sponsor type nor effort, consider two sponsors with operating costs $c_L \leq c_H$, any discount rates and the same cap V. On a revenue path on which the higher-cost sponsor does not exit before the lower-cost sponsor's concession ends, cumulative discounted disbursement at termination is at least as high for the higher-cost sponsor. Disbursement does not depend on the discount rate, except through the exit date.*

Proof. Let T^L denote the date on which the lower-cost sponsor's concession ends. Before settlement, the payment rule depends only on the revenue path and on cumulative support, so the two sponsors receive identical payments in every period before T^L. During each of those periods, the pre-settlement cash of the lower-cost sponsor is at least as high as that of the higher-cost sponsor, so its accrual is at least as great, and the higher-cost sponsor cannot have met its compensation target before time T^L. By assumption it does not exit before T^L and $T^L \leq \bar{T}$, so it remains through T^L. If the lower-cost concession ends at T^L, either by exiting or by reaching $\bar{T}$, then both sponsors receive the same payment in that period. If it ends at its target date, then its payment during that period is reduced first under settlement; the remaining higher-cost sponsor receives during that period either that unsettled payment, or if it too reaches its target, a settled payment that is at least as large because its remaining compensation is at least as large and its operating cash is no larger. Any payments made to the higher-cost sponsor after time T^L are nonnegative, so its total payments are at least as great. The discount rate enters neither the payment rule nor the accumulation of the compensation amount, which is determined using r_c. □

Corollary 2. *Suppose that on every revenue path the higher-cost sponsor does not exit before the lower-cost sponsor's concession ends, as when neither sponsor exits or both follow the same exit rule on*

state histories. Then expected expenditure at every common cap is at least as high for the higher-cost sponsor and, for given exit dates, does not depend on either discount rate. The minimum acceptable cap depends on both operating cost and discount rate, so a sponsor with a higher operating cost and a lower discount rate can have the lower minimum cap while its expected expenditure at every common cap is at least as high.

For every pair of sponsors with different operating costs that accept a given cap, the exit-ordering condition is checked on all 2^{20} revenue histories. In the design with arrears entry of 0.10 and $W = 30$, the sponsors of the example in the main text have expected disbursements of 8.55 and 6.81 at the common cap $\bar{V}$ (Table D4). Their exit probabilities at $\bar{V}$ are 0.06 and 0.03, and the higher-cost sponsor exits before the lower-cost concession ends on histories with total probability 0.02. The pair therefore lies outside the premise of Corollary 2 and illustrates its pattern numerically.

Under costless exit, the exit-ordering condition fails for each of the 138 pairs of participating types with different operating costs at $\bar{V}$, and for each of the 464 combinations of such a pair with a cap that the auction can award: in every case, there are histories on which the higher-cost type exits before the lower-cost concession ends (Table D7 for the pairs at $\bar{V}$; `c3_exit_ordering.csv` for all combinations of pair and cap). For each pair, these histories have a total probability between 0.003 and 0.07 at $\bar{V}$, and of at most 0.15 at the lower caps. At $\bar{V}$, they are histories of late exit: the higher-cost type exits in years 15 to 19 of the 20-year ceiling, and in more than 96.9 percent of the probability mass the lower-cost concession then ends by a later exit or at expiry (`c3_exit_diagnostic.py`). The diagnostic does not date the exits at the lower caps. Proposition 3 therefore covers none of the baseline comparisons. Under costless exit, no restricted award satisfies the exit-ordering condition for every pair of sponsors (Table D8), and in none of the contested awards whose qualifiers differ in operating cost does the condition hold for every pair at $\bar{V}$ (Table D9).

With the termination penalty, the exit-ordering condition holds for 60 of the 138 pairs at $\bar{V}$ and for 232 of the 464 pair and cap combinations; it holds for every pair in 31 to 55 percent of contested awards whose qualifiers differ in operating cost, evaluated at $\bar{V}$, and in at most 61 percent of restricted awards.

B. Numerical Implementation

Checks of the ledger, settlement and stopping outcomes. All 31 tests in the test file pass. Four tests, in two test functions, concern Proposition 3. Two check the per-history stopping dates used in the exit-ordering check of Appendix A.1 against the solver's outcome probabilities and the one-path ledger. In the worked example, support is 4.8 a year in years seven to nine, cumulative discounted support reaches 9.754, and the sponsor's cash in each arrears year is -1.2. Under complete nonpayment and a per-period limit that does not bind, a cap of 35 lies below the present value of complete nonpayment and therefore binds. Support is 20.8 in years seven and eight, the remaining amount in year nine and zero in year ten. Cumulative support never exceeds the cap. At settlement, support falls first (from 4.8 to 2.0 in the test case), excess operating cash passes to the asset owner (2.0 in the test case), and accumulated compensation equals W exactly; on 200 random paths it never exceeds W. The tree computation matches a recursive computation with its own one-period ledger to within 10^{-10} for the payoff, expected disbursement and the probabilities of each terminal event, in cases with and without exit. The terminal probabilities sum to one, expected disbursement never exceeds the cap, and U is constant above $V^{\max}$. In 1,500 random draws in the

tests, and 4,000 further draws in a separate probe, of contract terms, types, paths, exit dates and pairs of caps satisfying A2 and A4, a higher cap never lowered the payoff under a fixed exit date. The reference bound of Proposition 2 holds at the four caps in the tests and, as $U(V^{\max}) \leq U^{ref}$, in all 192 cells of exercise B in the two runs. When A2 or A4 is dropped, the ordering fails: in 4,000 draws with $\rho < r_c$ a higher cap lowered the payoff in 56 draws, in 4,000 draws with $L > \Lambda_W$ in 4 draws, and a test reproduces the negative-rate case of remark (ii) on Proposition 1.

The model component (version 2026-11-04a) of the replication package contains the contract implementation and exact solver (`lpvs_core.py`); its tests (`test_lpvs_core.py`, `test_c4.py` and `test_c4_sequential.py`, 44 tests in all); the participation exercises (`run_tractability.py`) and the probe of assumptions A2 and A4 (`probe_assumptions.py`); the selection exercises and the exit-ordering check (`c3_selection.py` and `c3_exit_diagnostic.py`); the posted-offer comparisons (`c4_posted_offer.py` and `c4_sequential.py`); and the table scripts. Manifests record code hashes, software versions and the assumptions of each run. The package is archived at https://doi.org/10.5281/zenodo.24013716.

C. Selection Shares and Ties

Comparisons. Each comparison classifies the winner as strictly lowest in expected disbursement (below every other sponsor in the comparison by more than 0.01), tied (within 0.01 of the lowest other sponsor) or not lowest. At the awarded cap, the comparison set consists of the sponsors in the profile that would accept that cap, each evaluated at that cap, and the comparison is restricted to contested awards with a positive cap in which those sponsors do not all have the same operating cost. At the common cap $\bar{V}$, the comparison set consists of all qualifying sponsors in a contested award.

Selection at the awarded cap. In the restricted awards in which the winner is not strictly the lowest, its expected disbursement exceeds the lowest by more than 0.01, so that no restricted award produces a tie (Table D8). With two bidders, the share of contested awards in which the winner is not lowest equals the share won by a higher-cost sponsor with a strictly lower minimum cap (Table D9); with three bidders it is no larger, and smaller in five of the six designs, since a lower-cost qualifier whose minimum cap exceeds the awarded cap does not accept that cap and leaves the comparison.

Selection evaluated at the common cap. Evaluating every qualifying sponsor at $\bar{V}$ answers a hypothetical question, since the awarded cap equals $\bar{V}$ only when a single sponsor qualifies. In contested awards, the expected disbursement of the winner at $\bar{V}$ is strictly the lowest in 24 to 56 percent of cases and tied in 34 to 44 percent; every tie is with sponsors of the winner's operating cost (Table D9). In the remaining 6 to 37 percent, a sponsor with a higher operating cost wins, and the mean excess over the lowest, across all contested awards, ranges from 0.09 to 0.52. With arrears entry of 0.05, 60 to 100 percent of these awards arise from ties at a bid of zero among sponsors that need no support, against 0 to 47 percent with arrears entry of 0.10; the awarded cap is then zero, and no support is disbursed whichever sponsor wins. Awards in which a higher-cost sponsor wins with a strictly lower minimum cap, the pattern of Corollary 2, account for 0 to 13 percent of contested awards. The qualifying sponsors differ in operating cost in 61 to 80 percent of contested

awards.

D. Additional Tables

Table D1. Smallest compensation target, as a multiple of I, at which the reference path meets the outside option

		Contractual rate r_c			
Operating cost c	Discount rate ρ	0.06	0.08	0.10	0.12
24	0.10	1.10	1.05	1.00	0.96
24	0.12	1.16	1.10	1.05	1.00
24	0.15	1.26	1.19	1.13	1.07
24	0.20	1.48	1.39	1.31	1.24
25	0.10	1.12	1.06	1.00	0.95
25	0.12	1.19	1.12	1.06	1.00
25	0.15	1.32	1.24	1.16	1.09
25	0.20	1.68	1.54	1.42	1.32
26	0.10	1.15	1.07	1.00	0.94
26	0.12	1.25	1.16	1.07	1.00
26	0.15	1.46	1.33	1.22	1.13
26	0.20	n.a.	n.a.	n.a.	n.a.

Notes: Reference path: normal payment in every period, revenue 30, no support paid. $I = 20$, $\bar{T} = 20$, $\Lambda_W = \Lambda_{\bar{T}} = 0$, $\omega = 0$. n.a.: the reference payoff is negative even if the concession runs to $\bar{T}$, so no target suffices. Where $\rho = r_c$ the multiple is 1.00, and where $\rho > r_c$ it exceeds 1, as Corollary 1 requires. Source: `run_tractability.py`, exercise A.

Table D2. Exposure, expenditure and stopping outcomes at the minimum acceptable cap (interior cases, baseline: costless exit)

Arrears	W	c	ρ	V^*	$E[S_{T^*}]$	Target	Exit	Expiry	Periods	Cap used
5%	27	25	0.15	11.4	2.9	0.90	0.03	0.06	9.8	0.15
5%	27	26	0.12	8.5	2.9	0.76	0.11	0.13	13.2	0.22
5%	30	24	0.20	13.4	2.8	0.94	0.02	0.03	8.4	0.10
5%	30	26	0.12	1.8	0.8	0.64	0.13	0.23	14.8	0.46
5%	34	26	0.15	10.8	4.4	0.62	0.12	0.26	17.1	0.24
10%	27	24	0.15	8.4	3.4	0.87	0.05	0.08	9.1	0.29
10%	27	25	0.10	1.9	1.1	0.61	0.19	0.19	12.4	0.57
10%	27	25	0.12	8.4	4.1	0.73	0.13	0.14	11.7	0.34
10%	27	26	0.10	14.8	7.6	0.63	0.21	0.16	14.9	0.32
10%	30	24	0.15	1.1	0.6	0.74	0.10	0.16	11.4	0.52
10%	30	25	0.12	3.0	1.8	0.57	0.20	0.22	13.5	0.61
10%	30	25	0.15	14.7	6.6	0.76	0.11	0.13	12.7	0.25
10%	30	26	0.10	9.7	6.0	0.49	0.28	0.23	16.0	0.48
10%	30	26	0.12	20.9	9.8	0.53	0.15	0.32	16.4	0.18
10%	34	24	0.20	11.4	5.1	0.81	0.06	0.12	11.6	0.30
10%	34	25	0.15	7.2	4.4	0.58	0.18	0.24	15.0	0.48
10%	34	26	0.10	6.5	4.8	0.34	0.35	0.31	17.6	0.64
10%	34	26	0.12	13.7	8.6	0.35	0.27	0.38	17.8	0.37

Notes: V^* is the committed exposure if the sponsor receives its own minimum cap. $E[S_{T^*}]$ is expected discounted disbursement, already discounted at r_f. Target, Exit and Expiry are the probabilities that the concession ends by reaching W, by the sponsor's exit, and at $\bar{T}$; Periods is the expected number of operating periods under the sponsor; Cap used is the probability that disbursement reaches V^*. Reaching W or $\bar{T}$ ends the concession and says nothing about service afterward. Source: `run_tractability.py`, exercise B, baseline.

Table D3. Minimum acceptable cap V^* by sponsor type, compensation target and shock process (sensitivity: termination penalty, $L = -2$)

		Arrears entry 5 percent a year				Arrears entry 10 percent a year			
c	ρ	$W = 20$	27	30	34	$W = 20$	27	30	34
24	0.10	–	0	0	0	–	0	0	0
24	0.12	–	0	0	0	–	0	0	0
24	0.15	–	0	0	0	–	8.4	1.1	0
24	0.20	–	–	13.4	0	–	–	–	11.4
25	0.10	–	0	0	0	–	2.0	0	0
25	0.12	–	0	0	0	–	8.6	3.1	0
25	0.15	–	11.5	0	0	–	–	14.9	7.3
25	0.20	–	–	–	–	–	–	–	–
26	0.10	–	0	0	0	–	15.2	10.0	6.8
26	0.12	–	8.7	1.9	0	–	–	21.3	14.0
26	0.15	–	–	–	11.0	–	–	–	–
26	0.20	–	–	–	–	–	–	–	–

Notes: As in Table 4, except that exit carries a termination penalty of 2 ($L = -2$). Source: `run_tractability.py`, exercise B, run with `L_EXIT=-2`.

Table D4. Types in one design: minimum cap, expenditure and operating outcomes at the minimum cap and at the common cap (arrears entry 0.10, $W = 30$, costless exit)

			At own minimum cap				At common cap $\bar{V} = 25$			
c	ρ	V^*	$E[S]$	Target	Exit	Expiry	$E[S]$	Target	Exit	Expiry
24	0.10	0.00	0.00	0.74	0.08	0.17	6.81	0.94	0.02	0.04
24	0.12	0.00	0.00	0.74	0.08	0.17	6.81	0.94	0.03	0.04
24	0.15	1.13	0.59	0.74	0.10	0.16	6.81	0.94	0.03	0.04
24	0.20	abstains								
25	0.10	0.00	0.00	0.57	0.17	0.26	8.55	0.84	0.06	0.10
25	0.12	2.99	1.83	0.57	0.20	0.22	8.55	0.84	0.06	0.10
25	0.15	14.74	6.58	0.76	0.11	0.13	8.55	0.84	0.06	0.10
25	0.20	abstains								
26	0.10	9.70	6.01	0.49	0.28	0.23	10.47	0.53	0.09	0.38
26	0.12	20.87	9.82	0.53	0.15	0.32	10.47	0.53	0.10	0.37
26	0.15	abstains								
26	0.20	abstains								

Notes: $E[S]$ is expected discounted disbursement. Target, Exit and Expiry are the probabilities that the concession ends by reaching W, by the sponsor's exit and at $\bar{T}$. Source: `c3_selection.py`.

Table D5. Second-price award: exposure and expenditure (costless exit)

Arrears	W	n	Award	One qualifier	Cap	Own V^*	$E[S]$ at cap	$E[S]$ at own V^*	$E[S]$ per opportunity
0.05	27	2	0.89	0.50	14.7	1.5	2.54	0.45	2.26
0.05	27	3	0.96	0.23	8.3	0.9	1.55	0.27	1.49
0.05	30	2	0.94	0.40	11.9	0.8	2.21	0.20	2.07
0.05	30	3	0.98	0.14	5.2	0.4	1.04	0.09	1.02
0.05	34	2	0.97	0.29	8.6	0.4	1.89	0.16	1.84
0.05	34	3	1.00	0.07	2.6	0.1	0.63	0.05	0.63
0.10	27	2	0.75	0.67	19.5	4.6	5.98	2.21	4.49
0.10	27	3	0.88	0.43	15.1	3.7	4.89	1.78	4.28
0.10	30	2	0.89	0.50	17.5	4.2	6.28	2.16	5.58
0.10	30	3	0.96	0.23	11.9	2.8	4.54	1.44	4.37
0.10	34	2	0.94	0.40	14.2	2.7	5.88	1.61	5.51
0.10	34	3	0.98	0.14	8.1	1.6	3.72	0.96	3.66

Notes: n sponsors drawn independently and uniformly from the twelve types; exact expectations over all 12^n profiles. Award: probability that at least one sponsor qualifies. One qualifier: probability, given an award, that a single sponsor qualifies and therefore receives $\bar{V} = 25$. Cap and Own V^*: expected awarded cap and expected minimum cap of the winner, given an award. $E[S]$ at cap and at own V^*: the winner's expected disbursement under the awarded cap and under its own minimum cap, given an award. $E[S]$ per opportunity counts zero when there is no award. Source: `c3_selection.py`.

Table D6. Operating outcomes of the winner under the awarded cap and under its own minimum cap (costless exit)

			Under the awarded cap			Under own minimum cap		
Arrears	W	n	Target	Exit	Expiry	Target	Exit	Expiry
0.05	27	2	0.90	0.03	0.07	0.85	0.05	0.11
0.05	27	3	0.88	0.03	0.08	0.85	0.04	0.11
0.05	30	2	0.87	0.03	0.10	0.82	0.05	0.14
0.05	30	3	0.84	0.04	0.12	0.82	0.05	0.14
0.05	34	2	0.78	0.05	0.17	0.73	0.07	0.20
0.05	34	3	0.75	0.07	0.18	0.73	0.07	0.19
0.10	27	2	0.86	0.06	0.09	0.75	0.11	0.14
0.10	27	3	0.84	0.06	0.10	0.75	0.11	0.14
0.10	30	2	0.77	0.08	0.15	0.66	0.14	0.20
0.10	30	3	0.75	0.09	0.15	0.67	0.13	0.20
0.10	34	2	0.68	0.11	0.22	0.57	0.17	0.26
0.10	34	3	0.65	0.13	0.22	0.58	0.16	0.26

Notes: Probabilities, given an award, that the winner's concession ends by reaching W (Target), by the sponsor's exit (Exit) and at $\bar{T}$ before reaching W (Expiry). Reaching W or $\bar{T}$ ends the concession and says nothing about service afterward. Source: `c3_selection.py`.

Table D7. Sponsor pool by design: participation categories and expected disbursement at the common cap by operating cost

		Number of types			$E[S]$ at $\bar{V}$ by operating cost			Pairs of types at $\bar{V}$		
Arrears entry	W	No support	Positive V^*	Abstain	$c = 24$	$c = 25$	$c = 26$	Pairs	Exit order holds	$E[S]$ ordered
0.05	27	6	2	4	3.12	4.00	5.17	21	0	21
0.05	30	7	2	3	3.53	4.42	5.53	26	0	26
0.05	34	9	1	2	3.98	5.11	6.17	33	0	33
0.10	27	2	4	6	6.07	7.75	9.86	11	0	11
0.10	30	3	5	4	6.81	8.55	10.47	21	0	21
0.10	34	5	4	3	7.68	9.70	11.27	26	0	26

Notes: Twelve types, $c \in \{24, 25, 26\}$ by $\rho \in \{0.10, 0.12, 0.15, 0.20\}$. No support: the sponsor participates at a cap of zero. Abstain: no cap up to $\bar{V} = 25$ meets the outside option. $E[S]$ at $\bar{V}$: expected discounted disbursement at the common cap, averaged over the participating types with that operating cost; across discount rates within a cost group, it varies by less than 0.0003 in every design. Pairs: pairs of participating types with different operating costs. Exit order holds: pairs for which, on every one of the 2^{20} revenue histories, the higher-cost type does not exit before the lower-cost type's concession ends, the condition of Proposition 3. $E[S]$ ordered: pairs in which the higher-cost type's expected disbursement at $\bar{V}$ is at least that of the lower-cost type. Baseline: costless exit. Source: `c3_selection.py`.

Table D8. Selection at the awarded cap under the second-price award (costless exit)

			Share			Restricted comparison at the awarded cap				
Arrears	W	n	Contested	Cap zero	Restricted	Strictly lowest	Tied	Not lowest	Excess	Exit order holds
0.05	27	2	0.50	0.56	0.31	0.80	0.00	0.20	0.16	0.00
0.05	27	3	0.77	0.67	0.23	0.82	0.00	0.18	0.14	0.00
0.05	30	2	0.60	0.60	0.27	0.55	0.00	0.45	0.46	0.00
0.05	30	3	0.86	0.74	0.18	0.61	0.00	0.39	0.40	0.00
0.05	34	2	0.71	0.81	0.14	1.00	0.00	0.00	0.00	0.00
0.05	34	3	0.93	0.91	0.07	1.00	0.00	0.00	0.00	0.00
0.10	27	2	0.33	0.11	0.61	0.91	0.00	0.09	0.06	0.00
0.10	27	3	0.57	0.15	0.57	0.90	0.00	0.10	0.07	0.00
0.10	30	2	0.50	0.14	0.59	0.89	0.00	0.11	0.09	0.00
0.10	30	3	0.77	0.21	0.53	0.88	0.00	0.12	0.08	0.00
0.10	34	2	0.60	0.31	0.49	0.75	0.00	0.25	0.33	0.00
0.10	34	3	0.86	0.45	0.40	0.76	0.00	0.24	0.32	0.00

Notes: Contested: share of awards in which at least two sponsors qualify, so that the winner and the runner-up both accept the awarded cap. Cap zero: share of contested awards with an awarded cap of zero, in which no sponsor would receive support; these awards remain in Table D5. Restricted: share of contested awards with a positive awarded cap in which the sponsors that would accept that cap do not all have the same operating cost. The remaining columns refer to the restricted awards, with every sponsor that would accept the cap evaluated at that cap. Strictly lowest: the winner's expected disbursement is below that of every other such sponsor by more than 0.01. Tied: it lies within 0.01 of the lowest among the other sponsors. Not lowest: it exceeds the lowest among the other sponsors by more than 0.01. Excess: mean amount by which the winner's expected disbursement exceeds the lowest among all such sponsors. Exit order holds: share of restricted awards in which the condition of Proposition 3 holds on every revenue history for every pair of such sponsors with different operating costs. Source: `c3_selection.py`.

Table D9. Selection evaluated at the common cap $\bar{V}$ in contested awards (costless exit)

Arrears	W	n	Strictly lowest	Tied	Not lowest	Excess	Higher cost	Strictly lower V^*	Different costs	Exit order holds
0.05	27	2	0.42	0.34	0.23	0.29	0.23	0.06	0.66	0.00
0.05	27	3	0.36	0.37	0.28	0.34	0.28	0.05	0.75	0.00
0.05	30	2	0.33	0.36	0.31	0.38	0.31	0.12	0.64	0.00
0.05	30	3	0.24	0.38	0.37	0.45	0.37	0.12	0.75	0.00
0.05	34	2	0.40	0.34	0.26	0.37	0.26	0.00	0.66	0.00
0.05	34	3	0.28	0.37	0.35	0.51	0.35	0.00	0.80	0.00
0.10	27	2	0.56	0.39	0.06	0.09	0.06	0.06	0.61	0.00
0.10	27	3	0.50	0.44	0.06	0.11	0.06	0.06	0.67	0.00
0.10	30	2	0.56	0.34	0.09	0.17	0.09	0.06	0.66	0.00
0.10	30	3	0.49	0.39	0.12	0.21	0.12	0.07	0.75	0.00
0.10	34	2	0.44	0.36	0.20	0.43	0.20	0.12	0.64	0.00
0.10	34	3	0.34	0.41	0.24	0.52	0.24	0.13	0.75	0.00

Notes: All contested awards; every qualifying sponsor is evaluated at $\bar{V} = 25$. Strictly lowest, Tied, Not lowest and Excess are defined as in Table D8, with all qualifying sponsors as the comparison set. Different costs: share of contested awards in which the qualifying sponsors do not all have the same operating cost. Exit order holds: share of those awards in which the condition of Proposition 3 holds on every revenue history for every pair of qualifying sponsors with different operating costs. Higher cost: share of contested awards won by a sponsor whose operating cost exceeds the lowest among the qualifiers. Strictly lower V^*: share won by such a sponsor with a minimum cap strictly below that of every lower-cost qualifier; the remainder of Higher cost consists of awards decided by a tie at the lowest bid. Source: `c3_selection.py`.

Table D10. Single offer, sequential offers and auction: outcomes per procurement opportunity (costless exit)

Arrears entry	W	Procedure	Award	Exposure	$E[S]$	Surplus	Target	Exit	Expiry
0.05	27	Single offer	0.667	16.7	2.64	1.87	0.624	0.009	0.033
0.05	27	Sequential, $n = 2$	0.889	22.2	3.52	2.50	0.832	0.013	0.044
0.05	27	Auction, $n = 2$	0.889	13.1	2.26	2.19	0.803	0.024	0.062
0.05	27	Sequential, $n = 3$	0.963	24.1	3.82	2.71	0.901	0.014	0.048
0.05	27	Auction, $n = 3$	0.963	7.9	1.49	2.13	0.849	0.033	0.081
0.05	30	Single offer	0.750	18.8	3.20	3.22	0.687	0.011	0.052
0.05	30	Sequential, $n = 2$	0.938	23.4	4.00	4.03	0.858	0.014	0.066
0.05	30	Auction, $n = 2$	0.938	11.1	2.07	3.50	0.812	0.031	0.095
0.05	30	Sequential, $n = 3$	0.984	24.6	4.20	4.23	0.901	0.014	0.069
0.05	30	Auction, $n = 3$	0.984	5.1	1.02	3.36	0.827	0.040	0.117
0.05	34	Single offer	0.833	20.8	4.15	5.03	0.700	0.013	0.120
0.05	34	Sequential, $n = 2$	0.972	24.3	4.84	5.86	0.817	0.015	0.140
0.05	34	Auction, $n = 2$	0.972	8.4	1.84	4.79	0.754	0.052	0.166
0.05	34	Sequential, $n = 3$	0.995	24.9	4.95	6.00	0.836	0.016	0.144
0.05	34	Auction, $n = 3$	0.995	2.6	0.63	4.44	0.745	0.067	0.184
0.10	27	Single offer	0.500	12.5	3.63	1.30	0.438	0.023	0.040
0.10	27	Sequential, $n = 2$	0.750	18.8	5.44	1.94	0.657	0.034	0.059
0.10	27	Auction, $n = 2$	0.750	14.6	4.49	1.74	0.643	0.041	0.066
0.10	27	Sequential, $n = 3$	0.875	21.9	6.35	2.27	0.766	0.040	0.069
0.10	27	Auction, $n = 3$	0.875	13.2	4.28	1.85	0.737	0.055	0.084
0.10	30	Single offer	0.667	16.7	5.58	2.33	0.531	0.037	0.098
0.10	30	Sequential, $n = 2$	0.889	22.2	7.44	3.11	0.708	0.050	0.131
0.10	30	Auction, $n = 2$	0.889	15.6	5.58	2.86	0.687	0.070	0.132
0.10	30	Sequential, $n = 3$	0.963	24.1	8.07	3.37	0.767	0.054	0.142
0.10	30	Auction, $n = 3$	0.963	11.5	4.37	2.86	0.724	0.090	0.149
0.10	34	Single offer	0.750	18.8	6.86	3.74	0.545	0.042	0.163
0.10	34	Sequential, $n = 2$	0.938	23.4	8.58	4.67	0.681	0.052	0.204
0.10	34	Auction, $n = 2$	0.938	13.3	5.51	4.01	0.636	0.099	0.202
0.10	34	Sequential, $n = 3$	0.984	24.6	9.01	4.90	0.715	0.055	0.214
0.10	34	Auction, $n = 3$	0.984	7.9	3.66	3.76	0.637	0.127	0.220

Notes: Per procurement opportunity: every measure counts zero when there is no award. Single offer: $\bar{V} = 25$ offered to one sponsor drawn from the twelve types. Sequential: $\bar{V}$ offered to up to n independently drawn sponsors in turn, with the award to the first that accepts; its award probability equals the auction's, and its outcomes conditional on award equal those of the single offer. Auction: n sponsors under the second-price award rule of Section 5.4 with the same $\bar{V}$. Exposure: expected awarded cap. Surplus: expected sponsor surplus $U - \omega$ of the recipient at the awarded cap, with $\omega = 0$, in present value at the recipient's own discount rate. Target, Exit and Expiry: probabilities that the opportunity ends in a concession that reaches W, ends by the sponsor's exit, or reaches $\bar{T}$ before W. Costless exit. Source: `c4_posted_offer.py`, `c4_sequential.py`.

Table D11. Posted offers and auction: outcomes conditional on award (costless exit)

Arrears entry	W	Offer	Award	One qualifier	Exposure	Expenditure		Surplus	Operating outcomes		
					Cap	$E[S]$	$E[S]$ at own V^*		Target	Exit	Expiry
0.05	27	Single offer	0.667	1.00	25.0	3.96	0.72	2.81	0.94	0.01	0.05
0.05	27	Auction, $n = 2$	0.889	0.50	14.7	2.54	0.45	2.46	0.90	0.03	0.07
0.05	27	Auction, $n = 3$	0.963	0.23	8.3	1.55	0.27	2.21	0.88	0.03	0.08
0.05	30	Single offer	0.750	1.00	25.0	4.27	0.40	4.30	0.92	0.01	0.07
0.05	30	Auction, $n = 2$	0.938	0.40	11.9	2.21	0.20	3.73	0.87	0.03	0.10
0.05	30	Auction, $n = 3$	0.984	0.14	5.2	1.04	0.09	3.41	0.84	0.04	0.12
0.05	34	Single offer	0.833	1.00	25.0	4.98	0.44	6.03	0.84	0.02	0.14
0.05	34	Auction, $n = 2$	0.972	0.29	8.6	1.89	0.16	4.93	0.78	0.05	0.17
0.05	34	Auction, $n = 3$	0.995	0.07	2.6	0.63	0.05	4.46	0.75	0.07	0.18
0.10	27	Single offer	0.500	1.00	25.0	7.26	2.70	2.59	0.88	0.05	0.08
0.10	27	Auction, $n = 2$	0.750	0.67	19.5	5.98	2.21	2.32	0.86	0.06	0.09
0.10	27	Auction, $n = 3$	0.875	0.43	15.1	4.89	1.78	2.12	0.84	0.06	0.10
0.10	30	Single offer	0.667	1.00	25.0	8.38	3.10	3.50	0.80	0.06	0.15
0.10	30	Auction, $n = 2$	0.889	0.50	17.5	6.28	2.16	3.22	0.77	0.08	0.15
0.10	30	Auction, $n = 3$	0.963	0.23	11.9	4.54	1.44	2.97	0.75	0.09	0.15
0.10	34	Single offer	0.750	1.00	25.0	9.15	2.54	4.98	0.73	0.06	0.22
0.10	34	Auction, $n = 2$	0.938	0.40	14.2	5.88	1.61	4.27	0.68	0.11	0.22
0.10	34	Auction, $n = 3$	0.984	0.14	8.1	3.72	0.96	3.82	0.65	0.13	0.22

Notes: Single offer: one sponsor, drawn from the twelve types, is offered $\bar{V} = 25$ and accepts if its minimum acceptable cap does not exceed $\bar{V}$. Auction: n sponsors under the second-price award rule of Section 5.4 with the same $\bar{V}$. Award: probability of an award per procurement opportunity. All other columns are conditional on award; sequential offers to n sponsors have the outcomes of the posted offer conditional on award. Cap: expected awarded cap (committed exposure). One qualifier: probability that a single sponsor qualifies and receives $\bar{V}$. $E[S]$: expected discounted disbursement under the awarded cap; at own V^*: under the recipient's own minimum cap, for reference. Surplus: sponsor surplus $U - \omega$ of the recipient at the awarded cap, with $\omega = 0$, in present value at the recipient's own discount rate. Target, Exit and Expiry: probabilities that the concession ends by reaching W, by the sponsor's exit and at $\bar{T}$ before reaching W. Exact expectations over all 12^n profiles; costless exit. Source: `c4_posted_offer.py`.

Table D12. Auction minus single offer, per opportunity: access and rules terms (costless exit)

Arrears entry	W	n	$E[S]$ Total	$E[S]$ Access	$E[S]$ Rules	Target Total	Target Access	Target Rules	Exit Total	Exit Access	Exit Rules
0.05	27	2	−0.38	0.88	−1.26	0.179	0.208	−0.029	0.015	0.003	0.012
0.05	27	3	−1.15	1.17	−2.32	0.225	0.277	−0.052	0.024	0.004	0.019
0.05	30	2	−1.14	0.80	−1.94	0.125	0.172	−0.047	0.020	0.003	0.018
0.05	30	3	−2.18	1.00	−3.18	0.140	0.215	−0.074	0.030	0.003	0.026
0.05	34	2	−2.31	0.69	−3.00	0.054	0.117	−0.063	0.039	0.002	0.037
0.05	34	3	−3.52	0.81	−4.33	0.045	0.136	−0.091	0.053	0.003	0.051
0.10	27	2	0.86	1.81	−0.96	0.205	0.219	−0.014	0.019	0.011	0.007
0.10	27	3	0.65	2.72	−2.07	0.299	0.328	−0.029	0.032	0.017	0.015
0.10	30	2	0.00	1.86	−1.87	0.156	0.177	−0.021	0.033	0.012	0.020
0.10	30	3	−1.21	2.48	−3.70	0.193	0.236	−0.043	0.053	0.017	0.036
0.10	34	2	−1.35	1.72	−3.07	0.091	0.136	−0.045	0.057	0.010	0.047
0.10	34	3	−3.20	2.14	−5.35	0.092	0.170	−0.078	0.086	0.013	0.073

Notes: Per procurement opportunity. Total: auction with n bidders minus the single offer. Access: sequential offers to n sponsors minus the single offer. Rules: auction minus sequential offers, which have the same award probability. The two terms sum to the total by construction. Costless exit. Source: `c4_sequential.py`.

Table D13. Posted offers minus auction, conditional on award: selection and cap terms (costless exit)

Arrears entry	W	n	$E[S]$ Difference	$E[S]$ Selection	$E[S]$ Cap	Target Difference	Target Selection	Target Cap	Exit Difference	Exit Selection	Exit Cap
0.05	27	2	1.42	0.07	1.35	0.033	−0.004	0.037	−0.013	0.001	−0.014
0.05	27	3	2.41	0.12	2.29	0.054	−0.008	0.062	−0.020	0.001	−0.022
0.05	30	2	2.07	0.02	2.05	0.050	−0.005	0.054	−0.019	0.000	−0.019
0.05	30	3	3.23	0.04	3.19	0.075	−0.008	0.084	−0.027	0.000	−0.027
0.05	34	2	3.09	0.09	3.00	0.064	−0.016	0.080	−0.038	0.001	−0.038
0.05	34	3	4.35	0.12	4.23	0.092	−0.021	0.113	−0.051	0.001	−0.052
0.10	27	2	1.28	0.21	1.07	0.018	−0.016	0.034	−0.010	0.004	−0.014
0.10	27	3	2.37	0.38	1.98	0.034	−0.029	0.063	−0.017	0.008	−0.025
0.10	30	2	2.10	0.30	1.80	0.024	−0.033	0.056	−0.023	0.006	−0.029
0.10	30	3	3.84	0.53	3.30	0.045	−0.058	0.103	−0.038	0.011	−0.048
0.10	34	2	3.27	0.21	3.07	0.048	−0.036	0.084	−0.050	0.004	−0.054
0.10	34	3	5.43	0.36	5.07	0.080	−0.063	0.142	−0.074	0.006	−0.080

Notes: Conditional on award, where single and sequential offers have the same outcomes. Difference: posted offer minus auction with n bidders. Selection: the recipient of the posted offer minus the auction winner, both evaluated at $\bar{V}$. Cap: the auction winner at $\bar{V}$ minus the same winner at the awarded cap. The two terms sum to the difference by construction, and the split assigns no causal weight. Costless exit. Source: `c4_posted_offer.py`.

Table D14. Single offer, sequential offers and auction: outcomes per procurement opportunity (termination penalty)

Arrears entry	W	Procedure	Award	Exposure	$E[S]$	Surplus	Target	Exit	Expiry
0.05	27	Single offer	0.667	16.7	2.64	1.87	0.625	0.003	0.038
0.05	27	Sequential, $n = 2$	0.889	22.2	3.52	2.50	0.834	0.004	0.051
0.05	27	Auction, $n = 2$	0.889	13.1	2.27	2.18	0.808	0.007	0.073
0.05	27	Sequential, $n = 3$	0.963	24.1	3.82	2.70	0.903	0.004	0.055
0.05	27	Auction, $n = 3$	0.963	8.0	1.50	2.12	0.857	0.009	0.096
0.05	30	Single offer	0.750	18.8	3.20	3.22	0.688	0.003	0.059
0.05	30	Sequential, $n = 2$	0.938	23.4	4.00	4.02	0.860	0.003	0.074
0.05	30	Auction, $n = 2$	0.938	11.2	2.07	3.49	0.817	0.008	0.112
0.05	30	Sequential, $n = 3$	0.984	24.6	4.20	4.23	0.903	0.003	0.078
0.05	30	Auction, $n = 3$	0.984	5.1	1.03	3.36	0.834	0.010	0.140
0.05	34	Single offer	0.833	20.8	4.15	5.02	0.701	0.004	0.129
0.05	34	Sequential, $n = 2$	0.972	24.3	4.84	5.86	0.817	0.005	0.150
0.05	34	Auction, $n = 2$	0.972	8.4	1.84	4.78	0.762	0.018	0.193
0.05	34	Sequential, $n = 3$	0.995	24.9	4.96	6.00	0.837	0.005	0.154
0.05	34	Auction, $n = 3$	0.995	2.6	0.63	4.43	0.755	0.022	0.218
0.10	27	Single offer	0.500	12.5	3.63	1.29	0.441	0.006	0.052
0.10	27	Sequential, $n = 2$	0.750	18.8	5.45	1.94	0.662	0.010	0.079
0.10	27	Auction, $n = 2$	0.750	14.7	4.50	1.74	0.652	0.008	0.090
0.10	27	Sequential, $n = 3$	0.875	21.9	6.36	2.26	0.772	0.011	0.092
0.10	27	Auction, $n = 3$	0.875	13.3	4.30	1.85	0.751	0.007	0.117
0.10	30	Single offer	0.667	16.7	5.59	2.33	0.534	0.011	0.121
0.10	30	Sequential, $n = 2$	0.889	22.2	7.45	3.10	0.712	0.015	0.162
0.10	30	Auction, $n = 2$	0.889	15.7	5.60	2.86	0.699	0.015	0.175
0.10	30	Sequential, $n = 3$	0.963	24.1	8.07	3.36	0.771	0.017	0.175
0.10	30	Auction, $n = 3$	0.963	11.6	4.40	2.86	0.743	0.013	0.207
0.10	34	Single offer	0.750	18.8	6.86	3.73	0.547	0.012	0.191
0.10	34	Sequential, $n = 2$	0.938	23.4	8.58	4.66	0.683	0.015	0.239
0.10	34	Auction, $n = 2$	0.938	13.4	5.54	4.00	0.649	0.021	0.267
0.10	34	Sequential, $n = 3$	0.984	24.6	9.01	4.89	0.718	0.015	0.251
0.10	34	Auction, $n = 3$	0.984	8.0	3.70	3.76	0.657	0.020	0.307

Notes: Per procurement opportunity: every measure counts zero when there is no award. Single offer: $\bar{V} = 25$ offered to one sponsor drawn from the twelve types. Sequential: $\bar{V}$ offered to up to n independently drawn sponsors in turn, with the award to the first that accepts; its award probability equals the auction's, and its outcomes conditional on award equal those of the single offer. Auction: n sponsors under the second-price award rule of Section 5.4 with the same $\bar{V}$. Exposure: expected awarded cap. Surplus: expected sponsor surplus $U - \omega$ of the recipient at the awarded cap, with $\omega = 0$, in present value at the recipient's own discount rate. Target, Exit and Expiry: probabilities that the opportunity ends in a concession that reaches W, ends by the sponsor's exit, or reaches $\bar{T}$ before W. Termination penalty. Bids are minimum acceptable caps. The sensitivity exercise reports allocations under threshold bids, without establishing an equilibrium. Source: `c4_posted_offer.py`, `c4_sequential.py`.

Table D15. Posted offers and auction: outcomes conditional on award (termination penalty)

Arrears entry	W	Offer	Award	One qualifier	Exposure	Expenditure		Surplus	Operating outcomes		
					Cap	$E[S]$	$E[S]$ at own V^*		Target	Exit	Expiry
0.05	27	Single offer	0.667	1.00	25.0	3.96	0.74	2.81	0.94	0.00	0.06
0.05	27	Auction, $n = 2$	0.889	0.50	14.7	2.55	0.47	2.46	0.91	0.01	0.08
0.05	27	Auction, $n = 3$	0.963	0.23	8.3	1.56	0.27	2.21	0.89	0.01	0.10
0.05	30	Single offer	0.750	1.00	25.0	4.27	0.41	4.29	0.92	0.00	0.08
0.05	30	Auction, $n = 2$	0.938	0.40	11.9	2.21	0.20	3.73	0.87	0.01	0.12
0.05	30	Auction, $n = 3$	0.984	0.14	5.2	1.05	0.09	3.41	0.85	0.01	0.14
0.05	34	Single offer	0.833	1.00	25.0	4.98	0.45	6.03	0.84	0.00	0.15
0.05	34	Auction, $n = 2$	0.972	0.29	8.6	1.89	0.16	4.92	0.78	0.02	0.20
0.05	34	Auction, $n = 3$	0.995	0.07	2.7	0.63	0.05	4.45	0.76	0.02	0.22
0.10	27	Single offer	0.500	1.00	25.0	7.26	2.75	2.58	0.88	0.01	0.10
0.10	27	Auction, $n = 2$	0.750	0.67	19.6	6.00	2.25	2.32	0.87	0.01	0.12
0.10	27	Auction, $n = 3$	0.875	0.43	15.2	4.92	1.81	2.11	0.86	0.01	0.13
0.10	30	Single offer	0.667	1.00	25.0	8.38	3.15	3.49	0.80	0.02	0.18
0.10	30	Auction, $n = 2$	0.889	0.50	17.6	6.30	2.19	3.22	0.79	0.02	0.20
0.10	30	Auction, $n = 3$	0.963	0.23	12.0	4.57	1.47	2.97	0.77	0.01	0.21
0.10	34	Single offer	0.750	1.00	25.0	9.15	2.59	4.97	0.73	0.02	0.26
0.10	34	Auction, $n = 2$	0.938	0.40	14.3	5.91	1.64	4.27	0.69	0.02	0.29
0.10	34	Auction, $n = 3$	0.984	0.14	8.2	3.75	0.99	3.82	0.67	0.02	0.31

Notes: Single offer: one sponsor, drawn from the twelve types, is offered $\bar{V} = 25$ and accepts if its minimum acceptable cap does not exceed $\bar{V}$. Auction: n sponsors under the second-price award rule of Section 5.4 with the same $\bar{V}$. Award: probability of an award per procurement opportunity. All other columns are conditional on award; sequential offers to n sponsors have the outcomes of the posted offer conditional on award. Cap: expected awarded cap (committed exposure). One qualifier: probability that a single sponsor qualifies and receives $\bar{V}$. $E[S]$: expected discounted disbursement under the awarded cap; at own V^*: under the recipient's own minimum cap, for reference. Surplus: sponsor surplus $U - \omega$ of the recipient at the awarded cap, with $\omega = 0$, in present value at the recipient's own discount rate. Target, Exit and Expiry: probabilities that the concession ends by reaching W, by the sponsor's exit and at $\bar{T}$ before reaching W. Exact expectations over all 12^n profiles; termination penalty. Bids as in Table D14: allocations under threshold bids, without an established equilibrium. Source: `c4_posted_offer.py`.

Table D16. Auction minus single offer, per opportunity: access and rules terms (termination penalty)

Arrears entry	W	n	$E[S]$ Total	$E[S]$ Access	$E[S]$ Rules	Target Total	Target Access	Target Rules	Exit Total	Exit Access	Exit Rules
0.05	27	2	−0.37	0.88	−1.25	0.183	0.208	−0.026	0.004	0.001	0.003
0.05	27	3	−1.14	1.17	−2.31	0.232	0.278	−0.046	0.007	0.001	0.005
0.05	30	2	−1.13	0.80	−1.93	0.129	0.172	−0.043	0.005	0.001	0.005
0.05	30	3	−2.17	1.00	−3.17	0.145	0.215	−0.070	0.008	0.001	0.007
0.05	34	2	−2.31	0.69	−3.00	0.061	0.117	−0.056	0.014	0.001	0.013
0.05	34	3	−3.52	0.81	−4.32	0.055	0.136	−0.081	0.018	0.001	0.017
0.10	27	2	0.87	1.82	−0.95	0.211	0.221	−0.010	0.001	0.003	−0.002
0.10	27	3	0.67	2.72	−2.05	0.310	0.331	−0.021	0.001	0.005	−0.004
0.10	30	2	0.02	1.86	−1.84	0.165	0.178	−0.013	0.003	0.004	−0.001
0.10	30	3	−1.18	2.48	−3.66	0.209	0.237	−0.028	0.002	0.005	−0.003
0.10	34	2	−1.32	1.72	−3.04	0.102	0.137	−0.034	0.009	0.003	0.006
0.10	34	3	−3.17	2.15	−5.31	0.110	0.171	−0.061	0.008	0.004	0.005

Notes: Per procurement opportunity. Total: auction with n bidders minus the single offer. Access: sequential offers to n sponsors minus the single offer. Rules: auction minus sequential offers, which have the same award probability. The two terms sum to the total by construction. Termination penalty. Bids as in Table D14: allocations under threshold bids, without an established equilibrium. Source: `c4_sequential.py`.

Table D17. Posted offers minus auction, conditional on award: selection and cap terms (termination penalty)

Arrears entry	W	n	$E[S]$ Difference	$E[S]$ Selection	$E[S]$ Cap	Target Difference	Target Selection	Target Cap	Exit Difference	Exit Selection	Exit Cap
0.05	27	2	1.41	0.07	1.34	0.029	−0.004	0.033	−0.004	0.000	−0.004
0.05	27	3	2.40	0.12	2.28	0.048	−0.007	0.056	−0.006	0.001	−0.006
0.05	30	2	2.06	0.02	2.04	0.046	−0.004	0.050	−0.005	0.000	−0.006
0.05	30	3	3.22	0.04	3.18	0.071	−0.008	0.079	−0.007	0.001	−0.008
0.05	34	2	3.08	0.09	3.00	0.057	−0.016	0.073	−0.013	0.001	−0.014
0.05	34	3	4.34	0.12	4.23	0.082	−0.021	0.103	−0.017	0.001	−0.018
0.10	27	2	1.26	0.21	1.05	0.013	−0.015	0.029	0.002	0.003	−0.001
0.10	27	3	2.35	0.39	1.96	0.024	−0.028	0.053	0.005	0.006	−0.001
0.10	30	2	2.08	0.30	1.78	0.014	−0.032	0.047	0.001	0.005	−0.004
0.10	30	3	3.81	0.53	3.27	0.029	−0.057	0.086	0.004	0.009	−0.006
0.10	34	2	3.24	0.21	3.04	0.037	−0.036	0.073	−0.007	0.005	−0.012
0.10	34	3	5.40	0.36	5.04	0.062	−0.063	0.125	−0.005	0.008	−0.013

Notes: Conditional on award, where single and sequential offers have the same outcomes. Difference: posted offer minus auction with n bidders. Selection: the recipient of the posted offer minus the auction winner, both evaluated at $\bar{V}$. Cap: the auction winner at $\bar{V}$ minus the same winner at the awarded cap. The two terms sum to the difference by construction, and the split assigns no causal weight. Termination penalty. Bids as in Table D14: allocations under threshold bids, without an established equilibrium. Source: `c4_posted_offer.py`.

Table D18. Number of types with outcomes monotone across awardable caps, by design

				$E[S]$	Target		Exit		Expiry
Exit rule	Arrears entry	W	Types	Nondecreasing	Nondecreasing	Largest fall	Nonincreasing	Largest rise	Nonincreasing
costless	0.05	27	8	8	8	0.000	6	0.012	6
costless	0.05	30	9	9	4	0.004	2	0.026	7
costless	0.05	34	10	10	10	0.000	10	0.000	7
costless	0.10	27	6	6	4	0.000	2	0.012	5
costless	0.10	30	8	8	5	0.028	4	0.053	3
costless	0.10	34	9	9	6	0.008	6	0.014	4
penalty	0.05	27	8	8	8	0.000	2	0.001	6
penalty	0.05	30	9	9	9	0.000	3	0.001	7
penalty	0.05	34	10	10	10	0.000	3	0.011	7
penalty	0.10	27	6	6	6	0.000	1	0.002	5
penalty	0.10	30	8	8	8	0.000	3	0.007	6
penalty	0.10	34	9	9	9	0.000	2	0.016	7

Notes: For each participating type, each outcome is evaluated at every cap that the auction can award in the design and that the type would accept, from its own minimum cap to $\bar{V}$. Types: participating types. The count columns give the number of types for which the outcome is monotone across those caps in the stated direction (tolerance 10^{-12}). Largest fall and Largest rise: the largest decrease in the target probability, and the largest increase in the exit probability, between adjacent awardable caps for any type. Proposition 1(a) establishes only that the payoff U is weakly increasing in the cap; U is nondecreasing for every type. Source: `c4_posted_offer.py`.

E. Data Construction for the Concessionality Exhibit

The unit of observation in Table 1 is a PSW transaction approved under IDA18 through IDA20. The sample frame is the list of PSW projects published on the IDA website, retrieved in October 2026 from the public data source that populates the list.

The list shows 204 Board-approved transactions. For each transaction, it shows the Board date, country, region, IFC industry group, sector, instrument, split of the allocation across the four facilities, fragility designation, replenishment cycle and links to the corresponding IFC and MIGA disclosure pages. The list includes no definition of a transaction. Its coverage differs from that of the results framework included in the IDA20 Mid-Term Review, which states that 238 transactions were approved through June 2023, including 166 IFC projects, 17 IFC programs and 55 executed MIGA guarantee contracts (International Development Association, 2023, p. 39). A list that IFC compiled from the same website as of April 30, 2024, excludes cancellations, shows platforms without their subprojects, shows notional amounts that are tagged not to be aggregated and notes a higher total volume approved for all PSW projects (International Finance Corporation, 2024, p. 14). The paper does not attempt to reconcile the list's transaction count with that of the Mid-Term Review. The exhibit only includes transactions from the published list.

The concessionality estimate is extracted from the disclosure document uploaded prior to Board approval. The boilerplate language used by IFC in its disclosures gives the level of concessionality as a percentage of total project cost, based on the difference between some reference price (market price if available; otherwise, a price calculated using IFC's pricing model or a negotiated price) and the concessional price that is actually charged. MIGA disclosures that provide an estimate report how much subsidy is being provided, as a percentage of total project costs over the guarantee period (usually provided as an upper bound). Links to IFC disclosures are listed for 152 transactions and links to MIGA disclosures are listed for 34 transactions. Pages for 151 IFC transactions and all MIGA transactions were successfully retrieved. Because three transactions list both links, a total of 182 unique transactions have at least one retrieved disclosure page. Concessionality estimates are listed for 119 transactions, 106 from IFC and 13 from MIGA. Of these 119 estimates, 105 report the estimate as a percentage of total project cost (92 IFC, 13 MIGA). Every extracted estimate was validated against the sentence in which it was contained. The sentence is stored with the observation.

Four attributes of the disclosures reduce their usefulness. The denominator lacks standardization: the 14 estimates not expressed against total project cost refer to facility size, fund size, targeted issuance amounts, component costs, the investment cost in a single country of a multi-country transaction, or expected incremental trade activity, while two state no denominator. Total project cost itself is inconsistently defined, as several of the Local Currency Facility disclosures report a total project cost equal to the amount of the investment, mechanically inflating the ratio; the two largest estimates in the sample, 60 and 51 percent, fall into this category. Eleven of the thirteen MIGA estimates are upper bounds. All MIGA estimates are cumulative over guarantee periods ranging from six to twenty years. The exhibit therefore excludes the MIGA estimates. Several disclosures contain bracketed values that appear to be unupdated template fields. One disclosure provides a range. The paper records this observation by its lower bound, upper bound, and midpoint.

The exhibit uses the 92 IFC transactions that state the estimate against total project cost; the ratio of the 90th to the 10th percentile of these estimates is 9.1. Disclosure rates differ by fragility. Among the 182 transactions with a retrieved disclosure, 58 percent of transactions in fragile and

conflict-affected situations state an estimate, against 70 percent of the others. The corresponding rates in the full registry are 56 and 60 percent. The exhibit therefore describes only the transactions that disclose an estimate.